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EVIDENCE FOR POSSIBLE O(6) SYMMETRY IN A=120-200 MASS REGION

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Abstract

The prediction of the O(6) symmetry limit of interacting boson model are reviewed for A=120-200 mass region. Evidence of functional relationship of the coefficient B and C from the energy expression of O(6) limit with N, Z and $R_{4/2}$(= E_{4g}/E_{2g}) are presented, B(E2; $2_g \rightarrow 0_g$) with the product of number of valence proton and number of valence neutron N_pN_n and neutron number N are also illustrated.

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1. Introduction

The phenomenological Interacting Boson Model (IBM) initially introduced by Arima and Iachello [1] has been rather successful in describing the collective properties of several medium and heavy mass nuclei. The basic idea of IBM is to assume that low lying collective states in even-even nuclei can be described by a system of interacting s-bosons and d-bosons carrying angular momentum 0 and 2, respectively. As originally formulated, it is a purely phenomenological model where the properties of nucleus are described in terms of interacting s(L = 0) and d(L = 2) bosons, such that the number of bosons are conserved. The interacting boson model provides the description of various symmetries involve in collective structure [2]. The model led to the U(6) group algebra which yields three dynamical symmetries: SU(5), SU(3) and O(6) which corresponds to anharmonic vibrator, deformed rotor and γ-unstable nuclei, respectively [3]. Casten's symmetry triangle (see Fig. 1) in its geometric representation describe the three limits. Each vertex represents a geometric limit and the legs represent the region where the structure undergoes a transition from one limit to other [4]. The nuclei in which neutron and proton are near closed shell can show the SU(5) behavior, while neutron and proton near the midshell would suggest SU(3) behavior . The occurrence of O(6) symmetry is expected in cases where neutrons are particle or proton are holes, or vice versa. The E(5) symmetry describes nuclei that at the critical point of the phase transition from anharmonic vibrator or SU(5) shape to a γ- unstable or O(6) shape and having energy ratio $R_{4/2}$ (=E_{4g}/E_{2g}) equal to 2.20. The X(5) symmetry describes nuclei at the critical point of the phase transition from anharmonic vibrator or SU(5) shape to a deformed rotor or SU(3) shape and having $R_{4/2}$ equal to 2.91. The measure of deformation is the energy ratio $R_{4/2}$ which is greater than 2.0 for SU(5) nuclei, greater and equal to 2.5 for O(6) nuclei and greater or equal to 3.0 for SU(3) nuclei. The structure of even-even nuclei and collectivity in low-energy nucleon structure is described by comparing the properties of the anharmonic vibrator, deformed rotor and γ-unstable rotor [5]. The SU(5) and O(6) symmetries have the subgroup O(5) and O(3) [6,7]. The most general Hamiltonian for the interacting boson model (IBM-1) [1], can be written as a linear combination of linear quadratic Casimir invariants of the groups

$$\begin{aligned} H = e_0 + e_1 b_1(U6) + e_2 b_2(U6) + \varepsilon b_1(U5) + \alpha b_2(U5) + \beta b_2(U5)\ldots \\ \ldots + \gamma b_2(O3) + \delta b_2(SU3) + \eta b_2(O6). \end{aligned} \tag{1}$$

Three chains symmetries are

$$U(6) \supset U(5) \supset O(5) \supset O(3),\ SU(5) \tag{2}$$

$$U(6) \supset O(6) \supset O(5) \supset O(3),\ \ O(6) \tag{3}$$

$$U(6) \supset SU(3)\ \supset O(3) \supset O(2),\ SU(3). \tag{4}$$

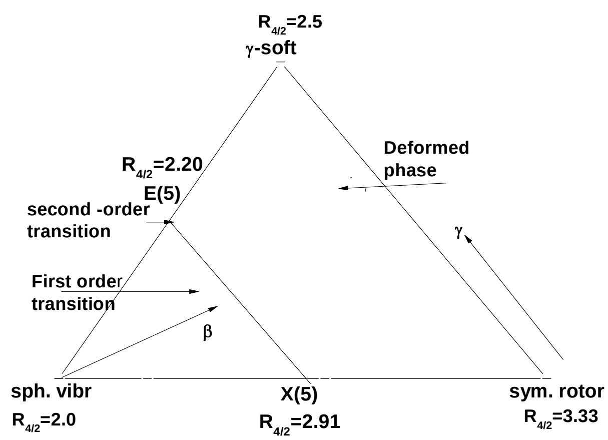

Fig. 1. Casten's symmetry triangle in its geometric representation with the γ-soft O(6) symmetries (see Ref. [9]).

