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On Multiple Hypotheses LAO Testing by Informed Statistician for Arbitrarily Varying Markov Source and on Such Source Coding Reliability Function

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Abstract

In this paper the problem of multiple hypotheses testing for arbitrarily varying Markov source (AVMS) with state sequence known to the statistician is solved from the point of view of logarithmically asymptotically optimal (LAO) testing. The matrix of asymptotic interdependencies of all possible pairs of the error probability exponents (reliabilities) in optimal testing for this model is studied. The LAO test, assuming that exponents of some number of the error probabilities are given, ensure the best asymptotic exponents for the rest of them. We find LAO test and the corresponding matrix of all error probability exponents. As an application to information theory, the E-optimal rate $R(E)$ (the minimum rate R of the source sequences compression when the decoding error probability is less than $\exp\{-NE\}$, $E > 0$) and the reliability function $E(R)$ of AVMS coding are determined.

1 Introduction

In statistics the problem of hypothesis testing is broadly known with its classical model of two hypotheses. Suppose that a $(N+1)$-vector $\mathbf{x} = (x_0, x_1, ..., x_N)$, $x_n \in \mathcal{X}$, $n = \overline{0, N}$, is emitted from an information source (discrete memoryless source (DMS), arbitrarily varying source (AVS), arbitrarily varying Markov source (AVMS), etc.), which is assumed to have a probability distribution (PD) either G_1 (hypothesis H_1) or G_2 (hypothesis H_2). Observing the data $\mathbf{x}$ the statistician must make a decision which of hypotheses is correct. The functional relation of the first and the second kind error probability exponents is in focus of research. There is a rich list of relevant references on this problem. The initial results for DMS were revealed by Hoeffding [1], then by Csiszár and Longo [2], Birgé [3] and others. Multiple hypothesis LAO tesing for DMS was studied by Haroutunian in [4]. There were found LAO test and the corresponding matrix of interdependencies of error probability exponents of all possible pairs of hypotheses. The LAO test, assuming that exponents of some number of the error probabilities are given, ensures the best asymptotic exponents for the rest of them.

Natarajan's [5] result concerns the Markov sources. Hypothesis testing for the model of AVS was investigated by Fu and Shen [6] for the case when side information is absent. The problem of multiple hypotheses LAO testing for discrete stationary Markovian source was solved by Haroutunian ([7]-[9]).

The hypothesis testing problem for AVS with side information was solved by Ahlswede, Aloyan and Haroutunian in [10]. Multiple hypotheses testing for AVS was examined by Haroutunian and Hakobyan in [11]. The latter generalizes the hypothesis testing model of Fu and Shen [6] to the case of more than two hypothesis, when side information is available for statistician.

We solve the problem of multiple hypotheses LAO testing for AVMS, with three hypotheses $G_l = \{G_l(x|u,s), \quad x,u \in \mathcal{X},\, s \in \mathcal{S}\}$, $l = \overline{1,3}$. The case of three hypotheses is considered solely with a view of brevity of exposition.

A formal representation of AVMS is as follows. Let $\mathcal{X}$ be the source alphabet, $\mathcal{S}$ be the finite set of source states.

Time-homogeneous Markov chain $X_0, X_1, X_2, \ldots$ is a stochastic process with transition probabilities

$$G_l(X_n = x|X_{n-1} = u, S_n = s) = G_l(x|u,s), \quad l = \overline{1,3}, \quad n = 1,2,\ldots.$$

The source studied in this paper is defined by homogeneous Markov chain with not necessarily unique stationary distributions Q_l corresponding to transition probabilities distribution G_l

$$Q_l = \{Q_l(u|s) = Q_l(X_0 = u|S_0 = s), \quad u \in \mathcal{X}, \quad s \in \mathcal{S}\},$$

such that

$$\sum_{u\in\mathcal{X}} Q_l(u|s)G_l(x|u,s) = Q_l(x|s), \quad s \in \mathcal{S}, \quad x \in \mathcal{X}. \tag{1}$$

The conditional probability of the vector $\mathbf{x} = (x_0, x_1, \ldots, x_N) \in \mathcal{X}^{N+1}$ of the Markov chain with transition probabilities G_l and stationary distribution Q_l, $l = \overline{1,3}$, with respect to known state vector $\mathbf{s} = (s_0, s_1, \ldots, s_N) \in \mathcal{S}^{N+1}$ is defined as follows

$$Q_l \circ G_l^N(\mathbf{x}|\mathbf{s}) = Q_l(x_0|s_0)\prod_{n=1}^{N} G_l(x_n|x_{n-1}, s_n), \quad l = \overline{1,3}. \tag{2}$$

The conditional probability of a subset $\mathcal{A} \subset \mathcal{X}^{N+1}$ is the sum

$$Q_l \circ G_l^N(\mathcal{A}|\mathbf{s}) = \sum_{\mathbf{x}\in\mathcal{A}} Q_l \circ G_l^N(\mathbf{x}|\mathbf{s}), \quad \mathbf{s} \in \mathcal{S}^{N+1}, \quad l = \overline{1,3}.$$

As we mentioned above the paper it is dedicated to the problem of multiple hypotheses LAO testing for AVMS with states sequence known to the statistician. We find LAO test and the corresponding matrix of error probability exponents (reliabilities) for this model. As an application to information theory, the E-optimal rate $R(E)$ (the minimum rate R of the source sequences compression when the decoding error probability is less than $\exp\{-NE\}$, $E > 0$) and the reliability function $E(R)$ of AVMS coding are determined. The rest of the paper is organized as follows. In Section 2 we describe the problem in detail and provide some definitions. In Section 3 the main theorem is formulated and the proof is exposed in Section 4. The application of this theorem to solution of the AVMS coding problem is presented in Section 5.

2 Basic Concepts and Definitions

Suppose that there are three alternative hypotheses about the transition PD of AVMS: $H_l : G_l = G_l(x|u,s), \quad x,u \in \mathcal{X},\, s \in \mathcal{S},\, l = \overline{1,3}$. A vector $\mathbf{x} = (x_0, x_1, \ldots, x_N)$ is emitted from the source, while the vector of states of the source was $\mathbf{s} = (s_0, s_1, ..., s_N) \in \mathcal{S}^{N+1}$. The statistician, knowing the data $\mathbf{x}$ and the states vector $\mathbf{s}$, attempts to decide which of hypotheses is correct.

