Armenian Journal of Physics, 2008, vol. 1, issue 4, pp. 229-233

THE FORCE ACTING ON A POLARIZABLE NANOPARTICLE IN THE QUANTIZED ELECTROMAGNETIC FIELD

V.E. Mkrtchian

Institute for Physical Research, Armenian National Academy of Sciences, Ashtarak, Republic of Armenia

Received December 14, 2008

We derive an expression for the force acting on a small (still macroscopic) particle in the field of the quantized electromagnetic radiation in any arbitrary quantum state. This result unifies in one simple formula all known expressions for the forces (i.e. van der Waals or frictional) acting on a small particle.

Key words: van der Waals interaction, quantum friction, nonequilibrium.

PACS numbers: 12.20.-m, 42.50.Ct, 34.50.Dy

1. Introduction

Growing successes in nanotechnologies and possibility of non-contact control of nanoparticles by optical tweezers [1,2] or by usage of effects of quantum friction [3] stimulate investigations of the force acting on a polarizable nanoparticle in the external electromagnetic field for the states of the field out of equilibrium. Inclusion in consideration of nonequilibrium states is stipulated by the possibility of tuning the interaction in both strength and sign [4,5].

The problem of the mechanical force acting on a small neutral particle or on an atom with the electromagnetic field in equilibrium is well understood [6]. Meanwhile the same problem for the nonequilibrium states of the field is solved only in a special case when usage of the fluctuation-dissipation theorem provides an opportunity in "construction" of a "hot" half-space [4,5].

In this paper we derive an expression for the force acting on a small (still macroscopic) particle (which we will call nanoparticle) in the field of the quantized electromagnetic radiation in any arbitrary quantum state. This result provides an opportunity to unify in one expression for the force the all known results related to the problem. As the simplest consequence of the generalization we recover expressions for particle-wall interaction force and expression for the frictional force acting on a moving particle in the field of blackbody radiation.

2. Expression for the force

Compared with the case of atom-field interaction problem the case of macroscopic nanoparticle permits a very important approximation, i.e., in this case for moderate intensities we can ignore dynamics of the particle under influence of the external electromagnetic field and suppose that the particle stays in its initial state (local thermodynamic equilibrium) during all the time of interaction with the external field.

Besides, in case of nanoparticles the characteristic size of the particle is much smaller than the wavelength of radiation (for thermal fields relevant wavelengths are of the order $2\pi c/T$), so we can solve the problem in the Rayleigh regime [7], i.e., we may ignore the change of external field in the volume of the particle and suppose that the particle behaves as a single electric dipole.

In electric-dipole approximation the force of radiation on a neutral particle located at the point $\vec{r}_A$ is given by [8]

$$\vec{F}(\vec{\mathrm{r}}_{\mathrm{A}},t)=\vec{\nabla}\left\langle \hat{\vec{d}}(t)\hat{\vec{E}}(\vec{\mathrm{r}},t)\right\rangle_{\vec{r}=\vec{r}_A}. \tag{1}$$

Taking interaction $\hat{V}$ of the electromagnetic field with a dipole in the form $\hat{V}=-\hat{\vec{d}}\hat{\vec{E}}(\vec{r}_A)$, we can evaluate $\left\langle \hat{\vec{d}}(t)\hat{\vec{E}}(\vec{\mathrm{r}},t)\right\rangle$ in formula (1) using the Keldysh technique [9]:

$$\left\langle \hat{\vec{d}}(t)\hat{\vec{E}}(\vec{\mathrm{r}},t)\right\rangle=\left\langle \hat{T}_C\hat{\vec{d}}(t^2)\hat{\vec{E}}(\vec{r},t^1)\hat{S}_C\right\rangle,$$

where

$$\hat{S}_C=\hat{T}_C\exp\left[\sum_{\sigma=1,2}(-1)^{\sigma}i\int_{-\infty}^{+\infty}\hat{V}(\tau^{\sigma})d\tau^{\sigma}\right].$$

Then, in the first order of interaction $\hat{V}$ we find

$$\left\langle \hat{\vec{d}}(t)\hat{\vec{E}}(\vec{\mathrm{r}},t)\right\rangle=\sum_{\sigma=1,2}(-1)^{\sigma+1}Tr\int_{-\infty}^{+\infty}d\tau\hat{D}'^{1\sigma}(\vec{r},t;\vec{r}_A,\tau)\hat{\alpha}^{\sigma2}(\tau,t), \tag{2}$$

where

$$\hat{D}'^{1\sigma}(\vec{r},t;\vec{r}_A,\tau)=\frac{i}{c^2}\partial_t\partial_\tau\hat{D}^{1\sigma}(\vec{r},t;\vec{r}_A,\tau),$$

$\hat{D}^{\lambda\sigma}(\vec{r},t;\vec{r}_A,\tau)$ is the photon propagator

$$\hat{D}_{ij}^{\lambda\sigma}(\vec{r},t;\vec{r}_A,\tau)=-i\left\langle \hat{T}_C\hat{A}_i(\vec{r},t^{\lambda})\hat{A}_j(\vec{r}_A,\tau^{\sigma})\right\rangle$$

in the Dzyaloshinskii gauge ($\varphi=0$) and

$$\alpha_{ij}^{\lambda\sigma}(t,\tau)=i\left\langle \hat{T}_C\hat{d}_i(t^{\lambda})\hat{d}_j(\tau^{\sigma})\right\rangle$$

is the particle propagator in the interaction picture.