Leviaten et al. [8] described the properties of the nuclei belonging to SU(5)-O(6) transition region. The value of $R_{4/2}$ for SU(5) is equal to the vibrational limit and approaches the axially symmetric rotational value of 3.33 in the SU(3) with increasing $R_{4/2}$. J. B. Gupta [9] described the unbroken SU(3) symmetries and the relation of interacting boson-model parameter with the shell model. P. von Brentano et al. [10] described the test of the O(6) character of nuclei in atomic mass A=130 region. Our aim is to study nuclei that show O(6) nature in A =120-200 mass region. The level energies can serve to prove the O(6) symmetry if levels with σ less than N are found, and obey the O(6) energy formula. For this the first necessary condition to show O(6) nuclei is that $R_{4/2}$ = 2.5, another condition is to test the validity of the expression for B(E2) ratios similar as

$$\frac{B(E2;\ 4_g \to 2_g)}{B(E2;2_g \to 0_g)} = \frac{10}{7}\frac{(N-1)(N-5)}{N(N+4)} \xrightarrow[N\to\infty]{} \frac{10}{7} \approx 1.4 \quad (5)$$

and

$$\frac{B(E2;0_\beta \to 2_g)}{B(E2;2_g \to 0_g)} = 0.$$

2. Calculation

The analytic solution can be obtained whenever the Hamiltonian can be written in terms of Casmir operators of one chain. For O(6) nuclei the Hamiltonian is

$$H = e_0 + e_1 b_1 (U6) + e_2 b_2 (U6) + \beta b_2 (O5). \quad (6)$$

The classification scheme for the O(6) nuclei is

$$\left| \begin{matrix} U(6) & \supset & O(6) & \supset & O(5) & \supset & O(3) & \supset & O(2) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ [N], & & \sigma, & & \tau,\ \tilde{v}_\Delta, & & L, & & M_L \end{matrix} \right\rangle . \quad (7)$$

Thus, the Hamiltonian is diagonal in the basis Eq. (7) with the eigenvalues

$$E([N]\,\sigma\,\tau\,\tilde{v}_\Delta\,L\,M) = A\frac{1}{4}(N\text{-}\sigma)(N+\sigma+4) + B\frac{1}{6}\tau(\tau+3) + CL(L+1) \quad (8)$$

The quantities A, B and C are constants and σ, τ and L are the quantum numbers. For low lying group of levels, with σ equal to N, the first term equals zero. The effect of C-term is also monotonic in breaking the degeneracy's of each τ-multiplet therefore at C=0 the energy spacing is given by the B-term, deformed γ-unstable oscillator which is the closest counterpart to the O(6) limit [6]. The γ-unstable rotor, O(6) shows the dependence of B(E2) values on spin within a band differing substantially in two dynamic symmetry [11]. By putting the values of N, σ, τ and L in Eq. (8) we calculated the values of B and C for which experimental data are taken from Sakai [12] and recent Nuclear Data Sheet [13] as follows:

$$B = \frac{3}{2}\left[E(2_g) - \frac{6}{14}[E(4_g) - E(2_\gamma)]\right] \quad (9)$$

and

$$C = \frac{1}{14}\left[E(4_g) - E(2_\gamma)\right] \quad (10)$$

Therefore from Eq. (9) we calculated the values of parameter B, which are listed in Table 1, the parameter B is fairly constant for fixed boson number N_B, except Nd and Sm. Therefore with decreasing N_B, the value of B is increasing and from Eq. (10) we calculated the values of parameter C (see Table 2).

Table 1. Energy scale parameter B.

B Parameter For N<82 Nuclei						For N>82	
N_B	**Xe**	**Ba**	**Ce**	**Nd**	**Sm**	**Pt**	**Hg**
11						362.4	
10	535.2					356.8	582.7
9	506.1	488.4	460.8	418.8		354.0	662.0
8	510.8	503.9	464.8	486.7	1.02	364.1	670.0
7	543.0	539.9	560.0	629.7	1.06	371.0	647.7
6	623.5	635.2	685.8			412.5	622.7
5		693.1				475.5	642.8
4							749.1

Table 2. Energy scale parameter C.

C Parameter For N<82 Nuclei						For N>82	
N_B	**Xe**	**Ba**	**Ce**	**Nd**	**Sm**	**Pt**	**Hg**
11						-8.3	
10	-5.7					4.6	4.0
9	-1.1	-11.6	-8.8	2.4		9.9	-4.1
8	2.2	-8.6	2.5	8.1	-25.2	12.2	-3.9
7	4.4	-0.4	5.9	16.8	9.5	13.5	-0.6
6	4.5	6.8	15.7	22.3		13.4	1.8
5		23.7	22.5			15.0	-2.7
4							-2.1

Here we also introduced an approximate formula of the energy spectra in the O(6) limit with three-body potential as

$$E([N]\,\sigma\,\tilde{\nu}_{\Delta}\;L\,M) = B'\,\tau\,(\tau+3) + C'\,L(L+1) - \frac{\kappa\left\{\,[\tau(\tau+3)/5]^2 + L(L+1)]^2\right\}}{N^2} \tag{11}$$

The values of B', C' and parameter κ are calculated by least square fit. From Eqs. (8) and (11) we calculated the theoretical energies Th.I and Th.II. Some data are presented in Table 4 for comparison between experimental and theoretical (Th.I & Th.II) values of energy. By using Eq. (5) we calculated the values of B(E2) ratios for those nuclei which show O(6) nature (Table 3). From the above table we see that the theoretical value of B(E2) $\approx$ 1.4 which satisfied the condition and the value of B(E2;$2_g \rightarrow 0_g$) is also $\approx$1.4. This means that these nuclei tend to show the O(6) limit.

Table 3. Comparison of experimental B(E2) ratios with the calculated O(6) values.

			B(E2)$_{exp}$			B(E2)$_{th}$
N_B	**Xe**	**Ba**	**Ce**	**Nd**	**Pt**	**Theor. Value**
11						1.39
10	1.46		1.35			1.38
9	1.47	1.12	1.39	1.42		1.38
8	1.29	1.03	0.56		1.51	1.35
7		0.90			1.35	1.34
6					1.39	1.31
5		1.5				1.30
4					1.27	1.21

Table 4. Comparison of ground state band energies of few nuclei (all in MeV).

Nuclei	Energy	2^+	4^+	6^+	8^+	10^+	12^+
^{120}Xe	Expt	0.322	0.795	1.396	2.047	2.871	3.674
	O(6)	0.322	0.767	1.334	2.023	2.835	3.955
^{122}Xe	Expt	0.330	0.827	1.466	2.216	3.038	3.819
	O(6)	0.330	0.818	1.463	2.264	3.222	4.379
^{124}Xe	Expt	0.354	0.878	1.548	2.330	3.170	3.881
	O(6)	0.354	0.905	1.655	2.603	3.748	4.992
^{130}Ba	Expt	0.357	0.901	1.592	2.394	3.259	3.983
	O(6)	0.357	0.885	1.585	2.455	3.496	4.746
^{132}Ba	Expt	0.464	1.127	1.932	2.796		
	O(6)	0.464	1.233	2.006	2.984		
^{132}Ce	Expt	0.325	0.857	1.540	2.326	3.154	3.724
	O(6)	0.325	0.810	1.453	2.255	3.215	4.355
^{138}Nd	Expt	0.520	1.249	2.134	3.105	3.171	
	O(6)	0.520	1.462	2.824	3.312	4.025	
^{196}Pt	Expt	0.356	0.877	1.527	2.255	3.045	
	O(6)	0.356	1.001	1.935	2.963	3.158	

3. Results and discussion

In Fig. 2 the variation of B depending on Z is shown for constant boson number N_B, the value of B increases with Z for N_B = 5, 6, 7 and the ratio $R_{4/2}$ of these N_B nuclei goes closer to the E(5) symmetry but with increasing N_B it moves towards the O(6) symmetry. Next, when the same data are plotted against the neutron number N (see Fig.3), the horizontal spread is less but the vertical spread is more. The dependence of neutron number N is better viewed on a graphical plot. We know that the collectivity depends on N_B and N, but for small N_B the collectivity is not fully attained. The collectivity increases for N=76-80 but decreases for N=66-74.

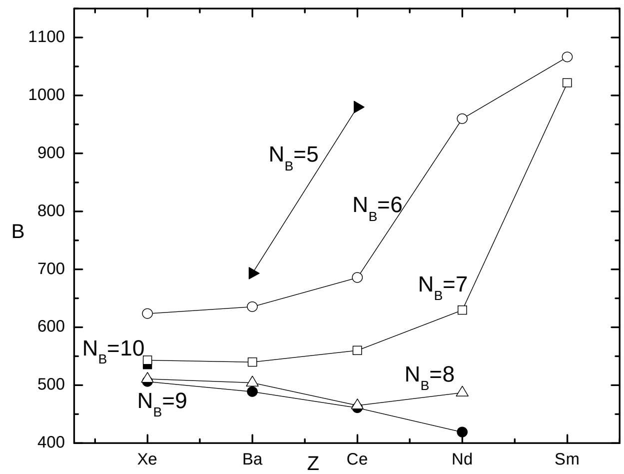

Fig. 2. Plot of the coefficient B in eq. (9) vs Z.

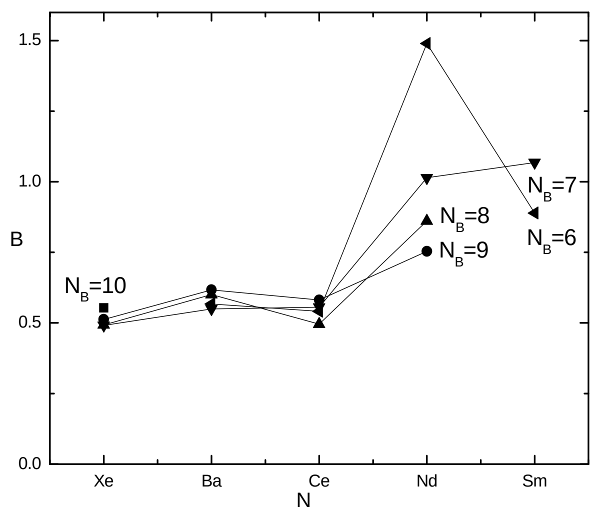

Fig. 3. Plot of the coefficient B in eq. (9) vs N.

In Fig.4 we show variation between the parameters C and N. In N=66-74 the energy of $E(2_\gamma)$ level is below $E(4_g)$ level, hence these are E(5) symmetrical nuclei and attain the negative value of the parameter C. However, with increasing neutron number N = 76-80 there is a decrease in $E(2_\gamma)$ energy but $E(4_g)$ energy increases and approaches O(6) symmetry. For ^{120}Xe the ratio $R_{4/2}$ is equal to 2.5 and have the same experimental and theoretical energy levels. Also B(E2) ratio is the same, i.e. $B(E2)_{expt} \approx B(E2)_{th}$. In ^{122}Xe the energy levels are the same almost up to $I^\pi = 10^+$. In case of ^{124}Xe the energy is the same up to $I^\pi = 8^+$ but the experimental and theoretical B(E2) ratios are equal as shown in Table 1. Next in ^{126}Xe the ratio $R_{4/2}$ is equal to 2.0-2.5 (see Fig.5), hence nuclei are not purely O(6) but it goes closer to the SU(5) symmetry. When we go towards higher state the irregularity comes across in Th.I energy values, but the energy level is better in Th.II. In ^{128}Xe nuclei $R_{4/2}$ is equal to 2.4 and energy $E(2_\gamma)$ lies below the $E(4_g)$ energy level and up to 6^+ state the theoretical energy is equal to the experimental energy level.