To find a test φ^N the statistician can create for each $\mathbf{s}$ partition of the set $\mathcal{X}^{N+1}$ into three disjoint subsets $\mathcal{A}_{l,\mathbf{s}}^{N+1}$, $l = \overline{1,3}$. if $\mathbf{x} \in \mathcal{A}_{l,\mathbf{s}}^{N+1}$ then the test adopts l-th hypothesis.

Making decision one can commit the following errors: the hypotheses H_l is adopted, but the correct was H_k, $l \neq k$. We consider the maximum probability $\alpha_{l|k}^N(\varphi^N)$ of such error over all states vectors $\mathbf{s}$, namely,

$$\alpha_{l|k}^N(\varphi^N) \triangleq \max_{\mathbf{s} \in \mathcal{S}^{N+1}} Q_k \circ G_k(\mathcal{A}_{l,\mathbf{s}}^{N+1}|\mathbf{s}), \quad l \neq k \quad l, k = \overline{1,3}.$$

The probability of rejecting the hypotheses H_k is defined as follows:

$$\alpha_{k|k}^N(\varphi^N) \triangleq \sum_{l \neq k} \alpha_{l|k}^N(\varphi^N). \tag{3}$$

For the infinite sequence of tests φ we study error probability exponents, which we call reliabilities (in the paper we use only log-s and exp-s to the base 2):

$$E_{l|k}(\varphi) \triangleq \limsup_{N \to \infty} -\frac{1}{N+1} \log \alpha_{l|k}^N(\varphi^N), \quad l, k = \overline{1,3}. \tag{4}$$

Using (3) and (4) we see that

$$E_{k|k}(\varphi) = \min_{l \neq k} E_{l|k}(\varphi). \tag{5}$$

We name the matrix $\mathbf{E} \triangleq \{E_{l|k}(\varphi)\}$ the reliability matrix for the sequence of tests φ.

Following Birgé [3] we call the sequence of tests logarithmically asymptotically optimal (LAO) if for given positive values of $L-1$ diagonal elements of the matrix E the procedure provides maximal values to other elements of it.

The goal of this work is to find out reliability matrix of LAO test for AVMS and reveal conditions of making positive of all its elements.

We use the method of types in the next sections. Therefore we remind following definitions and fundamental properties of types [14]. For defining types we don't take in consideration first element of the vector $\mathbf{s}$, so as a vector $\mathbf{s}$ we can consider $\mathbf{s} = (s_1, ..., s_N)$. Let $N(s|\mathbf{s})$ be the number of occurrences of a state $s \in \mathcal{S}$ in the $N-$ vector $\mathbf{s}$. The type of vector $\mathbf{s}$ is the PD

$$P_{\mathbf{s}} = \{P_{\mathbf{s}}(s) = \frac{1}{N} N(s|\mathbf{s}), \quad s \in \mathcal{S}\}.$$

The second order type of a Markov chain's vector $\mathbf{x} \in \mathcal{X}^{N+1}$ (see [7], [12], cf. [15], [16]) is the PD $Q_{\mathbf{x}}(u, x)$ defined by the square matrix of $|\mathcal{X}|^2$ relative frequencies $N^{-1}N(u, x|\mathbf{x})$ of simultaneous appearance of pair (u, x) in neighbor positions in $\mathbf{x}$.

The second order joint type of the pair of the vectors $\mathbf{x}$ and $\mathbf{s}$ is the PD

$$G_{\mathbf{x},\mathbf{s}}(u, x, s) = \{P_{\mathbf{s}}(s) Q_{\mathbf{x},\mathbf{s}}(u|s) G_{\mathbf{x},\mathbf{s}}(x|u, s) = \frac{1}{N} N(u, x, s|\mathbf{x}, \mathbf{s}), \ x, u \in \mathcal{X}, \ s \in \mathcal{S}\},$$

where $N(u, x, s|\mathbf{x}, \mathbf{s})$ is the number of n such that $(x_{n-1}, x_n) = (u, x)$ and $s_n = s$. $n = \overline{1, N}$, that is letters u and x appear in the vector $\mathbf{x} = (x_0, x_1, ...x_N)$ being neighbors with u preceding x and the states $s \in \mathcal{S}$ in vector $\mathbf{s}$ correspond to x.

We use as conditional second order type of vector $\mathbf{x}$ with respect to vector $\mathbf{s}$ the conditional PD $Q_{\mathbf{x},\mathbf{s}}(u|s)$ defined by relation

$$Q_{\mathbf{x},\mathbf{s}}(u|s) \triangleq N(u, s|\mathbf{x}, \mathbf{s})/N(s|\mathbf{s}), \qquad u \in \mathcal{X}, \quad s \in \mathcal{S},$$

where $N(u, s|\mathbf{x}, \mathbf{s})$ is the number of repetitions of u in $\mathbf{x}$ and s in $\mathbf{s}$, such that positions of u precede positions of s.

There is also the conditional type of the vector $\mathbf{x}$ with respect to vector $\mathbf{s}$ as conditional PD $G_{\mathbf{x},\mathbf{s}}(x|u, s)$ defined by

$$G_{\mathbf{x},\mathbf{s}}(x|u, s) = N(u, x, s|\mathbf{x}, \mathbf{s})/N(u, s|\mathbf{x}, \mathbf{s}), \qquad x \in \mathcal{X}, \quad s \in \mathcal{S}.$$

We apply notation $Q \circ G$ for the following PD

$$Q \circ G = \{Q(u|s)G(x|u, s), \quad x, u \in \mathcal{X}, s \in \mathcal{S}\},$$

similarly we will use PDs $Q_l \circ G_l$, $l = \overline{1,3}$ and

$$Q_{\mathbf{x},\mathbf{s}} \circ G_{\mathbf{x},\mathbf{s}} = \{Q_{\mathbf{x},\mathbf{s}}(u|s)G_{\mathbf{x},\mathbf{s}}(x|u, s) = \frac{N(u, x, s|\mathbf{x}, \mathbf{s})}{N(s|\mathbf{s})}, \quad x, u \in \mathcal{X}, \quad s \in \mathcal{S}\},$$

for the conditional type of $\mathbf{x}$ given $\mathbf{s}$.

