

Vacuum fluxes from a brane in de Sitter spacetime with compact dimensions

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Abstract. We investigate the vacuum expectation value of the energy flux density for a complex scalar field in de Sitter spacetime with an arbitrary number of toroidally compact spatial dimension and in the presence of a brane. Quasiperiodicity conditions with arbitrary phases are imposed along compact dimensions and on the brane the field obeys Robin boundary condition. Depending on the values of the parameters in the problem, the flux can be directed from the brane or to the brane. The behavior of the flux density in various asymptotic regions is investigated. It has been shown that the energy flux density is an even periodic function of magnetic fluxes enclosed by compact dimensions with the period equal to flux quantum.

Keywords: de Sitter spacetime, quantum scalar field, topological Casimir effect

1. Introduction

De Sitter spacetime is among the most frequently used background geometries in quantum field theory on curved spacetime. There are several reasons for that. First of all, de Sitter spacetime is maximally symmetric and a large number of field theoretical problems are exactly solvable on its background. This allows to form an idea of the effect of the gravitational field on various physical processes for more general background geometries. De Sitter spacetime plays an important role in cosmology. In inflationary cosmology the early stages of the cosmological expansion are described by de Sitter metric tensor and this provides a natural solution for a number of problems in standard cosmology (for review see [1]). The recent observations of the accelerated expansion of the Universe at present epoch [2] show that the properties of the source driving that expansion (dark energy) are very close to those for the cosmological constant. If the Universe expansion will continue without a phase transition to another dominating source of the expansion, then de Sitter spacetime will serve as a future attractor for the geometry of the Universe.

In the present paper we consider quantum effects for a scalar field in background geometry that is de Sitterian only locally, assuming that a part of spatial dimensions are toroidally compactified (for motivations of this type of geometry from the string theory perspective see [3]). The latter type of compactification leads to changes in global properties of the vacuum state for quantum fields and as a result of that the vacuum expectation values (VEVs) of various physical observables are changed as well. This effect is known as the topological Casimir effect and has been investigated for various types of topologies and fields. The VEVs of the energy-momentum tensor for scalar and fermionic fields in de Sitter spacetime with compact dimensions were studied in [4]-[6]. The compactification of spatial dimensions imposes periodicity conditions on the fields. Here we will consider an additional condition imposed by the presence of a flat brane. The latter gives rise to the shifts in the VEVs of local physical observables describing the vacuum properties. Note that the boundary-induced effects in de Sitter spacetime in the absence of compact dimensions have been previously discussed in [7]-[10] for planar and

spherical boundaries. The vacuum current densities for charged scalar and fermionic fields in dS spacetime with toroidally compact dimensions were investigated in [11]. The VEV of the energy-momentum tensor in this background is considered in [12].

2. Background geometry and the Hadamard function

By taking into account that a large number of physical models describing the fundamental interactions are formulated in higher dimensional spacetime, we will not specify the number D for spatial dimensions described by coordinates x^l . For de Sitter spacetime, the corresponding metric tensor in inflationary coordinates is given by the interval

$$ds^2 = dt^2 - e^{2t/\alpha} \sum_{l=1}^D (dx^l)^2, \quad (2.1)$$

where $\alpha = \sqrt{D(D-1)\Lambda/2}$ and Λ is the cosmological constant. Although the local geometry we plan to consider is described by (2.1), the topology will be different: for the coordinates $\mathbf{x}_p = (x^1, \dots, x^p)$ one has $-\infty < x^l < \infty$, $l=1, \dots, p$, and the coordinates $\mathbf{x}_q = (x^{p+1}, \dots, x^D)$, with $q = D - p$, are compactified to circles with the lengths L_l : $0 \leq x^l \leq L_l$, $l = p+1, \dots, D$. Hence, we consider the spatial topology $R^p \times (S^1)^q$.