After the Keldysh transformation [9,10]

$$G^{\lambda\sigma}=\frac{1}{2}\left[G^K+(-1)^{\lambda+1}G^A+(-1)^{\sigma=1}G^R\right],$$

in (2) we come to the expression

$$\left\langle \hat{\vec{d}}(t)\hat{\vec{E}}(\vec{\mathrm{r}},t)\right\rangle=\frac{1}{2}Tr\int_{-\infty}^{+\infty}d\tau\left[\hat{D}'^K(\vec{r},t;\vec{r}_A,\tau)\hat{\alpha}^A(\tau,t)+\hat{D}'^R(\vec{r},t;\vec{r}_A,\tau)\hat{\alpha}^K(\tau,t)\right],$$

which for stationary states of the electromagnetic field, i.e., when

$$\hat{D}^{K,R}(\vec{r},t;\vec{r}_A,\tau)=\int\frac{d\omega}{2\pi}e^{-i\omega(t-\tau)}\hat{D}^{K,R}(\omega;\vec{r},\vec{r}_A),$$

is reduced to a time-independent quantity and then for the force (1) we find

$$\vec{F}(\vec{r}_A)=i\int_{-\infty}^{+\infty}\frac{\omega^2 d\omega}{4\pi c^2}Tr\left[\hat{\alpha}^A(\omega)\vec{\nabla}\hat{D}^K(\omega;\vec{r},\vec{r}_A)+\hat{\alpha}^K(\omega)\vec{\nabla}\hat{D}^R(\omega;\vec{r},\vec{r}_A)\right]_{\vec{r}=\vec{r}_A} . \quad (3)$$

Further on we will suppose that the particle stays in a local thermodynamic equilibrium at temperature T during the interaction with electromagnetic field, therefore the Keldysh function of the particle is given by

$$\hat{\alpha}^K(\omega)=2i\coth\left(\frac{\omega}{2T}\right)\mathrm{Im}\hat{\alpha}^R(\omega). \quad (4)$$

Expression (3) (with (4)) for the force acting on a small particle in the external electromagnetic field in any arbitrary quantum state is the main result of this paper. This expression also contains all the previous results related to the problem.

3. Casimir--Polder interaction with the wall

As known, in the case of the global equilibrium

$$\hat{D}^K(\omega;\vec{r},\vec{r}_A)=2i\coth\left(\frac{\omega}{2T}\right)\mathrm{Im}\hat{D}^R(\omega;\vec{r},\vec{r}_A) . \quad (5)$$

Expressions (3)-(4) result in the force acting on the particle in equilibrium [11]:

$$\vec{F}(\vec{r}_A)=\int_{-\infty}^{+\infty}\frac{\omega^2 d\omega}{\pi c^2}\frac{1}{e^{-\omega/T}-1}\mathrm{Im}Tr\left[\hat{\alpha}^R(\omega)\vec{\nabla}\hat{D}^R(\omega;\vec{r},\vec{r}_A)\right]_{\vec{r}=\vec{r}_A} . \quad (6)$$

Using analytical properties of the retarded functions $\hat{\alpha}^R,\hat{D}^R$ in the integrand, we can simplify expression (6) representing it as a sum over imaginary frequencies. In fact, replacing the integration over real frequencies ω by the integration in the complex upper half-plane and then using the residue theorem, we come to the Matsubara representation of the force in equilibrium [12]:

$$\vec{F}(\vec{r}_A)=T\sum_{s=0}^{+\infty}(2-\delta_{s,0})k^2{}_s Tr[\hat{\alpha}(i\zeta_s)\vec{\nabla}\hat{D}(i\zeta_s;\vec{r},\vec{r}\,')]_{\vec{r}=\vec{r}_A} \qquad (k_s=\zeta_s/c) . \quad (7)$$