We denote by $\mathcal{T}^N_{P,Q\circ G}(X|\mathbf{s})$ the set of vectors $\mathbf{x}$ from $\mathcal{X}^{N+1}$ which have the conditional type given $\mathbf{s}$ such, that $Q_{\mathbf{x},\mathbf{s}} \circ G_{\mathbf{x},\mathbf{s}} = Q \circ G$ and $P_{\mathbf{s}} = P$.

Define the conditional entropy of X with respect to S:

$$H_{Q\circ G|P}(X|S) = -\sum_{x,u,s} P(s)Q(u|s)G(x|u, s) \log Q(u|s)G(x|u, s).$$

We define the Kullback-Leibler conditional divergence $D(Q \circ G||Q_l \circ G_l|P)$ of the distribution $P \circ Q \circ G$ from the distribution $P_l \circ Q_l \circ G_l$ as follows

$$D(Q \circ G||Q_l \circ G_l|P) = \sum_{x,u,s} P(s)Q(u|s)G(x|u, s) \log \frac{Q(u|s)G(x|u, s)}{Q_l(u|s)G_l(x|u, s)}, \quad l = \overline{1,3}.$$

Similarly $D(Q_k \circ G_k||Q_l \circ G_l|P)$, $l, k = \overline{1,3}$ are defined.

3 Formulation of Result

We denote by $\mathcal{P}^N(\mathcal{S})$ the space of all types on $\mathcal{S}$ for given N, and by $\mathcal{Q} \circ \mathcal{G}^N(\mathcal{X}, \mathbf{s})$ the set of all possible conditional types on $\mathcal{X}$ for given $\mathbf{s}$.

Suppose that positive numbers $E_{1|1}$, $E_{2|2}$ are given. We shall define LAO test with the following sets of types:

$$\mathcal{R}_l = \{Q \circ G \in \mathcal{Q} \circ \mathcal{G}^N(\mathcal{X}, \mathbf{s}), P \in \mathcal{P}^N(\mathcal{S}):$$

$$D(Q \circ G||Q \circ G_l|P) \leq E_{l|l}, \exists\, Q_l : \ D(Q||Q_l|P) < \infty\}, \quad l = 1, 2,$$

$$\mathcal{R}_3 = \{Q \circ G \in \mathcal{Q} \circ \mathcal{G}^N(\mathcal{X}, \mathbf{s}), P \in \mathcal{P}^N(\mathcal{S}):$$

$$D(Q \circ G||Q \circ G_1|P) > E_{1|1}, \quad D(Q \circ G||Q \circ G_2|P) > E_{2|2}\}.$$

Elements of reliability matrix corresponding to the LAO test will be the following:

$$E^*_{1|1}(E_{1|1}) \triangleq E_{1|1}, \qquad E^*_{2|2}(E_{2|2}) \triangleq E_{2|2}, \tag{6.a}$$

$$E^*_{l|k} \triangleq \inf_{P} \inf_{Q\circ G \in \mathcal{R}_l} D(Q \circ G \| Q \circ G_k | P), \quad k = \overline{1,3}, \quad l = 1,2, k \neq l, \tag{6.b}$$

$$E^*_{3|k} \triangleq \inf_{P} \inf_{Q\circ G \in \mathcal{R}_3} D(Q \circ G \| Q \circ G_k | P), \qquad k = 1,2, \tag{6.c}$$

$$E^*_{3|3} \triangleq \min_{k=1,2} E_{3|k}. \tag{6.d}$$

Theorem 1. If different conditional distributions G_l, $l = \overline{1,3}$, and positive numbers $E_{1|1}$, $E_{2|2}$ are given and the following inequalities hold

$$E_{1|1} < \min_{P} \min[\inf_{Q_2} D(Q_2 \circ G_2 \| Q_1 \circ G_1 | P), \inf_{Q_3} D(Q_3 \circ G_3 \| Q_1 \circ G_1 | P)], \tag{7.a}$$

$$E_{2|2} < \min_{P} \min[E^*_{1|2}, \inf_{Q_3} D(Q_3 \circ G_3 \| Q_3 \circ G_2 | P)], \tag{7.b}$$

then for AVMS

a) there exists a LAO sequence of tests φ^* such that elements of the reliability matrix $E(\varphi^*)$ are defined in (6),

b) if one of the inequalities (7) is violated, then at least one element of the matrix $E(\varphi^*)$ of any test is equal to 0.

This theorem is an generalization of the results of [6] and [11] to the case of AVMS.

4 Proof of Theorem 1

Our proof of Theorem 1 exploits the method of types. We are interested in the following properties of types.

For every conditional type of vector $\mathbf{x}$ given states vector $\mathbf{s}$ $\{Q_{\mathbf{x},\mathbf{s}}(u|s)G_{\mathbf{x},\mathbf{s}}(x|u,s), \quad x,u \in \mathcal{X}, s \in \mathcal{S}\}$ the upper estimate holds

$$|\mathcal{T}^N_{P,Q\circ G}(X|\mathbf{s})| \leq \exp\{N H_{P,Q\circ G}(X|S) + o(1)\}. \tag{8}$$

For a distribution G_l on $\mathcal{X}$, the vector $\mathbf{x} \in \mathcal{T}^N_{P,Q\circ G}(X|\mathbf{s})$ has the conditional probability with respect to $\mathbf{s}$ of the type $P_{\mathbf{s}} = P$

$$Q_l \circ G_l^N(\mathbf{x}|\mathbf{s}) = \exp\{-N(H_{Q\circ G|P}(X|S) + D(Q \circ G \| Q_l \circ G_l | P)) + o(1)\}. \tag{9}$$

Note also that (8) and (9) give the upper estimate for type class probability

$$Q_l \circ G_l^N(\mathcal{T}^N_{P,Q\circ G}(X|\mathbf{s})|\mathbf{s}) \leq \exp\{-ND(Q \circ G \| Q_l \circ G_l | P) + o(1)\}. \tag{10}$$

The lower bound of the probability of type class is the following:

$$Q_l \circ G_l^N(\mathcal{T}^N_{P,Q\circ G}(X|\mathbf{s})|\mathbf{s}) \geq \frac{1}{(N+1)^{|\mathcal{X}|^2|\mathcal{S}|}} \exp\{-ND(Q \circ G \| Q_l \circ G_l | P))\}.$$

The proof of the Theorem 1 consists of two parts. In the first part we prove the existence of sequence of tests. In the second part we prove that if one of the inequalities (7) is violated, then at least one element of the reliability matrix is equal to zero.

