No-z model: results and perspectives for accretion discs

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Abstract

Accretion discs surround different compact astrophysical objects such as black holes, neutron stars and white dwarfs. Also they are situated in systems of variable stars and near the galaxy center. Magnetic fields play an important role in evolution and hydrodynamics of the accretion discs: for example, they can describe such processes as the transition of the angular momentum. There are different approaches to explain the magnetic fields, but most interesting of them are connected with dynamo generation. As for disc, it is quite useful to take no-z approximation which has been developed for galactic discs to solve the dynamo equations. It takes into account that the disc is quite thin, and we can solve the equations only for two plane components of the field. Here we describe the time dependence of the magnetic field for different distances from the center of the disc. Also we compare the results with another approaches which take into account more complicated field structure.

Keywords: *accretion discs, dynamo, magnetic field*

1. Introduction

The accretion discs are situated near different compact objects in space. First of all, we should mention black holes, white dwarfs and neutron stars. Usually such discs are quite thin and rotate with keplerian law: their angular velocity is proportional to $r^{-3/2}$, where r is the distance from the center. Even pioneer works connected with accretion discs supposed that they have large-scale magnetic field Shakura & Sunyaev (1973). They can describe a lot of hydrodynamical processes. For example, they can explain transtion of the angular momentum. Also they are closely connected with generation of jets Balbus & Hawley (1991) and hydrodynamical winds Pudritz & Norman (1986). Another important examples of the accretion discs are associated with central parts of the galaxies: now there are interesting results about the Faraday rotation measurements in the central part of the M87 galaxy Kravchenko et al. (2020). It can describe the magnetic field in the accretion disc surrounding the central black hole in this object. The next important example is connected with the accretion discs which are associated with eruptive stars Andreasyan et al. (2021).

There are different approaches which try to describe the mechanism of generating these magnetic fields. First of all, some authors try to explain the field evolution using the transport of accreting matter Lubow et al. (1994). Unfortunately, it cannot describe the transition of the regular field structures because of the dissipative effects and mixing of the medium. Another important works describe the field generation using the interaction between the disc and the central object. However, Brandenburg with co-authors showed that the magnetic field in the accretion discs are connected with the dynamo action Brandenburg & Donner (1994).

The dynamo mechanism is connected with two basic effects. First of all, it is based on differential rotation which shows changing angular velocity of the object. Then, the dynamo mechanism contains alpha-effect which describes the helicity of the turbulent motions: the vorticity has non-zero average projection on the velocity vector. Both of these effects produce the magnetic field, and they compete with turbulent diffusion which tries to destroy the magnetic field. So the magnetic field generation is a threshold effect, and it can grow only for some special cases, else it decays Sokoloff (2015).

The magnetic field generation in dynamo theory is described by the Steenbeck – Krause – Rädler effect. It is obtained by averaging the magnetohydrodynamics equations on distances associated with turbulent

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lengthscales. They are quite difficult to be solved, and there are different models which take into account the geometrical shape of the astrophysical object. For example, for the Sun we can take the Parker dynamo model Parker (1955). As for accretion discs, we can take the no-z model which has been developed for galactic discs Moss (1995). It takes into account that the disc is quite thin. So, the plane components of the magnetic field are much larger than the vertical one. It leads us to neglecting the z-part of the magnetic field and reduce its z-derivative by the solenoidality condition. As for the z-dependence of the magnetic field components a cosine law is taken, so second z-derivatives of the main magnetic fields can be changed by the algebraic expressions. This approach is widely used for modeling magnetic fields of galaxies and give an opportunity to find fields in different cases Andreasyan et al. (2020). The accretion discs also are quite thin and we can take there no-z approximation, too. This approach was firstly used by Moss et al. (2016). It gave quite important results, but it was a problem connected with the maximum value of the field, which was larger than the equipartition value. (So, the electromagnetic force is too much and can destroy the structure of motions.) This problem was solved by Boneva et al Boneva et al. (2021), and it was shown that the no-z approximation is quite useful for accretion discs.