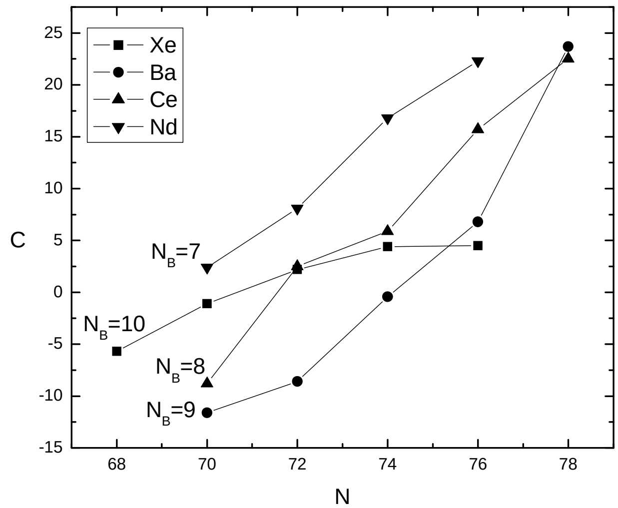

Fig. 4. Plot of the coefficient C in eq. (10) vs N.

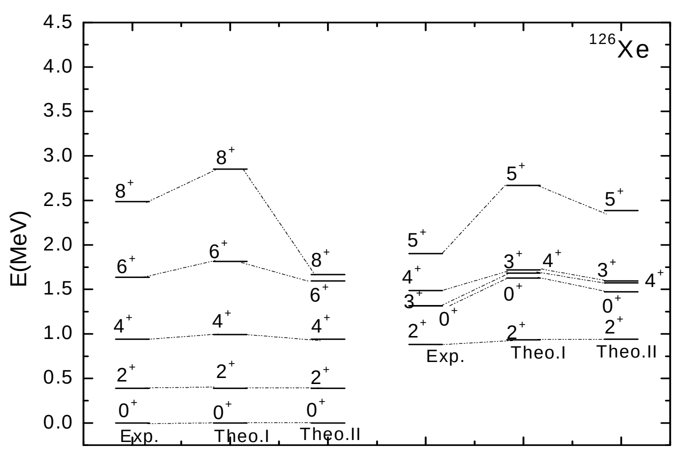

Fig. 5. Comparison of experimental and theoretical energy spectra of ^{126}Xe.

In case of $^{126-128}$Ba the energy levels are the same up to $I^\pi = 6^+$ state, above this state there is a rise in the theoretical energy values but still the energy ratio $R_{4/2}$ is equal to 2.5 and $B(E2)_{expt} \approx B(E2)_{th.}$. Hence it goes closer to the O(6) symmetry. For $^{130-132}$Ba, energies are the same but at higher state, i.e. at 10^+ a back banding was observed due to the band crossing. In ^{130}Ce $B(E2)_{expt}$ is equal to 1.35 [10] , $B(E2)_{th.}$ is equal to 1.38 and in ^{132}Ce and ^{134}Ce the experimental and theoretical energies (Th.I &Th.II) of O(6) limit are approximately equal (see Fig.6). But at higher spin there is a rise in the energy value of ^{134}Ce nuclei. In ^{134}Nd $B(E2)_{expt}$ is equal to 1.42 [14] and $B(E2)_{th.}$ is equal to 1.38 which are approximately the same. Similarly, as shown in Fig.7, ^{136}Nd nuclei have equal experimental and theoretical energies and the ratio $R_{4/2}$ is in the range of the O(6) nuclei. In ^{138}Nd the $R_{4/2}$ ratios are $\approx$ 2.5, but the energy levels do not show any regularity at higher spin state.

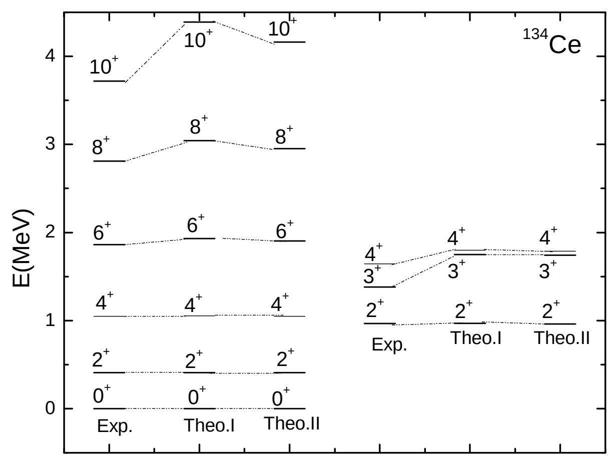

Fig. 6. Same as in Fig. 4, but for ^{134}Ce nuclei.