The test φ^N for every states vector $\mathbf{s} \in \mathcal{S}^N$ can be given by the set $\mathcal{B}_{l,\mathbf{s}}^N$, the parts of the space $\mathcal{X}^{N+1}$, in which the hypothesis H_l is adopted. We shall verify that the test for which

$$\mathcal{B}_{l,\mathbf{s}}^N = \bigcup_{Q\circ G \in \mathcal{R}_l} \mathcal{T}_{P,Q\circ G}^N(X|\mathbf{s}), \quad l = \overline{1,3}. \tag{11}$$

will be asymptotically optimal for given $E_{1|1}, E_{2|2}$. We can show that

$$\mathcal{B}_{l,\mathbf{s}}^N \bigcap \mathcal{B}_{k,\mathbf{s}}^N = \emptyset \quad l \neq k, \quad \text{and} \quad \bigcup_{l=1}^{3} \mathcal{B}_{l,\mathbf{s}}^N = \mathcal{X}^N.$$

In its order the first part consist of the following steps.

First we show that following inequality holds

$$E_{l|k}^* \leq \inf_P \inf_{Q\circ G \in \mathcal{R}_l} D(Q\circ G||Q\circ G_k|P), \quad k = \overline{1,3}, \quad l = 1,3 \quad k \neq l.$$

Then we will show that the converse inequality holds

$$E_{l|k}^* \geq \inf_P \inf_{Q\circ G \in \mathcal{R}_l} D(Q\circ G||Q\circ G_k|P), \quad k = \overline{1,3}, \quad l = 1,3, k \neq l.$$

Finally we will conclude that the determined sequence of the test is LAO.

For $\mathbf{x} \in \mathcal{T}_{P,Q\circ G}^N(X|\mathbf{s})$, $\mathbf{s} \in \mathcal{T}_P^N(S)$, $l = \overline{1,3}$ we show that the probability $Q_l \circ G_l^N(\mathbf{x}|\mathbf{s})$ satisfies equality (9):

$$Q_l \circ G_l^N(\mathbf{x}|\mathbf{s}) = Q_l(x_0|s_0) \prod_{x,u,s} G_l(x|u,s)^{NP(s)Q(u|s)G(x|u,s)} =$$

$$= Q_l(x_0|s_0) \prod_{x,u,s} \exp\{NP(s)Q(u|s)G(x|u,s)[\log G_l(x|u,s) -- \log G(x|u,s) + \log G(x|u,s)]\} =$$

$$= \exp\{N \sum_{x,u,s} P(s)Q(u|s)G(x|u,s)(-\log \frac{Q(u|s)G(x|u,s)}{Q(u|s)G_l(x|u,s)} +$$

$$+ \log G(x|u,s) + \log Q(u|s) - \log Q(u|s) + o_N(1))\} =$$

$$= \exp\{-N[D(Q\circ G||Q\circ G_l|P) + D(Q|Q_l) + H_{P,Q\circ G}(X|S)] + o_N(1)\} =$$

$$= \exp\{-N[D(Q\circ G||Q_l\circ G_l|P) + H_{P,Q\circ G}(X|S)] + o_N(1)\}. \tag{12}$$

Where

$$o_N(1) = \max\{\max_l \max_s |N^{-1} \log Q_l(u|s)| : Q_l(u|s) > 0\} \to 0, \quad \text{as} \quad N \to \infty.$$

The error probability $\alpha_{k|k}^N(\varphi^*)$ for $k = 1,2$ is the following

$$\alpha_{k|k}^N(\varphi^*) = \max_{\mathbf{s} \subset \mathcal{S}^N} Q_k \circ G_k(\overline{\mathcal{B}_{k,\mathbf{s}}^N}|\mathbf{s}) = \max_{\mathbf{s} \subset \mathcal{S}^N} Q_k \circ G_k\Big(\bigcup_{Q\circ G: D(Q\circ G||Q\circ G_k|P) > E_{k|k}} \mathcal{T}_{P,Q\circ G}^N(X|\mathbf{s})|\mathbf{s}\Big).$$

Now we have to show that $\alpha_{k|k}^N(\varphi^*)$ for $k = 1,2$ can be upper bounded. Using polynomial number of different types and (10) for probability $Q_k \circ G_k(\cdot|\mathbf{s})$ of the set $\bigcup_{Q\circ G: D(Q\circ G||Q\circ G_k|P) > E_{k|k}} \mathcal{T}_{P,Q\circ G}^N(X|\mathbf{s})$ we get

$$\alpha_{k|k}^N(\varphi^*) \leq \max_{\mathbf{s} \in \mathcal{S}^N} (N+1)^{|\mathcal{X}|^2|\mathcal{S}|} \sup_{Q\circ G: D(Q\circ G||Q\circ G_k|P) > E_{k|k}} Q_k \circ G_k(\mathcal{T}_{P,Q\circ G}^N(X|\mathbf{s})|\mathbf{s})$$

$$\leq (N+1)^{|\mathcal{X}|^2|S|} \sup_P \sup_{Q\circ G: D(Q\circ G||Q\circ G_k|P)>E_{k|k}} \exp\{-ND(Q\circ G||Q\circ G_k|P)\} =$$

$$= \sup_P \sup_{Q\circ G: D(Q\circ G||Q\circ G_k|P)>E_{k|k}} \exp\{-N[D(Q\circ G||Q\circ G_k|P) - o_N(1)]\} \leq$$

$$\leq \exp\{-N[E_{k|k} - o_N(1)]\},$$

where $o_N(1) = (N+1)^{-1}|\mathcal{X}|^2|S|\log(N+1) \to 0$, when $N \to \infty$.