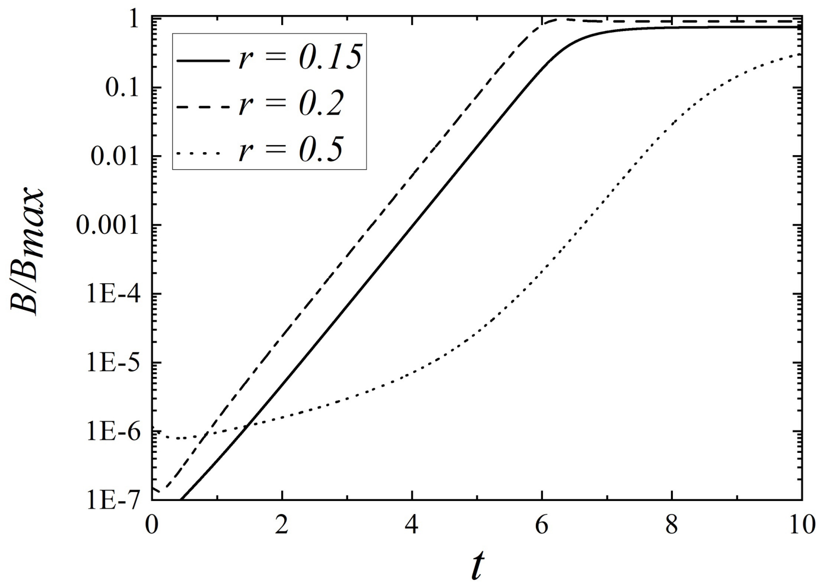

Figure 1. Magnetic field for different r obtained by no-z model.

It is also necessary to take into accont that for some cases the thickness of the disc can be quite comparable with its radius. So for this case it is quite difficult to apply the no-z approximation: we should use more difficult models for the field structure. For example, we can take the rz-model which uses the magnetic field as a combination of the azimuthal component and the rotor of azimuthal part of the vector potential of the magnetic field. This model was firstly developed as a torus dynamo model, which can be taken for outer rings of galaxies, where the width is comparable with thickness Mikhalov (2018). After that it was taken for main part of the galaxy Mikhalov (2021). Also we can take this model for the magnetic field of the accretion discs which can be very useful for the objects which have thickness comparable with the radius of the object.

In these paper we describe the magnetic field evolution using these two different models and compare their results.

2. Basic equations

The magnetic field evolution described by the dynamo theory is connected with the Steenbeck – Krause – Rädler equation Krause & Rädler (1980):

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\alpha \vec{B}\right) + \nabla \times \left(\vec{V} \times \vec{B}\right) + \eta \nabla^2 \vec{B};$$

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where $\vec{B}$ is the magnetic field induction, η is the turbulent diffusivity coefficient, $\vec{V}$ is the large-scale velocity and α characterizes the alpha-effect.

For large-scale velocity we can assume that $\vec{V} = r\Omega\vec{e}_\varphi$. As for the angular velocity we shall take the keplerian rotation law Shakura & Sunyaev (1973):

$$\Omega = \frac{\sqrt{GM}}{r^{3/2}};$$

where M is the mass of the central object.

The alpha-effect characterizes the helicity of the turbulent motions Krause & Rädler (1980):

$$\alpha = -\frac{\tau}{3} < \vec{v} \cdot \nabla \times \vec{v} >$$

where $\vec{v}$ is the small-scale velocity and τ is the timescale of the turbulence. As for disc objects it can be shown that:

$$\alpha \approx \frac{\Omega l^2}{h^2} \sin\left(\frac{\pi z}{2h}\right);$$

where l is the lengthscale of the turbulence.

The magnetic field cannot grow infinitely, so it is necessary to take the nonlinear saturation of the alpha-effect Boneva et al. (2021):

$$\alpha \sim 1 - \frac{B^2}{B^2_{max}(r)},$$

where $B_{max}(r)$ is the so-called equipartition value which shows the case when the kinetic energy of the turbulent motion is the same as the energy of the magnetic field.

This equation is quite difficult to be solved in 3D case. It needs quite large computational resources, and it is also difficult to make any analytical approximations. So, it is necessary to take the 2D-models which give us an opportunity to simplify the problem.

3. No-z model

One of the most important models which has been developed for thin discs is the no-z approximation Moss (1995). It takes the following approximation for the magnetic field:

$$B_{r,\varphi}(r,z,t) = B_{r,\varphi}(r,0,t)\cos\left(\frac{\pi z}{2h}\right).$$

So the magnetic field evolution is described by the equations Moss (1995):

$$\frac{\partial B_r}{\partial t} = -R_\alpha B_\varphi \left(1 - \frac{B^2}{B^2_{max}(r)}\right) - \frac{\pi^2 B_r}{4} + \lambda^2 \left(\frac{\partial^2 B_r}{\partial r^2} + \frac{1}{r}\frac{\partial B_r}{\partial r} - \frac{B_r}{r^2}\right);$$

$$\frac{\partial B_\varphi}{\partial t} = -\frac{3}{2r^{3/2}} R_\omega B_r - \frac{\pi^2 B_\varphi}{4} + \lambda^2 \left(\frac{\partial^2 B_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial B_\varphi}{\partial r} - \frac{B_\varphi}{r^2}\right).$$

Here time is measured in units of $\frac{h^2}{\eta}$ and distances are measured in units of radius of the disc. R_α characterizes the alpha-effect, R_ω shows the differential rotation and λ is connected with dissipation in the disc plane.