Because of the nontrivial topology, the periodicity conditions for quantum fields should be specified along compact dimensions. For a complex scalar field $\varphi(x)$, here we will consider the conditions

$$\varphi(t, \mathbf{x}_p, \mathbf{x}_q + L_l \mathbf{e}_l) = e^{i\alpha_l} \varphi(t, \mathbf{x}_p, \mathbf{x}_q), \quad (2.2)$$

where \mathbf{e}_l is the unit vector along the dimension x^l , $l=1, \dots, p$ and α_l are constant. The corresponding field equation reads

$$(D_\mu D^\mu + m^2 + \xi R)\varphi(x) = 0, \quad (2.3)$$

with $D_\mu = \nabla_\mu + ieA_\mu$ and R is the Ricci scalar. For a classical gauge field A_μ we assume the simplest configuration $A_\mu = \text{const}$. Though the corresponding strength is zero, due to the nontrivial topology of the background spacetime, this configuration will have effects on the vacuum properties. Additionally, the presence of a planar boundary located at $x^p = 0$ will be assumed. On the boundary we will take the condition of the Robin type

$$(1 + \beta D_p)\varphi = 0, \quad x^p = 0, \quad (2.4)$$

with a constant coefficient β with dimension of length. This can be considered as a simple model for a brane with the parameter β related to the brane thickness (for brane models in dS

spacetime see, for example, [13] and references therein). The constant gauge field is removed from the equations (2.3) and (2.4) by a gauge transformation $\phi'(x) = e^{-ie\omega(x)}\phi(x)$, $A'_\mu = A_\mu + \partial_\mu\omega(x)$ with the function $\omega = -A_\mu x^\mu$. Though in this gauge $A'_\mu = 0$, the field A_μ does not disappear from the problem. It appears in the quasiperiodicity conditions for the new field: $\phi'(t, \mathbf{x}_p, \mathbf{x}_q + L_l \mathbf{e}_l) = e^{i\tilde{\alpha}_l} \phi'(t, \mathbf{x}_p, \mathbf{x}_q)$ with the phase $\tilde{\alpha}_l = \alpha_l + eA_l L_l$. In what follows we will work in the new gauge omitting the primes for a scalar field.

The vacuum expectation values for physical observables bilinear in the field operator can be found by using the Hadamard function defined as

$$G^{(1)}(x, x') = \langle 0 | \phi(x)\phi^\dagger(x') + \phi^\dagger(x')\phi(x) | 0 \rangle, \quad (2.5)$$

where $|0\rangle$ is the vacuum state. The choice of the vacuum state in de Sitter spacetime is not unique. Here we assume that the field is prepared in the Bunch-Davies vacuum state [14]. For this state, in [15] the Hadamard function is decomposed as

$$G^{(1)}(x, x') = G_0^{(1)}(x, x') + G_b^{(1)}(x, x'), \quad (2.6)$$

where $G_0^{(1)}(x, x')$ is the Hadamard function in the absence of the brane and the part

$$G_b^{(1)}(x, x') = \frac{(\eta\eta')^{D/2}}{2^p \pi^p V_q \alpha^{D-1}} \int d\mathbf{k}_{p-1} e^{i\mathbf{k}_{p-1}\Delta\mathbf{x}_{p-1}} \sum_{\mathbf{n}_q} e^{i\mathbf{k}_q\Delta\mathbf{x}_q} \int_k^\infty du e^{-u(x^p+x^{p'})} \frac{\beta u + 1}{\beta u - 1} \times \left\{ K_\nu(\eta y) [I_{-\nu}(\eta' y) + I_\nu(\eta' y)] + [I_{-\nu}(\eta y) + I_\nu(\eta y)] K_\nu(\eta' y) \right\}_{y=\sqrt{u^2-k^2}}, \quad (2.7)$$

is induced by the presence of the brane. Here, $V_q = \prod_{l=p+1}^D L_l$ is the volume of the compact subspace, $I_\nu(x)$ and $K_\nu(x)$ are the modified Bessel functions, $\mathbf{k}_{p-1} = (k_1, \dots, k_{p-1})$, $\mathbf{x}_{p-1} = (x^1, \dots, x^{p-1})$, $\mathbf{k}_q = (k_{p+1}, \dots, k_D)$, $k = \sqrt{\mathbf{k}_{p-1}^2 + \mathbf{k}_q^2}$ and