For $l = 1,2$, $k = \overline{1,3}$, $l \neq k$, we can obtain similar inequalities

$$\alpha_{l|k}^N(\varphi^*) = \max_{\mathbf{s}\in S^N} Q_k \circ G_k(\mathcal{B}_{l,\mathbf{s}}^N|\mathbf{s}) = \max_{\mathbf{s}\in S^N} Q_k\circ G_k(\bigcup_{Q\circ G\in \mathcal{R}_l} \mathcal{T}_{P,Q\circ G}^N(X|\mathbf{s})|\mathbf{s}) \leq$$

$$\leq \max_{\mathbf{s}\in S^N}(N+1)^{|\mathcal{X}|^2|S|} \sup_{Q\circ G\in\mathcal{R}_l} Q_k\circ G_k(\mathcal{T}_{P,Q\circ G}^N(X|\mathbf{s})|\mathbf{s}) \leq$$

$$\leq (N+1)^{|\mathcal{X}|^2|S|} \sup_P \sup_{Q\circ G\in\mathcal{R}_l} \exp\{-ND(Q\circ G||Q\circ G_k|P)\} =$$

$$= \sup_P \sup_{Q\circ G\in\mathcal{R}_l} \exp\{-N[D(Q\circ G||Q\circ G_k|P) - o_N(1)]\} \leq \exp\{-N[E_{l|k} - o_N(1)]\}. \quad (13)$$

Now let us prove the inverse inequality

$$\alpha_{l|k}^N(\varphi^*) = \max_{\mathbf{s}\in S^N} Q_k \circ G_k(\mathcal{B}_{l,\mathbf{s}}^N|\mathbf{s}) = \max_{\mathbf{s}\in S^N} Q_k\circ G_k(\bigcup_{Q\circ G\in \mathcal{R}_l} \mathcal{T}_{P,Q\circ G}^N(X|\mathbf{s})|\mathbf{s}) \geq$$

$$\geq \max_{\mathbf{s}\in S^N} \sup_{Q\circ G\in\mathcal{R}_l} Q_k\circ G_k(\mathcal{T}_{P,Q\circ G}^N(X|\mathbf{s})|\mathbf{s}).$$

Using the bounds on $Q_k\circ G_k(\mathcal{T}_{P,Q\circ G}^N(X|\mathbf{s})$ derived in Theorem 1, we have

$$\max_{\mathbf{s}\in S^N} \sup_{Q\circ G\in\mathcal{R}_l} Q_k\circ G_k(\mathcal{T}_{P,Q\circ G}^N(X|\mathbf{s})) \geq$$

$$\geq (N+1)^{-|\mathcal{X}|^2|S|} \sup_P \sup_{Q\circ G\in R_l} \exp\{-ND(Q\circ G||Q\circ G_k|P)\} =$$

$$= \sup_P \sup_{Q\circ G\in\mathcal{R}_l} \exp\{-N[D(Q\circ G||Q\circ G_k|P) - o_N(1)]\}. \quad (14)$$

Taking into account (13) and (14) and the continuity of the functional $D(Q\circ G||Q\circ G_k|P)$ we obtain that $\lim_{N\to\infty}\{\sup -N^{-1}\log\alpha_{l|k}^N(\varphi^*)\}$ exists and in correspondence with 6.b equals to $E_{l|k}^*$. Similarly we can obtain upper and lower bounds for $\alpha_{3|k}^N(\varphi^*)$, $k = \overline{1,3}$. Applying the same reasons we get the reliability $E_{3|k}(\varphi^*) = E_{3|k}^*$.

The proof of the first part of the theorem will be accomplished if we demonstrate that the sequence of tests φ^* is LAO, that is for given $E_{1|1}, E_{2|2}$ doesn't exist such tests φ^{**} that for all $l,k = \overline{1,3}$ $E_{l|k}^{**} \leq E_{l|k}^*$. Let us consider any other sequence of tests φ^{**} which are defined for every $\mathbf{s}\in S^N$ by the sets $\mathcal{D}_{1,\mathbf{s}}^{(N)}, \mathcal{D}_{2,\mathbf{s}}^{(N)}, \mathcal{D}_{3,\mathbf{s}}^{(N)}$ such that

$$E_{l|k}^{**} \geq E_{l|k}^* \quad l,k = \overline{1,3}. \quad (15.a)$$

This condition for large enough N is equivalent to the following inequality:

$$\alpha_{l|k}^N(\varphi^{**}) \leq \alpha_{l|k}^N(\varphi^*). \quad (15.b)$$

Below we examine relation between $\mathcal{D}_{l,s}^N$ and $\mathcal{A}_{l,s}^N$, $l = \overline{1,3}$. There are the following possible cases:

$$\mathcal{D}_{l,s}^N \cap \mathcal{B}_{l,s}^N = \emptyset, \qquad \mathcal{D}_{l,s}^N \subset \mathcal{B}_{l,s}^N.$$

If we show that there exist some l_1, k_1, for which (15) doesn't held, we can say that the test φ^* is LAO and proof of the first part will be accomplished.

For the first case we have

$$\alpha_{l_1|l_1}^N(\varphi^{**}) = \max_{s \in S^N} Q_{l_1} \circ G_{l_1}(\overline{\mathcal{D}}_{l_1,s}^N|\mathbf{s}) \geq \max_{s \in S^N} Q_{l_1} \circ G_{l_1}(\mathcal{B}_{l_1,s}^N|\mathbf{s}) \geq \exp\{-N[E_{l_1|l_1} - o_N(1)]\},$$

Note that in this case the inequality (15) isn't satisfied.

The second case will be proved by the following way:

$$\alpha_{l_1|l_1}^N(\varphi^{**}) = \max_{s \in S^N} Q_{l_1} \circ G_{l_1}(\overline{\mathcal{D}}_{l_1,s}^N|\mathbf{s}) \geq \max_{s \in S^N} Q_{l_1} \circ G_{l_1}(\overline{\mathcal{B}}_{l_1,s}^N|\mathbf{s}) \geq$$

$$\geq \max_{s \in S^N} Q_{l_1} \circ G_{l_1}\Big(\bigcup_{Q \circ G: D(Q \circ G||Q \circ G_{l_1}|P) > E_{l_1|l_1}} \mathcal{T}_{P,Q \circ G}^N(X|\mathbf{s})|\mathbf{s}\Big) \geq$$

$$\geq \max_{s \in S^N} \sup_{Q \circ G: D(Q \circ G||Q \circ G_{l_1}|P) > E_{l_1|l_1}} Q_{l_1} \circ G_{l_1}(\mathcal{T}_{P,Q \circ G}^N(X|\mathbf{s})|\mathbf{s}) \geq$$

$$\geq (N+1)^{-|\mathcal{X}|^2|S|} \sup_P \sup_{Q \circ G: D(Q \circ G||Q \circ G_{l_1}|P_s) > E_{l_1|l_1}} \exp\{-ND(Q \circ G||Q \circ G_{l_1}|P)\} =$$