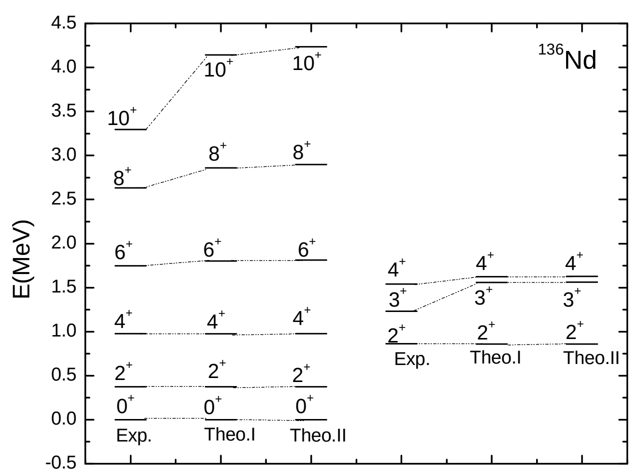

Fig. 7. Same as in Fig. 4, but for ^{136}Nd nuclei.

In case of N greater than 104 region the Pt and Hg nuclei also show the same energy level at low spin but there is a rise in the value of theoretical energy level at higher spin state. In ^{194}Pt $B(E2)_{expt}$ is equal to 1.38 [15] and $B(E2)_{th}$ also is equal to 1.34, therefore these isotopes also have a O(6) character. The experimental value of ratio B(E2)in ^{196}Pt is 1.46 and theoretical value is 1.31; similarly in ^{198}Pt $B(E2)_{expt}$ is equal to 1.27 [16] and $B(E2)_{th.}$ is equal to 1.30 . Both these nuclei are close to the O(6) symmetry. All these data show that with three parameter formula Eq. (11) the agreement between the theory and experiment is certainly better than with the O(6) limit (Th.II). We also calculated the B and C parameters from Eq. (8):

$$B = E(2_\gamma) - E(2_g) \tag{11}$$

and

$$C = \frac{1}{6}[E(2_g) - \frac{2}{3}(E(2_\gamma) - E(2_g)]. \tag{12}$$

There is no large difference in the calculated values. But we have used above formula for the calculation of the parameters B and C.

3.1 Variation of B(E2) values

Now we test the condition on B(E2;L+2→L). First we study the $B(E2; E_g \to 0_g)$ transition; data are taken from [17] and show a functional dependence of $B(E2; E_g \to 0_g)$ on N (see Fig.8). For a given series of isotopes at the first B(E2) values increase linearly with the neutron number. When graph is plotted with the number of valence protons and number of valence neutrons N_pN_n there is a faster rise in B(E2) values. Next we study the case of N greater than 104: we take Pt and Hg nuclei because the ratio $R_{4/2}$ is equal to 2.5. When we plot the graph between $B(E2; E_g \to 0_g)$ and N_pN_n, it shows a linear increase with B(E2) ratio (see Fig.9).

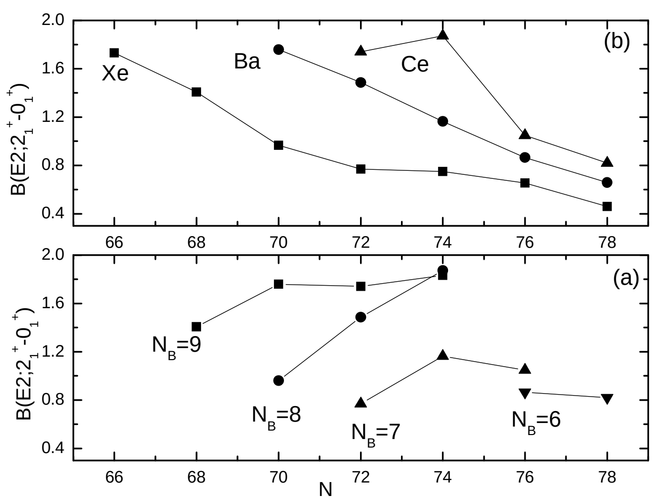

Fig.8 (a) Variation of $B(E2; 2_1^+ - 0_1^+)$ in e^2b^2 with neutron number and data linked for same boson number. (b) The data are linked for same proton number.