$$\nu = \sqrt{D^2/4 - D(D+1)\xi - m^2\alpha^2}. \quad (2.8)$$

The summation goes over $\mathbf{n}_q = (n_{p+1}, \dots, n_D)$ with $n_l = 0, \pm 1, \pm 2, \dots$, $l = p+1, \dots, D$. The eigenvalues of the components of the momentum along compact dimensions are quantized by the periodicity conditions (2.2) and are given by

$$k_l = (2\pi n_l + \tilde{\alpha}_l) / L_l. \quad (2.9)$$

In (2.7) we have introduced $\eta = |\tau|$, where $\tau = -\alpha e^{-t/\alpha}$, $-\infty < \tau < 0$, is the conformal time. In terms of the latter the line element is written in a conformal flat form $ds^2 = (\alpha/\tau)^2 \eta_{\mu\nu} dx^\mu dx^\nu$, with $\eta_{\mu\nu}$ being the Minkowskian metric tensor. Based on (2.7), the vacuum polarization induced

by a brane has been investigated in [16]. The boundary-induced effects for the electromagnetic field in dS spacetime have been discussed in [17].

3. Energy flux

Here we are interested in the VEV of the energy flux along the direction perpendicular to the brane. It is determined by the VEV of the component T_{0p} of the energy-momentum tensor $T_{\mu\lambda}$ for the scalar field $\varphi(x)$. The VEVs of the remaining off-diagonal components of the energy-momentum tensor vanish. In the problem under consideration the VEV is expressed in terms of the Hadamard function as

$$\langle T_{0p} \rangle = \frac{1}{2} \lim_{x' \rightarrow x} \partial_0 \partial'_p G^{(1)}(x, x') - \frac{1}{2} \xi \nabla_0 \nabla_p G^{(1)}(x, x). \quad (3.1)$$

It can also be presented in the form

$$\langle T_{0p} \rangle = -\frac{1}{2\eta} \partial_p \left[\left(\frac{1}{4} - \xi \right) \eta \partial_\eta - \xi \right] G^{(1)}(x, x). \quad (3.2)$$

In the geometry without the brane the Hadamard function $G_0^{(1)}(x, x')$ in the coincidence limit $x'^p \rightarrow x^p$ does not depend on x^p (this is a consequence of homogeneity of the corresponding geometry) and the corresponding energy flux vanishes, $\langle T_{0p} \rangle_0 = 0$. Hence, the energy flux is induced by the brane.

By using the expression (2.7) for the brane-induced part of the Hadamard function, in the conformal coordinates (τ, x^l) for the mixed component $\langle T_0^p \rangle$ we get the expression

$$\langle T_0^p \rangle = \frac{\pi^{-(p+1)/2} (\eta / \alpha)^{D+1}}{2^{p-2} \Gamma((p-1)/2) V_q} \sum_{\mathbf{n}_q} \int_0^\infty dy y F_\nu(\eta y) \times \int_0^\infty dw w^{p-2} \frac{\beta u + 1}{\beta u - 1} e^{-2x^p u} \Big|_{u=\sqrt{y^2+w^2+\mathbf{k}_q^2}}, \quad (3.3)$$

where $k_{\mathbf{n}_q}^2 = \mathbf{k}_q^2$ and

$$F_\nu(x) = \left[\left(\xi - \frac{1}{4} \right) x \partial_x + (D+1)\xi - \frac{D}{4} \right] [I_{-\nu}(x) + I_\nu(x)] K_\nu(x). \quad (3.4)$$