$$= \sup_P \sup_{Q \circ G: D(Q \circ G||Q \circ G_{l_1}|P) > E_{l_1|l_1}} \exp\{-N[D(Q \circ G||Q \circ G_{l_1}|P) - o_N(1)]\} \geq$$

$$\geq \exp\{-N[E_{l_1|l_1} - o_N(1)]\},$$

The statement of part b) of the theorem is evident, since in case of violation of one of the conditions (7) (for example (7.a)) at least one of the elements of reliability matrix $E_{l|k}^*$ defined in (6) reduces to the equality to 0. Suppose that (7.a) is violated and

$$\min_P \min[\inf_{Q_2} D(Q_2 \circ G_2||Q_1 \circ G_1|P), \inf_{Q_3} D(Q_3 \circ G_3||Q_1 \circ G_1|P)] = \inf_{Q_2} D(Q_2 \circ G_2||Q_1 \circ G_1|P),$$

that is

$$E_{1|1} \geq \inf_{Q_2} D(Q_2 \circ G_2||Q_1 \circ G_1|P).$$

According (6.b)

$$E_{1|2}^* \triangleq \inf_P \inf_{Q \circ G \in \mathcal{R}_1} D(Q \circ G||Q \circ G_2|P).$$

So $E_{1|2}^* = 0$.

5 Application to Source Coding

The tight connection of the hypothesis testing problem and the problem of estimating the optimum probability of incorrect decoding, when vectors of messages of length N from a discrete memoryless source are block coded, was emphasized by many authors. In source coding problem the reliability function $E(R)$ expresses the dependence of the reliability (exponent of the optimal error probability) on the code rate R ([14]). This function can be specialized from Marton ([17]) best component $E(R,\Delta)$ (when $\Delta = 0$), Δ standing for distortion constraint. Sometimes it is important to present the same dependence by the rate-reliability function $R(E)$ ([12]) for $0 < E < \infty$. Beside the hypothesis testing problem for AVS, Fu and Shen [6] examined also the application of their result in source coding problem, in particular the dual functions of $E(R)$ and $R(E)$ in source coding are obtained. Our goal is to derive the those functions for AVMS with side information.

In order to proceed with details, we give some definitions from information theory. Let we aim to transmit the AVMS X through a errorless channel subject to reconstruct at the decoder with a maximally lower probability. This model of communication is depicted in Fig. 1.

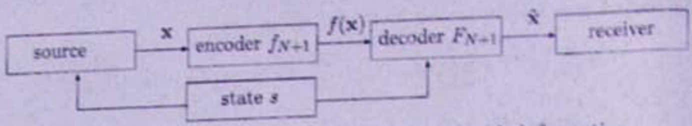

Fig. 1. Communication model in source coding with side information.

Mathematically the problem of source coding can be formulated as follows. A block code (f_{N+1}, F_{N+1}) in Fig. 1 is composed by a couple of mappings. The encoder f_N maps the space $\mathcal{X}^{N+1}$ into a finite set of labels

$$f_{N+1} : \mathcal{X}^{N+1} \to \{0, 1, 2, 3, ..., M_{N+1}\}$$

and the decoder knowing states vector $\mathbf{s}$

$$F_{N+1} : \{0, 1, 2, 3, ..., M_{N+1}\} \times \mathcal{S}^{N+1} \to \mathcal{X}^{N+1}$$

chooses a vector $\hat{\mathbf{x}}$ from $\mathcal{X}^{N+1}$.

Given an AVMS defined by G_1 in Section 2, let

$$\mathcal{A} = \{\mathbf{x} : F_{N+1}(f_{N+1}(\mathbf{x}), \mathbf{s}) = \mathbf{x}\}$$

be the set of those messages which are decoded correctly. Then the error probability in AVMS coding for the code (f_{N+1}, F_{N+1}) is determined by the following way

$$e(f_{N+1}, F_{N+1}) = 1 - \min_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_1 \circ G_1^N(\mathcal{A}|\mathbf{s}) = \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_1 \circ G_1^N(\overline{\mathcal{A}}|\mathbf{s}). \qquad (16)$$

The cardinality of the finite subset $\mathcal{A} \subset \mathcal{X}^{N+1}$ is called the volume of the code and is denoted by M_{N+1}. The code rate is $\frac{1}{N+1}\log|\mathcal{A}|$.

Definition 1. $R \geq 0$ is called an E-achievable code rate for reliability $E > 0$ and sufficiently large N if for any $\varepsilon > 0$ there exist a sequence of codes (f_{N+1}, F_{N+1}) such that

$$e(f_{N+1}, F_{N+1}) \leq \exp(-(N+1)E) \qquad \text{and} \qquad \frac{1}{N+1}\log M_{N+1} \leq R + \varepsilon. \qquad (17)$$

The infimum of all E-achievable code rates is called optimal and denoted by $R(E)$ (rate-reliability function). It can be defined as follows:

$$R(E) \stackrel{\Delta}{=} \overline{\lim_{N\to\infty}} \frac{1}{N+1} \log \min_{(f_{N+1}, F_{N+1}):\, e(f_{N+1}, F_{N+1}) \leq \exp(-NE)} M_{N+1}.$$

The minimal error probability among the codes with $2^{(N+1)R}$ codewords with side information is defined as

$$e(G_1^N,R)\triangleq \min_{\mathcal{A}\subseteq\mathcal{X}^{N+1}:\,|\mathcal{A}|\leq\exp\{NR\}}\ \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_1\circ G_1^N(\overline{\mathcal{A}}|\mathbf{s}).$$

Definition 2. The reliability function $E(R)$ of the AVMS for rate R is defined as follows:

$$E(R)\triangleq \overline{\lim_{N\to\infty}} -\frac{1}{N+1}\log e(G_1^N,R)=$$

$$\overline{\lim_{N\to\infty}} -\frac{1}{N+1}\log \min_{\mathcal{A}\subseteq\mathcal{X}^{N+1}:\,|\mathcal{A}|\leq\exp\{NR\}}\ \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_1\circ G_1^N(\overline{\mathcal{A}}|\mathbf{s}).$$

Now for estimation of $R(E)$ and $E(R)$ we apply the result of hypothesis testing from Theorem 1 for $L=2$.