The results of the magnetic field modelling is shown on Figure 1. We describe the field evolution for different distances from the center of the discs.

4. Model with z-dependence

If we take into account more complicated z-structure of the field, the magnetic field can be described as:

$$\vec{B} = B\vec{e}_\varphi + \nabla \times (A\vec{e}_\varphi)$$

The field equations will be Mikhalov (2021):

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$$\frac{\partial A}{\partial t} = -R_\alpha z \frac{\partial B}{\partial z}\left(1 - \frac{B^2}{B^2_{max}(r)}\right) + \lambda^2 \left(\frac{\partial^2 A}{\partial z^2} + \frac{\partial^2 A}{\partial r^2} + \frac{1}{r}\frac{\partial A}{\partial r} - \frac{A}{r^2}\right);$$

$$\frac{\partial B}{\partial t} = -\frac{3}{2r^{3/2}} R_\omega \frac{\partial A}{\partial z} + \lambda^2 \left(\frac{\partial^2 B}{\partial z^2} + \frac{\partial^2 B}{\partial r^2} + \frac{1}{r}\frac{\partial B}{\partial r} - \frac{B}{r^2}\right).$$

The results for this case are shown on Figure 2.

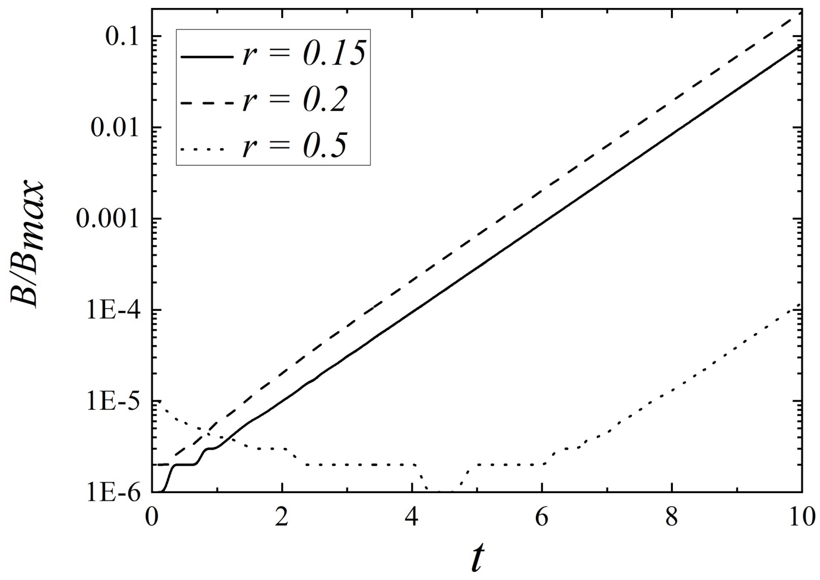

Figure 2. Magnetic field for different r obtained by rz-model.

5. Conclusions

We have studied the magnetic field evolution using the no-z model and the rz-approximation. The typical rate of the magnetic field growth differs, but basic features are the same. Also we have shown that the magnetic field can be generated by the dynamo model.

6. References

References

Andreasyan R. R., Mikhailov E. A., Andreasyan H. R., 2020, Asronomy Reports, 64, 189

Andreasyan H. R., Magakian T. Y., Movsessian T. A., Moiseev A. V., 2021, Astrophysics, 64, 187

Balbus S. A., Hawley J. F., 1991, Astrophysical Journal, 376, 214

Boneva D., Mikhalov E., Pashentseva M., Sokoloff D., 2021, Astronomy and Astrophysics, 652, A38

Brandenburg A., Donner K. J., 1994, MNRAS, 288, L29

Krause F., Rädler K. H., 1980, Mean-field magnetohydrodynamics and dynamo theory. Pergamon, Oxford

Kravchenko E., Giroletti M. F., Hada K., et al. 2020, Astronomy and Astrophysics, 637, L6

Lubow S. H., Papaloizou J. C. B., Pringle J. E., 1994, MNRAS, 267, 235

Mikhalov E. A., 2018, Astrophysics, 61, 147

Mikhalov E. A., 2021, Research in Astronomy and Astrophysics, 21, 056

Moss D., 1995, MNRAS, 191, 275

Moss D., Sokoloff D., Suleimanov V., 2016, Astronomy and Astrophysics, 588, A18

doi: doi.org/10.52526/25792776-2021.68.2-490

Parker E. N., 1955, Astrophysical Journal, 122, 293

Pudritz R. E., Norman C. A., 1986, Astrophysical Journal, 301, 571

Shakura N. I., Sunyaev R. A., 1973, Astronomy and Astrophysics, 500, 33

Sokoloff D. D., 2015, Physics-Uspekhi, 58, 601

doi: doi.org/10.52526/25792776-2021.68.2-490