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# ELECTROABSORPTION IN A NARROW GAP InSb QUANTUM RING: ROTATOR MODEL

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**Abstract -** For the case of the two-dimensional rotator model a theoretical investigation of electronic states and interband dipole transitions in narrow gap *InSb* cylindrical layer quantum dot under the influence of both magnetic and electric fields has been done. The dispersion law for light hole and conduction electron is considered within the framework of two-band Kane model while for the heavy hole the parabolic dispersion law is used. An expression for the energy spectrum of cylindrical quantum rings is found. It is shown that in the case of non-parabolic dispersion law the energy spectrum is proportional to $\phi/\phi_0, 1/L^2$ and $1/R_{eff}^2$ under square root unlike to the case of parabolic dispersion law where the energy has the same dependence but without root ($\phi$ is the magnetic flux and $\phi_0$ is the quantum of magnetic flux).The threshold frequencies were calculated and the selection rules are found. The influences of external homogeneous electric field on the system are considered within the framework of perturbation theory for two different orientations. Direct calculation showed that the Stark shift in both cases seems to be quadratic. The effect of existence of the electric field on the interband dipole transition produces a new quantum number selection rules.

**Key words**: quantum dot, quantum ring, Stark shift, rotator model

## 1. Introduction

Investigation of the optical characteristics of semiconductor nanostructures continue to play an important role in the process of band structure ascertainment of examined samples. Changing the 2-dimensionality of a semiconductor to zero-dimensional nanostructures or as it is referred to as quantum dots (QD's) results in essential rebuilding of optical absorption character due to changing of density of states. Another most important factor brightly expressed in QD is strong sensitivity of charge carrier's to the changing of dimensions and geometry of samples to be studied. Controlling its dimensions and geometry, in the process of the QD controllable growth, one can flexibly manipulate with charge carriers energy levels thus creating systems with preassigned physical properties. Note that the geometrical form of the QD defines the symmetry of the one-particle Hamiltonian, which describes the particle behavior in the system under discussion. In terms of simplicity of description for one-particle states in QD's, the most suitable of objects are QD's with a spherical shape. However, such systems have only one geometrical parameter – the radius of the QD – which can be used to adjust its energetic spectrum. In this consideration, systems with cylindrical symmetry are preferred, since for these elements one may manipulate two parameters: the height of the cylinder and its radius. Recently, the authors of [1] reported the first results on the growth of InSb quantum dots by liquid phase epitaxy on InAs substrates. They obtained QD arrays with an average height of $L = (3.4\pm1)\times10^{-7}$ cm. And an average radius of $R = (27.2\pm7.5)\times10^{-7}$cm.

One of the first articles, where one and two-electron states in a quantum ring were studied, is the work of Chakraborty and Pietilainen [2]. The authors
simulating the one- and two-electron states in circular quantum structures have proposed to consider the confining potential of a system as a two-dimensional shifted oscillator. Such a model allows one to take into account the presence of boundaries both at the outer and inner radii. However, even the single particle Schrödinger equation for such confining potential is not solved exactly, which considerably hampers the analytical study of physical characteristics of such structures.

In the last few years, a large number of works is devoted to the theoretical investigation of physical properties of ring shape and layered structures [3–10]. The electronic and optical characteristic energies of semiconductor quantum rings in the presence of magnetic fields were investigated in [5] by adopting distinct potential models to describe the ring confinement and consider different geometric confinement parameters defining the quantum ring. The optical spectra for large and narrow quantum rings exhibit a set of resonances governed basically by the strength of the radial confinement. The presence of a magnetic field produces a notable enhancement of the resonance intensities.

In the mentioned works the dispersion law for charge carriers was considered as parabolic. However, recently the narrow-band InSb quantum dots were realized where the charge carrier's dispersion is essentially nonparabolic. In [11] the interband transitions in cylindrical InSb quantum dots were considered within the framework of the Kane band model. According to this model, the valence band consists of light-hole band, heavy-hole band, and spin-orbital splitting band. In consideration of electronic states and interband transitions, neglecting spin effects, one can restrict himself only to the light-hole and heavy-hole bands. It is remarkable that in InSb the dispersion laws for electrons and light holes coincide formally with the relativistic one:

$$E_{e(lh)} = \sqrt{p_{e(lh)}^2 s^4 + \mu_{e(lh)}^2 s^4} - \mu_{e(lh)} s^2, \tag{1}$$

where $s \sim 10^8$ cm/s is the parameter of nonparabolicity, and $\mu_{e(lh)}$ is the effective mass of an electron (light hole). In this case the dispersion law for heavy holes remains parabolic. The problem of ring shape nanostructures was solved numerically but in order to get an analytical solution for such a problem we consider the rotator model.