The problem of source coding, subject to exact reconstruction of the source messages, can be parallelized with a hypothesis testing. Let the equiprobable distribution $G_0\in\mathcal{P}(\mathcal{X})$ be given by $G_0(x)=\frac{1}{|\mathcal{X}|}$, $x\in\mathcal{X}$ and let $G_2(.|\mathbf{s})=G_0$ for every $\mathbf{s}\in\mathcal{S}^{N+1}$. So we have following hypotheses: $H_1:\ G_1$ and $H_2:\ G_2$ with corresponding error probabilities. The first kind error probability is

$$\alpha_{1|2}^N(\varphi^N)\triangleq \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_1\circ G_1(\overline{\mathcal{A}}|\mathbf{s}),$$

and the second order error probability is

$$\alpha_{2|1}^N(\varphi^N)\triangleq \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_0\circ G_0(\mathcal{A})=\max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_2\circ G_2(\mathcal{A}|\mathbf{s}).$$

The optimal (minimum) coding rate subject to error exponent $E>0$ is given by (17). Coming from this parallel we are able to derive the functions $R(E)$ and $E(R)$ by specializing Theorem 1. The analytics for those functions are given by the following theorems.

Theorem 2. In the presence of side information the rate-reliability function of AVMS with conditional PD G_1, for any $E>0$ has the following presentation:

$$R(E)=\max_P \max_{Q\circ G:D(Q\circ G\|Q\circ G_1|P)\leq E,\ \exists\, Q:\, D(Q\|Q_1|P)<\infty} H_{P,Q\circ G}(X|S).$$

Theorem 3. If

$$\max_P H_{P,Q_1\circ G_1}(X|S)<R\leq\log|\mathcal{X}|$$

and the conditional probability distribution G_1 is given then the reliability function of AVMS can be presented in the following form:

$$E(R)=\min_P \min_{Q\circ G:\,H_{P,Q\circ G}(X|S)\geq R} D(Q\circ G\|Q\circ G_1|P).$$

Proof of Theorem 2. Applying Theorem 1, with G_1 and G_2 and taking $L=2$ we get the following representation for (6) (which implies $E_{1|1}=E_{1|2}=E$ and $E_{2|2}=E_{2|1}$)

$$E_{2|1}=\inf_P \inf_{Q\circ G:D(Q\circ G\|Q\circ G_1|P)\leq E,\ \exists\, Q_1:\, D(Q\|Q_1|P)<\infty} D(Q\circ G\|Q\circ G_2|P). \tag{18}$$

According to the definition of $E_{2|1}$ we have

$$E_{2|1}=\max_{\varphi:\,\alpha_{1|1}^N(\varphi)\leq\exp\{-NE\}} \overline{\lim_{N\to\infty}} -\frac{1}{N+1}\log\alpha_{2|1}^N(\varphi)=$$

$$= \sup_{\mathcal{A}\subseteq\mathcal{X}^{N+1}:\ \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_1\circ G_1^N(\overline{\mathcal{A}}|\mathbf{s})\leq\exp(-NE)} \overline{\lim_{N\to\infty}} -\frac{1}{N+1}\log \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_2\circ G_2^N(\mathcal{A}|\mathbf{s}) =$$

$$= \overline{\lim_{N\to\infty}} -\frac{1}{N+1}\log \inf_{\mathcal{A}\subseteq\mathcal{X}^{N+1}:\ \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_1\circ G_1^N(\overline{\mathcal{A}}|\mathbf{s})\leq\exp(-NE)} \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_2\circ G_2^N(\mathcal{A}|\mathbf{s}). \tag{19}$$

Then

$$\inf_{\mathcal{A}\subseteq\mathcal{X}^{N+1}:\ \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_1\circ G_1^N(\overline{\mathcal{A}}|\mathbf{s})\leq\exp(-NE)} \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_2\circ G_2^N(\mathcal{A}|\mathbf{s}) =$$

$$= \inf_{\mathcal{A}\subseteq\mathcal{X}^{N+1}:\ \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_1\circ G_1^N(\overline{\mathcal{A}}|\mathbf{s})\leq\exp(-NE)} \frac{M_{N+1}}{|\mathcal{X}|^{N+1}}. \tag{20}$$

Because

$$D(Q\circ G\|Q_0\circ G_0|P) = \log|\mathcal{X}| - H_{P,Q\circ G}(X|S),$$

then

$$\inf_{P}\ \inf_{Q\circ G:\ D(Q\circ G\|Q\circ G_1|P)\leq E,\ \exists Q_1:\ D(Q\|Q_1|P)<\infty} D(Q\circ G\|Q\circ G_2|P) =$$

$$= \inf_{P}\ \inf_{Q\circ G:\ D(Q\circ G\|Q\circ G_1|P)\leq E,\ \exists Q_1:\ D(Q\|Q_1|P)<\infty} [\log|\mathcal{X}| - H_{P,Q\circ G}(X|S)] =$$

$$= \log|\mathcal{X}| - \max_{P}\ \max_{Q\circ G:\ D(Q\circ G\|Q\circ G_1|P)\leq E,\ \exists Q_1:\ D(Q\|Q_1|P)<\infty} H_{P,Q\circ G}(X|S). \tag{21}$$

By definition of $R(E)$, according to Theorem 1 and (18), (20), (21) we obtain

$$R(E) = \overline{\lim_{N\to\infty}} \frac{1}{N+1}\log \min_{\mathcal{A}\subseteq\mathcal{X}^{N+1}:\ \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_1\circ G_1^N(\overline{\mathcal{A}}|\mathbf{s})\leq\exp(-NE)} M_{N+1} =$$

$$= \log|\mathcal{X}| - \overline{\lim_{N\to\infty}} -\frac{1}{N+1}\log \inf_{\mathcal{A}\subseteq\mathcal{X}^{N+1}:\ \max_{\mathbf{s}\in\mathcal{S}^{N}} Q_1\circ G_1^N(\overline{\mathcal{A}}|\mathbf{s})\leq\exp(-NE)} \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_2\circ G_2^N(\mathcal{A}|\mathbf{s}) =$$

$$= \max_{P}\ \max_{Q\circ G:\ D(Q\circ G\|Q\circ G_1|P)\leq E,\ \exists Q_1:\ D(Q\|Q_1|P)<\infty} H_{P,Q\circ G}(X|S).$$

Theorem is proved.