For the component $\langle T_{(s)0}^p \rangle$ in synchronous coordinates (τ, x^l) one has $\langle T_{(s)0}^p \rangle = (\eta / \alpha) \langle T_0^p \rangle$. The energy flux through a $(D-1)$ -dimensional hypersurface ∂M enclosing the volume M is given by the expression

$$\int_{\partial M} d^{D-1} x \sqrt{h} n_l \langle T_{(s)0}^l \rangle, \quad (3.5)$$

where n_l is the external normal to the boundary ∂M normalized by the condition $\gamma^{nl}n_n n_l = 1$ with γ^{nl} being the spatial metric tensor. In (3.5), h is the determinant of the induced metric $h_{nl} = \gamma_{nl} - n_n n_l$. In the special case of a planar boundary at $x^p = x_{(0)}^p$, parallel to the brane (with the volume M corresponding to $0 \leq x^p \leq x_{(0)}^p$), one has $n_l = (\alpha/\eta)\delta_l^p$ and the flux is expressed as $(\alpha/\eta)^D S_{\partial M} \langle T_{(s)0}^p \rangle_{x^p=x_{(0)}^p} = (\alpha/\eta)^{D-1} S_{\partial M} \langle T_0^p \rangle_{x^p=x_{(0)}^p}$, where $S_{\partial M}$ is the coordinate surface area. Note that the proper surface area, measured by an observer at $x^p = x_{(0)}^p$ is given as $(\alpha/\eta)^{D-1} S_{\partial M}$. Hence, the VEV $\langle T_0^p \rangle_{x^p=x_{(0)}^p}$ determines the energy flux density per unit proper surface area.

An alternative expression for the energy flux density is obtained introducing polar coordinates (r, θ) in the plane (y, w) and passing to the integration over $u = \cos \theta$ instead of θ :

$$\begin{aligned} \langle T_0^p \rangle &= \frac{\pi^{-(p+1)/2} (\eta/\alpha)^{D+1}}{2^{p-2} \Gamma((p-1)/2) V_q} \sum_{\mathbf{n}_q} \int_0^\infty dr r^p e^{-2x^p \sqrt{r^2 + k_{\mathbf{n}_q}^2}} \\ &\times \frac{\beta \sqrt{r^2 + k_{\mathbf{n}_q}^2} + 1}{\beta \sqrt{r^2 + k_{\mathbf{n}_q}^2} - 1} \int_0^1 du u (1-u^2)^{\frac{p-3}{2}} F_\nu(\eta r u). \end{aligned} \quad (3.6)$$

The corresponding energy flux density is an even periodic function of the magnetic fluxes enclosed by compact dimensions with a period equal to the flux quantum. The flux density (3.6) depends on L_l , β , x^p and η through the dimensionless ratios L_l/η , β/η , x^p/η . Note that the proper length of the l th compact dimension is given by $L_l^{(p)} = \alpha L_l/\eta$. Hence, the ratio L_l/η is the proper length of the compact dimension in units of the dS curvature scale α . In a similar way, x^p/η is the proper distance from the brane in units of α . For a conformally coupled massless field one has $\nu = 1/2$ and $[I_\nu(y) + I_{-\nu}(y)]K_\nu(y) = 1/y$. It is seen from (3.6) that in this case the energy flux vanishes. This result could also be directly obtained on the base of that for a conformally coupled field the problem in locally dS spacetime is conformally related to the corresponding problem in locally Minkowskian spacetime and in the latter problem the energy flux is zero.

In special cases of Dirichlet and Neumann boundary conditions it is convenient to use the representation (3.3). The integral over w is expressed in terms of the Macdonald function [18] and we get

$$\langle T_0^p \rangle = \mp \frac{4(\eta/\alpha)^{D+1}}{(2\pi)^{p/2+1} V_q (2x^p)^{p-1}} \sum_{\mathbf{n}_q} \int_0^\infty dy y g_{p/2}(2x^p \sqrt{y^2 + \mathbf{k}_q^2}) F_\nu(\eta y), \quad (3.7)$$

where the upper/lower signs correspond to Dirichlet/Neumann boundary conditions and we have introduced the notation

$$g_u(x) = x^u K_u(x). \quad (3.8)$$

Depending on the values of the parameters the flux densities (3.6) and (3.7) can be either positive or negative. For the positive (negative) sign the energy flux is directed from (to) the brane.

Let us consider the asymptotic behavior of the flux density in the limiting cases. For large values of the lengths of compact dimensions, $L_l \gg x^p$, the dominant contribution to (3.3) comes from the terms of the series with large values of $|n_l|$, $l = p+1, \dots, D$. To the leading order, we can replace the summation over \mathbf{n}_q by integration in accordance with

$$\sum_{\mathbf{n}_q} \rightarrow \frac{V_q}{(2\pi)^q} \int d\mathbf{k}_q, \quad (3.9)$$

where the integrations go over ranges $-\infty < k_l < \infty$, $l = p+1, \dots, D$. After integration over the angular part of \mathbf{k}_q , the leading term in the asymptotic expansion of the flux density is presented as

$$\langle T_0^p \rangle \approx \frac{\pi^{-(D+1)/2} (\eta/\alpha)^{D+1}}{2^{D-2} \Gamma((D-1)/2)} \int_0^\infty dy y F_\nu(\eta y) \int_y^\infty du u (u^2 - y^2)^{\frac{D-3}{2}} \frac{\beta u + 1}{\beta u - 1} e^{-2x^p u}. \quad (3.10)$$

The expression in the right-hand side coincides with the flux density from a planar brane in $(D+1)$ -dimensional dS spacetime in the absence of compact dimensions [7].

At early stages of the expansion, one has $\tau \rightarrow -\infty$ and η is large. By taking into account that for $y \gg 1$ one has $[I_\nu(y) + I_{-\nu}(y)] K_\nu(y) \approx 1/y$, to the leading order we get

$$\langle T_0^p \rangle \approx \frac{2D(\xi - \xi_D)(\eta/\alpha)^D}{(4\pi)^{p/2} \Gamma(p/2) \alpha V_q} \sum_{\mathbf{n}_q} \int_{k_{\mathbf{n}_q}}^\infty du u (u^2 - k_{\mathbf{n}_q}^2)^{p/2-1} e^{-2x^p u} \frac{\beta u + 1}{\beta u - 1}. \quad (3.11)$$

For Dirichlet and Neumann boundary conditions, after evaluation of the integral, the leading terms is presented as

$$\langle T_0^p \rangle \approx \mp \frac{2D(\xi - \xi_D)(\eta/\alpha)^D}{(2\pi)^{(p+1)/2} \alpha V_q (2x^p)^p} \sum_{\mathbf{n}_q} g_{(p+1)/2}(2x^p k_{\mathbf{n}_q}). \quad (3.12)$$

For a conformally coupled field the next term should be kept in the asymptotic expansion of the function $F_\nu(y)$.

At late stages of the expansion one has $\eta \rightarrow 0$. The asymptotic behavior of the flux density is essentially different for positive and purely imaginary values of the parameter ν . For $\nu > 0$ we use the asymptotic expression

$$[I_\nu(y) + I_{-\nu}(y)] K_\nu(y) \approx \frac{\pi}{2 \sin(\nu\pi)} \frac{(y/2)^{-2\nu}}{\Gamma^2(1-\nu)}. \quad (3.13)$$

For the leading term this gives

$$\begin{aligned} \langle T_0^p \rangle &\approx 2^{2\nu+1} \Gamma(\nu) (\eta / \alpha)^{D+1-2\nu} \frac{2\nu(1/4 - \xi) + (D+1)\xi - D/4}{(4\pi)^{(p+1)/2} \Gamma((p+1)/2 - \nu) \alpha^{2\nu} V_q} \\ &\times \sum_{\mathbf{n}_q} \int_{k_{\mathbf{n}_q}}^{\infty} du u (u^2 - k_{\mathbf{n}_q}^2)^{\frac{p-1}{2} - \nu} e^{-2x^p u} \frac{\beta u + 1}{\beta u - 1}. \end{aligned} \quad (3.14)$$

This expression is further simplified for Dirichlet and Neumann boundary conditions:

$$\langle T_0^p \rangle \approx \mp 2^{p/2+\nu+2} \Gamma(\nu) (\eta / \alpha)^{D+1-2\nu} \frac{2\nu(1/4 - \xi) + (D+1)\xi - D/4}{(4\pi)^{p/2+1} \alpha^{2\nu} V_q} \sum_{\mathbf{n}_q} g_{p/2+1-\nu}(2x^p k_{\mathbf{n}_q}). \quad (3.15)$$

In this case, the energy flux density decays monotonically. In terms of the synchronous time coordinate t one has $\langle T_0^p \rangle \propto \exp[-(D+1-2\nu)t]$ for $t \rightarrow +\infty$.

For imaginary values of $\nu = i|\nu|$ we use the asymptotic expression

$$[I_\nu(y) + I_{-\nu}(y)] K_\nu(y) \approx \frac{\pi}{\sinh(\pi|\nu|)} \operatorname{Im} \left[\frac{(y/2)^{-2i|\nu|}}{\Gamma^2(1-i|\nu|)} \right]. \quad (3.16)$$

For the leading term of the asymptotic expansion of the flux density this gives

$$\langle T_0^p \rangle \approx -\frac{(\eta / \alpha)^{D+1}}{2^{p-1} \pi^{(p+1)/2} V_q} \sum_{\mathbf{n}_q} A(x^p, k_{\mathbf{n}_q}) \sin[2|\nu| \ln(\eta/2) - \phi(x^p, k_{\mathbf{n}_q})], \quad (3.17)$$

where $A(x^p, k_{\mathbf{n}_q})$ and $\phi(x^p, k_{\mathbf{n}_q})$ are defined by the relation

$$\begin{aligned} A(x^p, k_{\mathbf{n}_q}) e^{i\phi(x^p, k_{\mathbf{n}_q})} &= \Gamma(i|\nu|) \frac{(D+1)\xi - D/4 - 2i|\nu|(\xi - 1/4)}{\Gamma((p+1)/2 - i|\nu|)} \\ &\times \int_{k_{\mathbf{n}_q}}^{\infty} du u (u^2 - k_{\mathbf{n}_q}^2)^{\frac{p-1}{2} - i|\nu|} e^{-2x^p u} \frac{\beta u + 1}{\beta u - 1}, \end{aligned} \quad (3.18)$$

with $A(x^p, k_{\mathbf{n}_q})$ being the modulus of the right-hand side in (3.18). As seen, in this case the decay of the energy flux, as a function of η , is oscillatory.

Now we turn to numerical examples for behavior of the flux density in the case of conformally coupled scalar field with Dirichlet boundary condition. These examples are considered for $D = 4$ model with a single compact dimension of the length L and with the phase $\tilde{\alpha}_D = \tilde{\alpha}$. Figure 1 displays the dependence of the flux density on the proper distance from the brane in units of the curvature scale α . The graphs are plotted for $m\alpha = 1$, $\tilde{\alpha} = \pi/4$ and for several values of the ratio L/η (numbers near the curves). As seen, for the example considered

the energy flux is directed from the brane. For Neumann boundary condition, $\langle T_0^p \rangle$ has opposite sign and the flux is directed to the brane.

In Figure 2 we have plotted the flux density versus the phase $\tilde{\alpha}$ in the range of one period, for a fixed proper distance from the brane corresponding to $x^p/\eta=1$. The numbers near the curves are the values of L/η and the values for the other parameters are the same as those for Figure 1. For small values of L/η , the flux density is strongly peaked near $\tilde{\alpha}=0$. For large values of L/η , the dependence of the flux density on the phase is weak and it tends to the corresponding value in 5-dimensional dS spacetime in the absence of compact dimensions.

The dependence of the flux density on the proper length of the compact dimension (in units of the curvature radius α) is shown in Figure 3 for $x^p/\eta=1$ and $m\alpha=1$. The numbers near the curves correspond to the values of the phase $\tilde{\alpha}/2\pi$. As seen, the behavior of the flux for small values of L/η is essentially different for the cases $\tilde{\alpha}=0$ and $\tilde{\alpha}\neq 0$. In the first case, for small L/η , the dominant contribution comes from the zero mode (corresponding to the zero momentum along the compact dimension, $k_D=0$ or $n_D=0$) and from the expressions given above it can be seen that the leading term coincides with the corresponding flux in the model with $D=3$ multiplied by L . For $\tilde{\alpha}\neq 0$ the zero mode is absent and the flux density for small values of L/η is exponentially suppressed. This feature can also be seen from the formulas for the flux density given above.

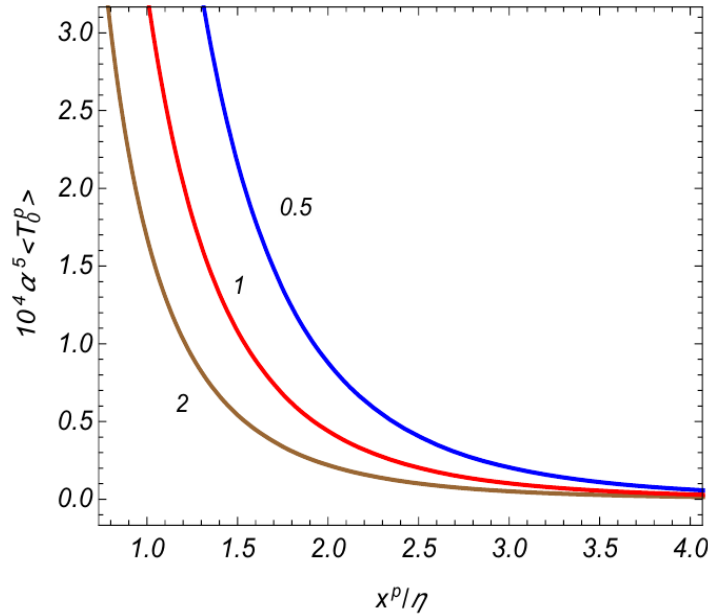


Figure 1: The energy flux density versus the distance from the brane for a conformally coupled scalar field with Dirichlet boundary condition. The graphs are plotted for the $D=4$ model with a single compact dimensions and for $m\alpha=1$, $\tilde{\alpha}=\pi/4$. The numbers near the curves correspond to the values of L/η .

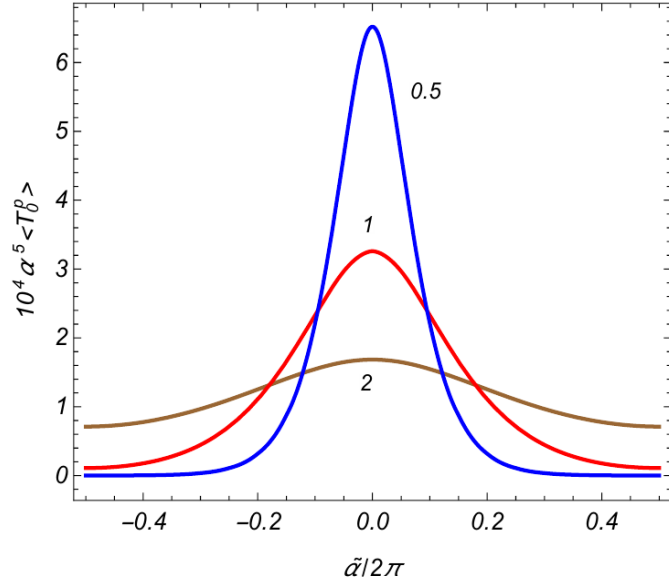


Figure 2: The flux density at a fixed distance from the brane $x^p / \eta = 1$, as a function of the phase in the periodicity condition along compact dimensions. The numbers near the curves are the values of L/η and the remaining parameters are the same as those for Figure 1.