In this work, within the framework of the spatial rotator model, we study the interband dipole transitions in a narrow band InSb quantum ring. We consider the transitions between the light-hole and conduction bands, as well as between the heavy-hole and conduction bands. The effect of a uniform electric filed on the behavior of interband transitions is also discussed.

## 2. Theory

**a. Electronic states in magnetic field**

In the case of a very thin layer we can use the two-dimensional rotator model with effective moment of inertia $I_{eff} = \mu R_{eff}^2$ and effective radius $R_{eff} = (R_1 + R_2)/2$. Consider one-particle states in a cylindrical layer QD with the inner radius $R_1$, outer radius $R_2$ and height $L$. The condition of layer thickness can be presented as

$$d = R_2 - R_1 << (R_1, R_2), \tag{2}$$

Let us determine the energy spectrum and the wave function assuming the existence of a homogeneous magnetic field, directed along OZ-axis. The confining potential of the layer is approximated with the infinitely high rectangular walls:

$$V_{conf}(\rho, z) = \begin{cases} 0, & R_1 \le \rho \le R_2,\ -\dfrac{L}{2} \le z \le \dfrac{L}{2}, \\ \infty, & \rho < R_1, \rho > R_2,\ |z| > \dfrac{L}{2}. \end{cases} \tag{3}$$

According to Kane's approximation for *InSb* the dispersion law for the electron in the conduction band and *lh* in a magnetic field is

$$E(H) = \sqrt{(\mathbf{p} - \frac{e}{c}\mathbf{A})^2 s^2 + \mu^2 s^4} - \mu s^2, \tag{4}$$

After some transformations Schrödinger equation can be presented in the following form:

$$\frac{1}{2\mu}(\mathbf{p} - \frac{e}{c}\mathbf{A})^2 \psi(\rho, \varphi, z) = \varepsilon \psi(\rho, \varphi, z), \tag{5}$$

where $\varepsilon = [(E + \mu s^2)^2 - \mu^2 s^4]/2\mu s^2$ for *hh* we will consider the cylindrical rotator model with parabolic dispersion law and with the effective moment of inertia $I_{eff}^{hh} = \mu^{hh} R_{eff}^2$. Our wave function will take the following form:

$$\psi_{n,m}(\rho, \varphi, z) = f_0(\rho)\phi_m(\varphi)\chi_n(z), \tag{6}$$

where $f_0(\rho)$ is the ground state radial wave function.

$$\phi_m(\varphi) = \frac{1}{\sqrt{2\pi}} e^{im\varphi} \quad \text{and} \quad \chi_n(z) = \begin{cases} \sqrt{2/L}\sin\left(\dfrac{\pi n}{L} z\right), & n = 2k, \\ \sqrt{2/L}\cos\left(\dfrac{\pi n}{L} z\right), & n = 2k+1. \end{cases} \tag{7}$$

The wave function must also obey the following boundary conditions:

$$\psi(\pm L/2) = \psi(R_1) = \psi(R_2) = 0, \tag{8}$$

According to boundary conditions (8) and using (7) Schrödinger equation has the form:

$$\left[-\frac{\hbar^2}{2\mu}\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\frac{\partial}{\partial\rho})+\frac{\mu\omega_H^2}{8}\rho^2\right]f_0(\rho)=\varepsilon_{\rho_0}f_0(\rho), \tag{9}$$

We obtain the following expression for the *hh*:

$$E_{n,m}^{hh}=\varepsilon_{n,m}^{hh}=\varepsilon_{\rho_{0hh}}+\frac{\hbar^2 m_{hh}}{2I_{eff_{hh}}}+\frac{\hbar\,\omega_{H_{hh}}m_{hh}}{2}+\frac{\pi^2\hbar^2 n_{hh}{}^2}{2\mu_{hh}L^2}=\varepsilon_{\rho_{0hh}}+\frac{\hbar^2}{I_{eff_{hh}}}\left(\frac{m_{hh}{}^2}{2}-m_{hh}\frac{\varphi_{hh}}{\phi_0}\right)+\frac{\pi^2\hbar^2 n_{hh}{}^2}{2\mu_{hh}L^2}, \tag{10}$$

where $\phi=\pi R_{eff}^2 H$ is the magnetic flux and $\phi_0=2\pi\hbar c/e$ is the quantum of magnetic flux. Substituting. (10) in. (4) we get the following expression for *lh* and electron of the conduction band:

$$E_{n,m}^{e(lh)}=\sqrt{2\mu s^2\left(\varepsilon_{\rho_0}+\frac{\hbar^2}{I_{eff}}\left(\frac{m^2}{2}-m\frac{\phi}{\phi_0}\right)+\frac{\pi^2\hbar^2 n^2}{2\mu L^2}\right)+\mu^2 s^4}-\mu s^2\,. \tag{11}$$

It is necessary to mention that the energy for non-parabolic dispersion law for the electron and the *lh* proportional to $\phi/\phi_0$, $1/L^2$ and $1/R_{eff}^2$ under square root unlike to the case of parabolic dispersion law of the *hh*, where the energy has the same dependence but without root [6].

## b. Interband transition in magnetic field

Now let's consider the interband transitions in frame of rotator model in the magnetic field. As in the previous case, we will consider the transition between both *lh* and *hh* states to the electron states of the conduction band. The exact form of wave functions and energy levels of electrons and holes allows the calculation of the coefficient of interband absorption caused by the transition between the valence band and the conduction band. To calculate the corresponding absorption coefficient's $K^{lh\to e}(\omega)$ and $K^{hh\to e}(\omega)$, we use the following formula [3]:

$$K(\omega)=\frac{A_0}{V}\sum_{\nu,\nu'}\left|\int\psi_\nu^e\psi_{\nu'}^h dV\right|^2\delta\left(\hbar\omega-E_g-E_\nu^e-E_{\nu'}^h\right). \tag{12}$$

Here $\psi_\nu^e$ is the electron wave function, $\psi_\nu^h$ is the hole wave function, $\nu$ is the complete set of quantum numbers $(m,n_e,n_\rho)$, $E_g$ is the band gap of InSb, ω is the incident light frequency, $E_\nu^e$ and $E_\nu^h$ are electron and hole energy respectively, $A_0$ is the coefficient proportional to the square of the dipole matrix elements taken in the Bloch function. The $\delta$ function provides the energy conservation law during the transition. From the arguments of $\delta$ function we can find the threshold frequency for *lh*-electron and *hh*-electron transitions :

$$\hbar\omega_{00}^{lh\to e}=E_g+2\left\{\sqrt{2\mu_{e(lh)}s^2\left(\varepsilon_{\rho_{0e(lh)}}+\frac{\hbar^2}{I_{eff_{e(lh)}}}\left(\frac{m_{e(lh)}{}^2}{2}-m_{hh}\frac{\phi_{e(lh)}}{\phi_0}\right)+\frac{\pi^2\hbar^2n_{e(lh)}^2}{2\mu_{e(lh)}L^2}\right)+\mu_{e(lh)}{}^2s^4}-\mu_{e(lh)}s^2\right\},$$

$$\hbar\omega_{00}^{hh\to e}=E_g+\left\{\sqrt{2\mu_e s^2\left(\varepsilon_{\rho_{0_{hh}}}+\frac{\hbar^2}{I_{eff_e}}\left(\frac{m_e{}^2}{2}-m_e\frac{\phi_e}{\phi_0}\right)+\frac{\pi^2\hbar^2n_e^2}{2\mu_eL^2}\right)+\mu_e{}^2s^4}-\mu_e s^2\right\}+ \qquad (13)$$

$$+\varepsilon_{\rho_{0_{hh}}}+\frac{\hbar^2}{I_{eff_{hh}}}\left(\frac{m_{hh}{}^2}{2}-m_{hh}\frac{\phi_{hh}}{\phi_0}\right)+\frac{\pi^2\hbar^2n_{hh}{}^2}{2\mu_{hh}L^2}.$$

Finally and after integration we find the following selection rules for the rotator model: $n_e=n_{lh(hh)}$, $m_e=-m_{lh(hh)}$.

**c. Electronic states and interband transitions in the electric field**

Now let us consider our system in an external homogeneous electric field **F**. Let us discuss the influence of the field in the framework of the perturbation theory for two different orientations of **F**: 1) along the axis OZ, and 2) in the XOY plane, when the angle between **F** and X-axis is φ. Let us discuss the first case. The perturbation has the following form:

$$V(z)=-eFz, \qquad (14)$$

Direct calculations show that the first order correction $\langle n|V(z)|n\rangle$ vanishes [12]. Thus, we must discuss the second approximation of the perturbation theory. For the corresponding energy correction we have:

$$\Delta_2E_n^{(z)}=\sum_{n'\neq n}\frac{\left|\langle n'|V(z)|n\rangle\right|^2}{E_{n,m}-E_{n',m}}. \qquad (15)$$

After calculations for the corresponding correction of energy, we obtain

$$\Delta_2E_n=(eFL)^2\sum_{k=1}^{\infty}\left|\left(\frac{\sin[\pi(2k+n)/2]}{(2k+n)^2}+\frac{\sin[\pi(2k-n)/2]}{(2k-n)^2}\right)\right|^2\frac{1}{E_{n,m}-E_{2k,m}} \quad \text{for odd } n, \qquad (16)$$

$$\Delta_2E_n=(eFL)^2\sum_{k=1}^{\infty}\left|\left(\frac{\sin[\pi((2k-1)+n)/2]}{((2k-1)+n)^2}+\frac{\sin[\pi((2k-1)-n)/2]}{((2k-1)-n)^2}\right)\right|^2\frac{1}{E_{n,m}-E_{2k-1,m}} \quad \text{for even } n, \qquad (17)$$

For the wave function, we have for even *n*

$$\psi_n^{(1)}(z)=\psi_n^{(0)}(z)+\sum_{k=1}^{\infty}\psi_{k-1}^{(0)}(z)\left(\frac{\sin[\pi((2k-1)+n)/2]}{((2k-1)+n)^2}+\frac{\sin[\pi((2k-1)-n)/2]}{((2k-1)-n)^2}\right)\frac{1}{E_n-E_{2k-1}}. \qquad (18)$$

Similarly, for odd *n*

$$\psi_n^{(1)}(z)=\psi_n^{(0)}(z)+\sum_{k=1}^{\infty}\psi_{k-1}^{(0)}(z)\left(\frac{\sin[\pi(2k+n)/2]}{(2k+n)^2}+\frac{\sin[\pi(2k-n)/2]}{(2k-n)^2}\right)\frac{1}{E_n-E_{2k}}. \qquad (19)$$

Now let us consider the second case when the electric field lies in the XOY plane and its angle with the X-axis is φ. In this case, the perturbation potential has the form

$$V(\varphi) = -P_{eff} F \cos\varphi, \tag{20}$$

where $P_{\text{eff}}$ is the effective electric dipole momentum of a charge carrier. As in previous case, the shift of unperturbed levels in the first approximation is given by

$$\Delta_1 E_m = 0, \tag{21}$$

In the second approximation, we have for the shift:

$$\Delta_2 E_m = -\frac{P_{eff}^2 F^2}{4}\left(\frac{1}{E_m - E_{m+1}} + \frac{1}{E_m - E_{m-1}}\right), \tag{22}$$

For the wave function, we obtain:

$$\Psi_m^1(\varphi) = \Psi_m^0(\varphi) - \frac{P_{eff}{}^2 F^2}{2}\left(\frac{\Psi_{m+1}^0(\varphi)}{E_m - E_{m+1}} + \frac{\Psi_{m-1}^0(\varphi)}{E_m - E_{m-1}}\right), \tag{23}$$

The Stark shift in both cases seems to be quadratic. Basing on the obtained result and using definition (11) for the absorption coefficient, for the case of OZ-direction, we obtain

$$K(\omega) \sim \sum_{\substack{m_c, n_c \\ m_v, n_v}} \left( \delta_{m_c, n_v} \delta_{m_c, n_v} + \left| \frac{\langle n_v | V | n_c \rangle}{E_{n_v} - E_{n_c})} \right|^2 \delta_{m_v, n_c} \right) \times \delta(\hbar\omega - E_g - E_{m_v, n_v} - E_{m_c, n_c}). \tag{24}$$

So, the existence of the electric field leads to the taking down the selection rule in the direction of OZ (selection rule by the quantum number $n$ vanishes).

For the case of XOY-plane, after integrating for absorption coefficient we can find:

$$K(\omega) \sim \delta_{n_c, n_v} \delta_{m_c, m_v} + B \delta_{n_c, n_v} \delta_{m_c, m_v \pm 1}, \tag{25}$$

where B is a coefficient depending on the electric field. So, the presence of the electric field produces new magnetic quantum number selection rules: $m_c = m_v \pm 1$.

## 3. Discussion

For the quantitative analysis of the obtained result let us consider an InSb cylindrical layer for which $s \approx 10^8 Cm/Sec$, $\mu_e = 0.015\mu_e$ ($\mu_e$ is the mass of free electron). Fig.1. shows the dependence of the threshold energy on the applied electric field for transition from band of (a) light hole and (b) heavy hole to the conduction band, when $R_1 = 800$Å and $R_2 = 1000$Å and magnetic field (B = 5T).

As seen, with increasing electric field in both cases the threshold energy of excited state (n = 2) increases unlike the case of ground state (n = 1), which decreases with the increase in the electric field. The reason for such behavior is in good agreement with [13], where the Stark shift in quantum film was studied when the electric field is directed perpendicularly to the plane of a film. The authors showed that the behavior of electron energy levels for ground and first excited states have opposite characters with increase in the electric field. If the ground state energy at once is rolled down, the first excited level, at first, rises and then starts to fall. In our work, we consider only week electric fields and the influence of the electric field are discussed within the framework of the perturbation theory. Thus, it is clear that the behaviors of curves 1 and 2 will have opposite characters. Also we note that the value of the threshold energy for *hh*-electron transition is less than that for *lh*-electron transition. This is connected with the dispersion law and effective masses of a heavy hole in comparison with the mass of the light hole.

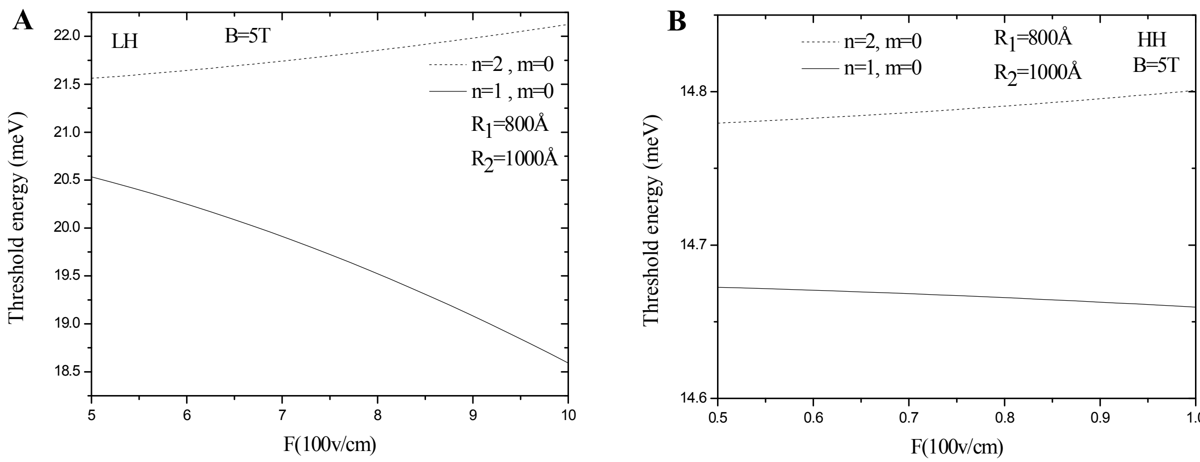


**Fig. 1.** Dependence of the threshold energy on the applied electric field F (a) for $(lh \to e)$ transition. (b) for $(hh \to e)$ transition at $R_1$ = 800Å and $R_2$ = 1000Å.

## 4. Conclusion

Within the framework of rotator model we investigate the influence of non-parabolicity of the dispersion law on the electronic states and optical properties in a narrow band *InSb* cylindrical quantum ring in the presence of magnetic and electric fields. It is shown that the energy levels of the electron with Kanes dispersion are less responsible to the change of geometrical parameters compared to those in the case of parabolic dispersion. It should be underlined that the Stark splitting in both cases is quadratic in electric field. When the electric field is applied in the Z direction, the selection rule by the quantum number n vanishes. So, a transition between states with different values of $n_v$ and $n_c$ can take place. If an external field is applied in the plane of cylindrical layer cross-section, a new selection rules for magnetic quantum number are obtained: $m_c = m_v \pm 1$.

The absorption coefficient for *lh* are proportional to the square root magnetic field H unlike the case of *hh*, where we have the same dependence but without root. The perturbation theory is acceptable only for small values of the electric field $(F \le 50V/cm)$ for *hh* due to the large value of the effective mass.

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