Proof of Theorem 3. $E_{2|1}(E_{1|1})$ is defined in (17), it can also be written as follows:

$$E_{2|1}(E_{1|1}) = \overline{\lim_{N\to\infty}} -\frac{1}{N+1}\log \inf_{\mathcal{A}\subseteq\mathcal{X}^{N+1}:\ \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_1\circ G_1^N(\overline{\mathcal{A}}|\mathbf{s})\leq\exp(-NE_{1|1})} \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_2\circ G_2^N(\mathcal{A}|\mathbf{s}). \tag{22}$$

Let $G_1 = G_0$ and $E_{1|2} = \log|\mathcal{X}| - R$. From this notations and (22) we have

$$E_{2|1}(\log|\mathcal{X}| - R) =$$

$$= \overline{\lim_{N\to\infty}} -\frac{1}{N+1}\log \inf_{\mathcal{A}\subseteq\mathcal{X}^{N+1}:\ \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_0\circ G_0^N(\overline{\mathcal{A}}|\mathbf{s})\leq\exp\{-N(|\mathcal{X}|-R)\}} \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_2\circ G_2^N(\mathcal{A}|\mathbf{s}) =$$

$$= \overline{\lim_{N\to\infty}} -\frac{1}{N+1}\log \inf_{\mathcal{A}\subseteq\mathcal{X}^{N+1}:\ \frac{|\overline{\mathcal{A}}|}{|\mathcal{X}|^{N+1}}\leq\exp\{-N(|\mathcal{X}|-R)\}} \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_2\circ G_2^N(\mathcal{A}|\mathbf{s}) =$$

$$= \overline{\lim_{N\to\infty}} -\frac{1}{N+1}\log \inf_{\mathcal{A}\subseteq\mathcal{X}^{N+1}:\ |\mathcal{A}|\leq\exp(NR)} \max_{\mathbf{s}\in\mathcal{S}^{N+1}} Q_2\circ G_2^N(\mathcal{A}|\mathbf{s}) = E(R). \tag{23}$$

As $G_1 = G_0$

$$D(Q\circ G||Q\circ G_0|P) = \sum_{u,x,s} P(s)Q(u|s)G(x|u,s)\log\frac{Q(u|s)G(x|u,s)}{Q(u|s)G_0(x|u,s)} = \log|\mathcal{X}| - H_{P,Q\circ G}(X|S), \tag{24}$$

and

$$\min_P D(Q\circ G||Q\circ G_0|P) = \min_{P\in\mathcal{P}(S)} \sum_{x,u,s} P(s)Q(u|s)G(x|u,s)\log\frac{Q(u|s)G(x|u,s)}{Q(u|s)G_0(x|u,s)} =$$
$$= \log|\mathcal{X}| - \max_P H_{P,Q\circ G}(X|S). \tag{25}$$

From (24) and our notations we obtain that

$$\min_P \inf_{Q\circ G: D(Q\circ G||Q\circ G_1|P)\le E_{1|1},\, \exists Q_1: D(Q||Q_1|P)<\infty} D(Q\circ G||Q\circ G_2|P) =$$
$$= \min_P \inf_{Q\circ G: D(Q\circ G||Q\circ G_0|P)\le \log|\mathcal{X}|-R,\, \exists Q_1: D(Q||Q_1|P)<\infty} D(Q\circ G||Q\circ G_2|P) =$$
$$= \min_P \inf_{Q\circ G: H_{P,Q\circ G}(X|S)\ge R,\, \exists Q_2: D(Q||Q_2|P)<\infty} D(Q\circ G||Q\circ G_2|P). \tag{26}$$

By the first part of Theorem 1, when $E_{1|2} < \min_P D(G_2||G_1|P)$ then

$$E_{2|1}(E_{1|1}) = \min_P \inf_{Q\circ G: D(Q\circ G||Q\circ G_1|P)\le E_{1|1}\, \exists Q_1: D(Q||Q_1|P)<\infty} D(Q\circ G||Q\circ G_2|P).$$

According this fact, or notations,(23),(25) and (26) we have that when $\max_{P\in\mathcal{P}(S)} H_{P,Q\circ G}(X|S) < R \le \log|\mathcal{X}|$, then

$$E(R) = \min_P \inf_{Q\circ G: H_{P,Q\circ G}(X|S)\ge R,\, \exists Q_2: D(Q||Q_2|P)<\infty} D(Q\circ G||Q\circ G_2|P).$$

By the second part of Theorem 1, if $E_{1|2} \ge \min_{P\in\mathcal{P}(S)} D(G_2||G_1|P)$, then $E_{2|1}(E_{1|1}) = 0$. From here, (23), (25) and (26) it follows that when $R \le \max_{P\in\mathcal{P}(S)} H_{P,Q\circ G}(X|S)$ then $E(R) = 0$. Theorem is proved.

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Տեղեկացված վիճակագրի կողմից կամայականորեն փոփոխվող մարկովյան աղբյուրի վերաբերյալ բազմակի վարկածների ստուգումը և աղբյուրի կոդավորման հուսալիության ֆունկցիայի գնահատումը

Ն. Գրիգորյան

Ամփոփում

Լուծված է կամայականորեն փոփոխվող կողմնակի ինֆորմացիայով մարկովյան աղբյուրի մոդելի համար բազմակի վարկածների ստուգման խնդիրը: $M(\geq\ .2)$ հավանականային բաշխումները հայտնի են, և օրյենքը կամայականորեն ընդունում է դրանցից որևէ մեկը: Այս մոդելի համար ուսումնասիրվել է բոլոր հնարավոր զույգերի սխալների հավանականությունների ցուցիչների (հուսալիությունների) փոխկախվածությունը: Երկու վարկածների դեպքը ընդհատ առանց հիշողության կապուղու համար երբ վիճակների հաջորդականությունը անհայտ է ընդունում որոշողին, դիտարկվել է Ֆուի և Շենի կողմից: Իսկ նույն մոդելի համար, երբ վիճակների հաջորդականությունը հայտնի է վիճակագրին դիտարկվել է Ալսվեդեի, Հարությունյանի և Ալոյանի կողմից: Ինչպես Ֆուն և Շենը, մենք նույնպես ստացել ենք կողմնակի ինֆորմացիայով կամայականորեն փոփոխվող աղբյուրի համար արագություն-հուսալիություն և հուսալիություն-արագություն ֆունկցիաները: