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Dependence of the Transmission and Reflection Bands of an Oblique Incident Electromagnetic Wave on the Parameters of One Dimensional Periodic Structure

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Abstract. We investigate the behavior of the transmission and reflection bands for a plane electromagnetic wave falling obliquely on an ideal layered structure. The width of allowed bands as function of the structure parameters and the wave incident angle is considered. It is shown, that in general case the bandwidth is a non-monotonous function of the problem parameters. The conditions defining the contact of the transmission bands for the both types of wave polarizations are derived. It is proved that irrespective of the wave polarization, the transmission coefficient equals to unit at the contact points. It is also shown that in the case of Brewster angle the contact of the allowed *p* bands is also take place.

Keywords: one-dimensional periodic structure, transmission band, *s* and *p* waves

The problem of wave propagation through a one-dimensional periodic structure has a long-term history and the forms of transmission and reflection coefficients for waves of different natures are well-known [1-6]. To introduce the formulas of the scattering amplitudes let us consider a periodic system consisting of N identical and periodically ranged layers. Below we consider electromagnetic wave propagation, where, as it is known, any plane wave process can be presented as a superposition of so-called s (TE – tangential electric) and p (TM – tangential magnetic) waves. For the first of them, the field electric component is perpendicular to the incident plane and for the second one, the magnetic component is perpendicular to the same plane.

We denote the transmission and reflection amplitudes of the system as T_N and R_N, correspondingly. In case of need for different types of polarization we will make corresponding notations in a quantity index: T_N^s, T_N^p and so on. Since the structure elements, i.e. the layers of the periodic system are identical they have a same value of transmission amplitudes and their reflection amplitudes differ from each other by the phase factor (see, for example [7]):

$$t = t_1 = t_2 \cdots = t_N \ , \ r = r_1 = r_2 \exp\{-\mathrm{i}\,2kL\} = \cdots = r_N \exp\{-\mathrm{i}\,2NkL\}, \tag{1}$$

where t_j, r_j ($j = 1, 2, \cdots, N$) are the transmission and reflection amplitudes of j-th layer of a periodic structure, L is its period. Here

$$k = \omega \cos\alpha / c, \tag{2}$$

where ω is the field frequency, c - is light velocity and α is the incident angle.

For the general case of an absorbing media, the analytic expressions for scattering amplitudes are the following [8, 9]:

$$\frac{1}{T_N(k)}=\exp\{ikLN\}\left(\cos(\mu N)+\frac{1}{2}\left(\frac{\exp\{-ikL\}}{t(k)}-\frac{\exp\{ikL\}}{t(-k)}\right)\frac{\sin(\mu N)}{\sin(\mu)}\right), \tag{3}$$

$$\frac{R_N(k)}{T_N(k)}=\exp\{ikL(N-1)\}\frac{r(k)}{t(k)}\frac{\sin(\mu N)}{\sin(\mu)}, \tag{4}$$

where the following notation is made:

$$\cos\mu=\frac{1}{2}\left(\frac{\exp\{-ikL\}}{t(k)}+\frac{\exp\{ikL\}}{t(-k)}\right). \tag{5}$$

It is important to note that for the case of a non-dissipative medium the presented formulas are transformed to the well-known classical result [1]. For this case the change of wave numbers sign on opposite is equivalent to the complex conjugate action [10]:

$$t(-k)=t^*(k)\text{ and } r(-k)=r^*(k). \tag{6}$$

Indeed, by using Eq. (6), for Eq. (5) one can get:

$$\cos\mu=\mathrm{Re}\left(\exp\{-ikL\}/t(k)\right). \tag{7}$$

It is easy to see that for a non-dissipative medium the dimensionless quantity μ (see Eq. (7)) takes real or imaginary value since the expression $\mathrm{Re}\left(\exp\{-ikL\}/t(k)\right)$ is a purely real quantity. However, in the general case of an absorbing medium (see Eq.(5)) the expression $\exp\{-ikL\}/t(k)+\exp\{ikL\}/t(-k)$, which is an even function k ,can take the complex value, therefore, μ is a complex quantity.

Below we consider the transmission and reflection bands for a non-absorbing media for that case when from Eqs. (3), (4) and (7) one can write down [1]

$$|T_N(k)|^2+|R_N(k)|^2=1$$

and

$$|T_N(k)|^2=\frac{\sin^2(\mu)/\sin^2(\mu N)}{|r(k)/t(k)|^2+\sin^2(\mu)/\sin^2(\mu N)}. \tag{8}$$

As it follows from Eq.(8), the transmission and reflection bands are defined by means of the quantity μ. Indeed, when μ is an imaginary quantity ($\mu=i\gamma$, γ is a real quantity) when (8) takes the form of:

$$|T_N(k)|^2=\frac{\sinh^2(\gamma)/\sinh^2(\gamma N)}{|r(k)/t(k)|^2+\sinh^2(\gamma)/\sinh^2(\gamma N)} \tag{9}$$

and it is easy to see that when $N \to \infty$ the transmission coefficient vanishes. Given Eqs.(8) and (9) take place for the transmission coefficients of both s and polarized waves.

We conduct our consideration for a periodic system having a structure element in the form of a one homogenous layer with a width equals to d. It is clear that d should be less that the system period L. $b = L - d$ is the distance between the structure layers. We assume that the space outside of the periodic structure and the regions between the layers are homogeneous media and they have the same dielectric permeability equals to ε_0. Directing the X-axis perpendicular to the structure layers and taking a coordinate of the initial point of the first layer as $x = 0$, the dielectric permeability of the structure can be written as:

$$\varepsilon(x) = \begin{cases} \varepsilon_0, \;\; x < 0, \\ \;\;\dots\dots\dots \\ \varepsilon, \;\; wnen \;\; (n-1)L < x < (n-1)L + d, \\ \varepsilon_0, \;\; when \;\; (n-1)L + d < x < nL, \\ \\ \varepsilon_0, x > NL - b. \end{cases} \tag{10}$$

where ε is a dielectric permeability of the homogenous layer.

For s and p polarized waves the transmission amplitudes of a homogenous layer, which is the structure of the chosen periodic system, are defined by formulas [1]:

$$\frac{1}{t^s(k)} = \exp(ikd)\left[\cos(iqd) - i\frac{q^2 + k^2}{2qk}\sin(iqd)\right], \tag{11}$$

$$\frac{1}{t^p} = \exp(ikd)\left[\cos(iqd) - i\frac{(\varepsilon_0/\varepsilon)q^2 + (\varepsilon/\varepsilon_0)k^2}{2qk}\sin(iqd)\right], \tag{12}$$

where q is determined by using the Snell's low:

$$q = \frac{\omega}{c}\sqrt{\varepsilon}\cos\gamma \text{ and } \sqrt{\varepsilon_0}\sin\alpha = \sqrt{\varepsilon}\sin\gamma. \tag{13}$$

It should be noted that when α is larger than the critical angle of the total reflection ($\alpha > \arcsin\sqrt{\varepsilon}$), then q is released as $q = i\chi$:

Inserting Eqs.(11) and (12) into Eq. (7) one can get (see, for example, [7]):

$$\cos\mu^s = \cos(kb)\cos(qd) - \frac{q^2 + k^2}{2kq}\sin(kb)\sin(qd), \tag{14}$$

$$\cos\mu^s = \cos(kb)\cos(qd) - \frac{q^2(\varepsilon_0/\varepsilon) + (\varepsilon/\varepsilon_0)k^2}{2kq}\sin(kb)\sin(qd). \tag{15}$$

As it was mentioned, the transmission and reflection bands are defined by mean of the quantity μ (see Eqs. (8), (9). So, when $|\cos\mu| \le 1$ (μ is a real) we have a transmission, in the case of $|\cos\mu|<1$ the structure reflects the radiation. In accordance with the above mentioned the band borders are defined by means of the following equations:

$$\cos\mu^{s,p}=1 \text{ and } \cos\mu^{s,p}=-1 \quad (16)$$

Using Eqs.(14), the borders of the s bands can be written down by following equations:

$$\cos(kb)\cos(qd)-\frac{q^2+k^2}{2kq}\sin(kb)\sin(qd)=1, \quad (17)$$

$$\cos(kb)\cos(qd)-\frac{q^2+k^2}{2kq}\sin(kb)\sin(qd)=-1 \quad (18)$$

and for the borders of the p bands one gets:

$$\cos(kb)\cos(qd)-\frac{q^2(\varepsilon_0/\varepsilon)+(\varepsilon/\varepsilon_0)k^2}{2kq}\sin(kb)\sin(qd)=1, \quad (19)$$

$$\cos(kb)\cos(qd)-\frac{q^2(\varepsilon_0/\varepsilon)+(\varepsilon/\varepsilon_0)k^2}{2kq}\sin(kb)\sin(qd)=-1. \quad (20)$$

It can be shown that for both s and p waves each the above written equations (17) -(20) is equivalent to two equations. Therefore, from Eq. (17) and Eq. (18) one can get the following equations determining the band borders for the s waves:

$$tg\left(kb/2\right)=-\frac{k}{q}tg\left(qd/2\right), tg\left(kb/2\right)=-\frac{q}{k}tg\left(qd/2\right), \quad (21)$$

and

$$tg\left(kb/2\right)=\frac{k}{q}ctg\left(qd/2\right), tg\left(kb/2\right)=\frac{q}{k}ctg\left(qd/2\right). \quad (22)$$

For the reflection bands of the p waves, from Eqs.(19) and (20) the following equations are obtained:

$$tg\left(kb/2\right)=-\frac{\varepsilon k}{\varepsilon_0 q}tg\left(qd/2\right), tg\left(kb/2\right)=-\frac{\varepsilon_0 q}{\varepsilon k}tg\left(qd/2\right), \quad (23)$$

and

$$tg\left(kb/2\right)=\frac{\varepsilon k}{\varepsilon_0 q}ctg\left(qd/2\right), tg\left(kb/2\right)=\frac{\varepsilon_0 q}{\varepsilon k}ctg\left(qd/2\right), \quad (24)$$

When the wave incidence angle exceeds the critical angle of total reflection, then $q=i\chi$ (see (13)) and Eqs. (21), (22) correspondingly take the form of:

$$tg\left(kb/2\right)=-\frac{k}{\chi}th\left(\chi d/2\right), tg\left(kb/2\right)=\frac{\chi}{k}th\left(\chi d/2\right), \quad (25)$$

$$tg\left(kb/2\right)=-\frac{k}{\chi}cth\left(\chi d/2\right), tg\left(kb/2\right)=\frac{\chi}{k}cth\left(\chi d/2\right). \tag{26}$$

Similarly, if in Eqs. (23), (24) q is replaced on $i\chi$ one gets:

$$tg\left(kb/2\right)=-\frac{\varepsilon k}{\varepsilon_0\chi}th\left(\chi d/2\right), tg\left(kb/2\right)=\frac{\varepsilon_0\chi}{\varepsilon k}th\left(\chi d/2\right), \tag{27}$$

$$tg\left(kb/2\right)=-\frac{\varepsilon k}{\varepsilon_0\chi}cth\left(\chi d/2\right), tg\left(kb/2\right)=\frac{\varepsilon_0\chi}{\varepsilon k_{0z}}cth\left(\chi d/2\right). \tag{28}$$

It is interesting to mention, that for values of the incidence angle larger than the total reflection one, when the layer width d increases $\left(\chi d>>1\right)$, the equations that determine borders of odd and even bands of reflection, for both cases of s and p waves, turn into each other. Indeed, if $\chi d>>1$, it is possible to put $th\left(\chi d/2\right)\approx cth\left(\chi d/2\right)\approx 1$, Eqs. (25), (26) take the same form:

$$tg\left(kb/2\right)=-\chi,\ tg\left(k_{0z}b/2\right)=\frac{\chi}{k}. \tag{29}$$

In the case of Eqs.(27), (28) when $\chi d>>1$ ($th\left(\chi d/2\right)\approx cth\left(\chi d/2\right)\approx 1$) one gets:

$$tg\left(kb/2\right)=-\frac{\varepsilon k}{\varepsilon_0\chi},\ tg\left(kb/2\right)=\frac{\varepsilon_0\chi}{\varepsilon k}. \tag{30}$$

Note that Eqs. (29) and (30) determine the spectrum of antisymmetric and symmetric modes for s and p polarized waves, which propagate in a wave-guide regime inside a homogeneous layer with a dielectric constant ε and thickness b. This result becomes clear if one takes into account that the condition $\left|\cos\mu^{s,p}\right|\le 1$ defines not only the waves transmission bands, but it also gives the region of frequencies when a wave can exist inside the ideal and both sides infinite structure without damping. Usually this region of frequencies is called as allowed band. In the total reflection regime, the wave energy is mainly concentrated in optically more dense layers. When the width of the layers increases, the filed energy concentrated inside the vacuum regions decreases. Due to this fact, the field inside an infinite periodic structure represents itself an infinite number of separate waves propagating in wave-guide regime inside the optical more dense layers.

Below we bring the results of numerical calculations, which show the dependence of the band boarders on the problem parameters. Figure 1 shows the locations of transmission and reflection bands, depending on the layer dielectric constant at normal incidence for four different values of dimensionless parameter b/d. As it is seen from the figure, the increase of the dielectric constant of the layer shifts the both transmission and reflection bands to the lower frequency region. From the given graphs, it also follows that when the distance between the layers becomes larger compared to their thickness the bandwidths have a tendency to decrease.

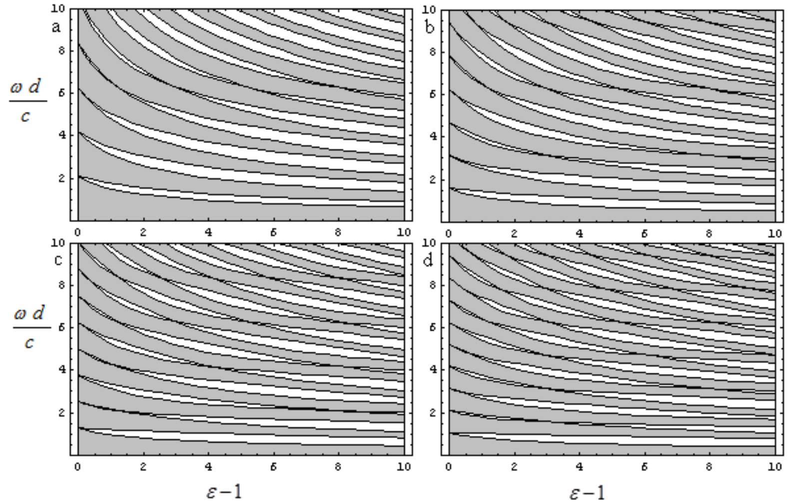

Fig. 1. Location of the transmission bands (darkened regions) depending on dielectric constant of the layer at normal incidence of wave in case of eight different values of b / d *:* 0.5, 1, 1.5, 2 (see figures from a) to d), correspondingly) and $\varepsilon_0 = 1$.

On Fig. 2, the band locations are represented depending on the dimensionless parameter d/b for four different values of layer dielectric constant. As it is seen from the figure, the increase of d/b (as in the case of an increase of ε (see Fig. 1)), leads to the shift of the transmission and reflection bands to the lower frequency region. It should be mentioned that at all considered cases the band contact effect is presented (see below).

On Fig. 3, locations of bands as a function of $\varepsilon - 1$ for s and p waves are plotted for two values of b / d and for two different values of the incident angle. The value of ε_0 was chosen more than the values of ε ($\varepsilon < \varepsilon_0 = 6$). Therefore, the regions of the layers correspond to optically less dense mediums than others. As it is seen from the graphs, generally the reflection bands decrease when the dielectric constant $\varepsilon - 1$ increases. However, there are small regions where this dependence has an inverse behavior. When ε tends to ε_0 all transmission bands merge and only one transmission band is obtained. Indeed, when $\varepsilon = \varepsilon_0$ then the layered system disappears and we have an infinite media where a wave of arbitrary frequency and polarization can freely propagate. Additionally, for the small values of ε the transmission or allowed bands transform to line and these lines correspond to the frequency spectrum for wave propagation in the wave-guide regime.

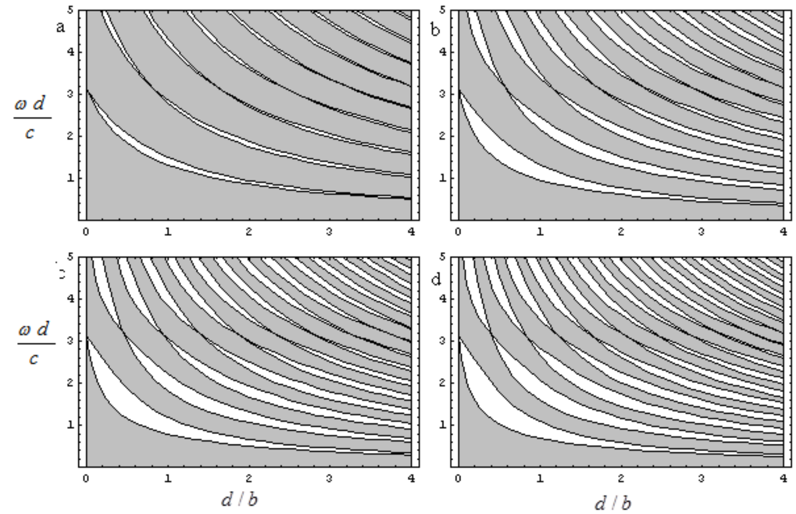

Fig. 2. The location of the transmission bands (darkened regions) at normal incidence of wave depending on dimensionless parameter d / b in case of eight different values of ε = 1.5, 3, 4 (see figures from a) to d), correspondingly) and $\varepsilon_0 = 1$.

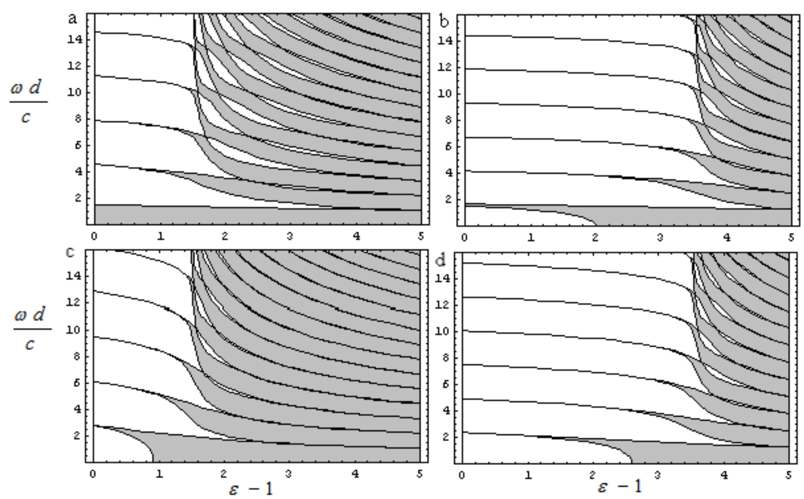

Fig. 3. The location of the transmission bands (darkened regions) as a function of $\tilde{\varepsilon}$ for s and p waves at two different values of b / d and for two different values of the incident angles: a) s- wave and b / d= 0.5 , $\alpha = 40^0$; b) s- wave and b / d = 1, $\alpha = 60^0$; c) p – wave and b / d = 0.5 , $\alpha = 40^0$; d) p- wave and b / d = 1, $\alpha = 60^0$ and $\varepsilon_0 = 6$.

Finally, let us consider the contact effect of transmission (allowed) bands, which are presented in all given Figures. First, let us note that a contact of allowed bands means that the forbidden bandwidth is zero, i.e. its boarders are contracted into a point. Each of Eqs.(21), (22) (s waves) or Eqs.(23), (24) (p wave) depending on values of the problem, parameters can define both the upper and lower band borders. It should be noted, that if an equation defines the lower (upper) forbidden band boarder on the left hand side of a contact point, then the same equation would define the upper (lower) boarder on the right hand side of the contact point.