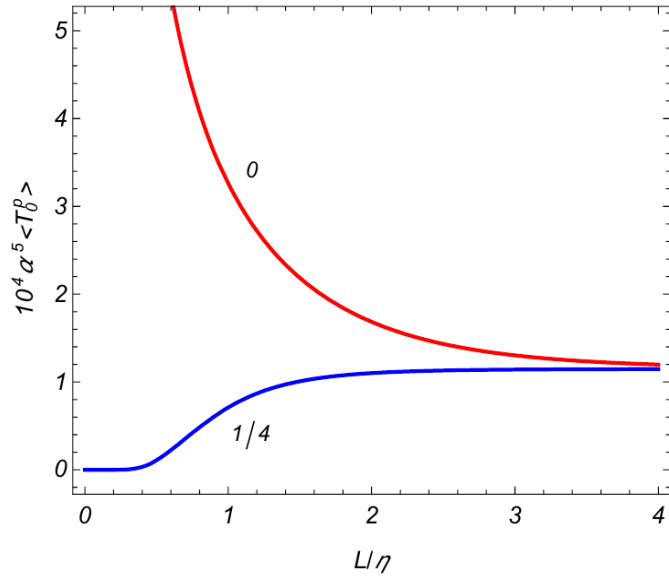


Figure 3: The dependence of the energy flux on the length of the compact dimension for $\tilde{\alpha} / 2\pi = 0, 1/4$. The values of the remaining parameters are the same as those in Figures 1 and 2.

4. Conclusion

Thus, on the example of a complex scalar field with an arbitrary curvature coupling parameter, we have shown that the spatial expansion and the presence of a planar brane give rise the energy flux in the vacuum state along the direction normal to the brane. In order to have an exactly solvable problem, highly symmetric background geometry is considered corresponding to locally dS spacetime with toroidally compact spatial dimensions. Along those dimensions

quasiperiodicity conditions were imposed with arbitrary phases and on the brane the field is constrained by Robin boundary condition. As important special cases, the latter includes Dirichlet and Neumann conditions. In addition, we have assumed the presence of a constant gauge field. Though the corresponding field tensor is zero, because of the nontrivial spatial topology, the VEVs of physical observables depend on the components of the gauge potential along compact dimensions.

Two alternative expressions for the VEV of the flux density are provided, given by (3.3) and (3.6). These expressions are further simplified for Dirichlet and Neumann boundary conditions (corresponding to upper and lower signs in (3.7)). The flux is an even periodic function of magnetic fluxes enclosed by compact dimensions with the period equal to flux quantum. Depending on the values of the parameters, the flux can be either positive or negative, corresponding to the flux direction from or to the brane, respectively. In the early stages of the cosmological expansion, corresponding to large values of the parameter η , the flux density $\langle T_0^p \rangle$ behaves as $(\eta/\alpha)^D$. At late stages of the expansion the parameter η is small the behavior of the flux density is essentially different for positive and imaginary values of ν , defined by (2.8). In the first case, corresponding to relatively small values of $m\alpha$ (the mass of the field quanta measured in units of the inverse curvature scale $1/\alpha$), the flux density decays monotonically, like $(\eta/\alpha)^{D+1-2\nu}$, $\eta \rightarrow 0$. For purely imaginary ν , the decay is oscillatory. The numerical examples, presented in Figures 1-3 were given for a conformally coupled field with Dirichlet boundary conditions in the $D=4$ model with a single compact dimensions. In particular, these data show that for small proper length of compact dimension (compared with the curvature radius and the distance from the brane) the behavior of the energy flux essentially differs for zero and nonzero values of the phase $\tilde{\alpha}$. In the latter case the flux density is exponentially small.

Author Contributions

The authors equally contributed to all steps of the paper preparation.

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Conflicts of interest

The authors declare no conflict of interest.

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