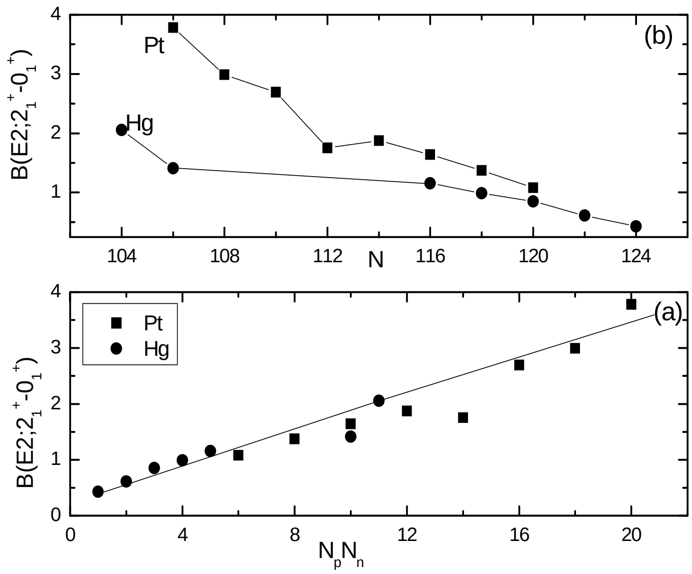

Fig. 9 (a) The B(E2; $2_1^+ - 0_1^+$) plotted versus the product N_pN_n. (b) The same data plotted versus N.

4. Conclusion

There are many nuclei in the Xe, Ba, and Ce region described in terms of γ-soft rotational nuclei [18]. The ratio $R_{4/2}$ shifts towards the lower value when goes closer to the spherical nuclei. When N is equal to 74, 76, the ratio $R_{4/2}$ is equal to the 2.5 which goes close to the O(6) symmetry [19]. The ^{128}Xe is a new example of the E(5) symmetry, while $^{186\text{-}188}$Pt shows the O(6) symmetry [18]. It is shown that B(E2) ratios satisfy the condition and the experimental energies are equal to the theoretical values for a given series of isotopes and the $R_{4/2}$ ratios are also ≈ 2.5. A small systematic downward deviation of the data from O(6) values still is not understood. From the above consideration we see that B(E2) values agree with the experimental data. We also find that the energy spectra are improved when three-body potential is added to the O(6) equation, hence Th.II shows good agreement than Th.I with experimental energy spectra.

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References

1. A. Arima and F. Iachello, The Interacting Boson Model, Cambridge Univ. Press, 1987.
2. Nureddin Turkan, J. Phys G: Nucl. Part Phys. **34**, 2235 (2007).
3. F. Iachello, Phys. Rev. Lett. **87**, 052502 (2001).
4. D. H. Feng, R. Gilmore, and S. R. Deans, Phys. Rev**.** C **23**, 1254 (1981).
5. E. A. McCutchan, and R. F. Casten, Phys. Rev. C **74**, 057302 (2006).

6. A. Arima and F. Iachello, Ann. Phys. (N.Y) **99**, 253 (1976).
7. A. Arima and F. Iachello, Ann. Phys. (N.Y.) **123**, 468 (1979).
8. A. Leviatan, A. Novoselsky, and I. Talmi, Phys. Lett. B **172**, 144 (1986).
9. J. B. Gupta, Phys. Rev. C **33**, 1505 (1986).
10. P. von. Brentano, A. Gelberg, and S. Harissopulos, Phys. Rev. C **38**, 2386 (1988).
11. R. F. Casten and P. von. Brentano, Phys. Lett. B **152**, 22 (1985).
12. M. Sakai, Atomic Data and Nuclear Data Table **31**, 399 (1984).
13. Nuclear Data Sheets, National Nuclear Data Centre (BNL-Upton N. Y.), Relevant Volumes for A=120-200 used.
14. M. A. J. Mariscotti, G. S. Goldhaber, Buck et al., Phys. Rev. **178**, 1864 (1969).
15. M. O. Kortelahti et al., Phys. Rev. C **42,** 1267 (1990).
16. C. Baktash et al., Phys. Rev. C **18**, 131(1978).
17. S. Raman, C. W. Nestor, and P. Tikkanen, Atomic Data and Nuclear Data Tables, **78**, 1(2001).
18. Liao Ji-Zhi and Wang Huang-sheng, Phys. Rev. C **49**, 2465 (1994).
19. P. Breantono et al. J. Phys. **G11**, L85 (1985).