The condition of a forbidden bandwidth being zero means, that the equations, defining different band boarders, give the same solution for certain parameters of the problem. From this follows, that for both *s* and *p* waves the mentioned condition has a same form. This condition is represented by two equations:

$$kb = n\pi\,,\ qd = m\pi\ , \tag{31}$$

where n, m are integer. From this result is followed that the contact of bands is possible if the problem parameters are connected only by the following expression:

$$\frac{\cos\alpha}{\sqrt{\varepsilon/\varepsilon_0 - \sin^2\alpha}} = \frac{d}{b}\frac{n}{m}, \tag{32}$$

The first equation of (31) shows that the contact of bands takes place at frequencies:

$$\omega_n = \frac{\pi n c}{b\sqrt{\varepsilon_0}\cos\alpha}\,. \tag{33}$$

Note, that when the parameters of the periodic structure are given then the value of $\cos\alpha$ should be found from Eq. (32) and it should be also mentioned that if both *n* and *m* are even or add simultaneously then Eqs. (21) and (23) have the same solution, otherwise the same solution have Eqs.(22) and (24).

To understand the physical consequences of allowed bands contact effect let us consider the behavior of the transmission coefficient Eq. (8) at the contact point. As it is well known, from the theory of wave propagation in periodic systems, the system length increases when band boarders leads to the total reflection. For the considered structure the transmission coefficient at the allowed band boarders, which are defined by equations $\cos\mu^{s,p} = \pm 1$ (see Eq. (16)), has the form [11]:

$$\left|T_N^{s,p}(k_n)\right|^2 = \frac{1}{1 + N^2\left|r^{s,p}(k_n)/t^{s,p}(k_n)\right|^2}\,. \tag{34}$$

where $k_n = \omega_n / c$ (see Eq. (33)). This expression obviously tends to zero when *N* increases, if only at the band contacts points $r^{s,p}(k_n) \neq 0$ .If $r^{s,p}(k_n) = 0$ the transmission coefficient is equal to the unit for an arbitrary *N*. It is easy to check that the reflection amplitudes of the homogenous layer vanishes ($r^{s,p}(k_n) = 0$) for the both *s* and *p* waves if $qd = m\pi$ (see for example [11]). On the other hand, according to Eq. (31), equations $qd = m\pi$ and $kb = n\pi$ determine the band contact points. Therefore, it is easy to conclude, that for any polarization the periodic structure becomes an absolutely transparent at the band contact points. The obtained result does not contradict the well-known Bragg reflection condition for the band boarders at all. It is necessary to note that the contact point cannot be considered as a boarder point for contacted allowed bands, since after a contact one large allowed band is created. It is formed by mean of a margin of two bands and now the contact point is an interior one for the new allowed band.

For the *p* waves, an analogous effect of a complete transparency is also known when the incident angle is that of Brewster [11]. In this case, the reflection amplitude for *p* waves vanishes ($r_p = 0$). From Eqs.(23), (24), which define the band boarders for *p* waves, the fact that in the case of Brewster`s condition the allowed bands contact may be easily shown as well. Indeed, equations (23), (24) have a same solution not only when the condition in Eq. (31) takes place, but also when

$$\frac{\varepsilon k}{\varepsilon_0 q} = \frac{\varepsilon_0 q}{\varepsilon k}. \tag{35}$$

Using Snell's law it is easy to see that Eq. (35) is nothing else than Brewster law ($tg\alpha = \sqrt{\varepsilon / \varepsilon_0}$).

Note, that in the case of Brewster angle, the periodic structure is transparent for *p* waves of an arbitrary frequency. In the case of Eq. (31) the structure can be transparent for the both polarizations, however for certain values of the incidence angle and the wave frequency.

Conclusion

Thus, the problem of reflection and transmission of a plane electromagnetic wave for a one-dimensional periodic layered structure is considered. Dependence of the transmission bands on the problem parameters is considered for both *s* and *p* polarized wave. Using the standard theory, the equations separately defining the even and odd band boarders are obtained. The graphs for the band location dependence on various structural and material parameters of layered structure are presented. It is proved that at a certain choice of the problem parameters the contact of allowed bands is possible. An analytical condition defining the possibility of the contact is obtained. This condition has the same form for two polarizations. Numerical calculation shows that the bandwidth dependence on the structure parameters is not a monotonous function. This non-monotony appears at the band contact regions. The behavior of the transmission coefficient at the contact points is considered. It is shown that for these points of the spectrum the periodic structure becomes transparent.

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