As a simple application of this expression let us consider the interaction force of the particle with a dielectric half-space $z<0$. Inserting the expression for the temperature Green's function $\hat{D}(i\zeta_s;\vec{r},\vec{r}_A)$ [13], we find for the unique nonzero z-component of the force (7)

$$F_z(z_A)=\frac{T}{\pi}\sum_{s=0}^{+\infty}\left(1-\frac{1}{2}\delta_{s,0}\right)\int\vec{k}_\perp d\vec{k}_\perp\left[R-\overline{R}\right]e^{-2w_0 z_A} , \quad (8)$$

$$R=\frac{w_0-w}{w_0+w}k_s^{\,2}\left[n_y^{\,2}\alpha_{xx}(i\zeta_s)+n_x^{\,2}\alpha_{yy}(i\zeta_s)\right], \quad (9.a)$$

$$\overline{R}=\frac{w_0\varepsilon(i\zeta_s)-w}{w_0\varepsilon(i\zeta_s)+w}\{k_\perp^{\,2}\alpha_{zz}(i\zeta_s)+w^2{}_0\left[n_x^{\,2}\alpha_{xx}(i\zeta_s)+n_y^{\,2}\alpha_{yy}(i\zeta_s)\right], \quad (9.b)$$

where $w_0=\sqrt{k_s^{\,2}+k_\perp^{\,2}}, w=\sqrt{\varepsilon(i\zeta_s)k_s^{\,2}+k_\perp^{\,2}}$ and $\vec{n}=\vec{k}_\perp/k_\perp$. For the isotropic particle $\alpha_{ij}=\delta_{ij}\alpha$ and expressions (8), (9) coincide with the well-known result [14]

$$F_z(z_A) = T\sum_{s=0}^{+\infty}(2-\delta_{s,0})k^4{}_s\alpha(i\zeta_s)\int_0^{+\infty} pdpe^{-2k_s p z_A}\left[r + (1-2p^2)\bar{r}\right],$$

where

$$r = \frac{p-s}{p+s},\quad \bar{r} = \frac{\varepsilon(i\zeta_s)p - s}{\varepsilon(i\zeta_s)p + s},\quad s = \sqrt{\varepsilon(i\zeta_s) - 1 + p^2}\ .$$

4. Frictional force in free space

As an application of our result (3) to the case of frictional forces let us consider the simplest problem: frictional force acting on the moving particle in the field of blackbody radiation [15]. In the case of free electromagnetic field the retarded (advanced) function depends on the difference of coordinates [16] and

$$\operatorname{Im}\hat{D}^R(\omega;\vec{k}) = \hat{T}(\vec{k})[\delta(\omega+ck) - \delta(\omega-ck)], \tag{10}$$

$$T_{ij}(\vec{k}) = \frac{2\pi c^2}{k}\left(\delta_{ij} - \frac{k_i k_j}{k^2}\right).$$

In the reference system of the particle moving uniformly with a velocity $\vec{v}$ (relative to the blackbody radiation) the photon distribution function depends also on the photon momentum $\vec{k}$ and is given by

$$h(\omega,\vec{k}) = \coth\left(\frac{\omega - \vec{k}\vec{v}}{2T}\right) \tag{11}$$

and for the Keldysh function we can use the expression

$$\hat{D}^K(\omega;\vec{k}) = 2ih(\omega,\vec{k})\operatorname{Im}\hat{D}^R(\omega;\vec{k})\ . \tag{12}$$

In this case we get for the force (3):

$$F_i = \frac{v}{2\pi c^5 T}\int_0^{+\infty}\frac{\omega^5 d\omega}{\sinh^2(\omega/2T)} f_i(\omega)\ , \tag{13.a}$$

$$f_i(\omega) = \frac{2}{15}\operatorname{Im}\left\{2\delta_{iz} Tr\hat{\alpha}(\omega) - \left[\delta_{ix}\alpha_{xz}(\omega) + \delta_{iy}\alpha_{yz}(\omega) + \delta_{iz}\alpha_{zz}(\omega)\right]\right\}\ . \tag{13.b}$$

For isotropic polarization $\alpha_{ij} = \delta_{ij}\alpha$

$$f_i(\omega) = \delta_{iz}\frac{2}{3}\operatorname{Im}\alpha(\omega),$$

and for the force acting on the particle (in the frame of particle) we find [17,18]

$$\vec{F} = \frac{\vec{v}}{3\pi c^5 T}\int_0^{+\infty} d\omega \frac{\omega^5 \operatorname{Im}\alpha(\omega)}{\sinh^2(\omega/2T)}.$$

To conclude, we derive an expression for the interaction force of a small particle with the quantized electromagnetic field in nonequilibrium regime. This result could be used in the problems of noncontact control and alignment of nanoparticles in the thermal fields out of equilibrium. Besides, the result provides an opportunity to unify in one expression for the force all known results related to the problem, i.e. the Van der Waals interactions of the particle with surroundings and frictional forces acting on the moving particle.

Acknowledgements

I am very grateful to Professors L.Novotny and C.Henkel for stimulating discussions.

References

1. L.Novotny, R.Bian, and X.Xie, Phys. Rev. Lett., **79**, 645 (1997).
2. D.Grier, Nature, **424**, 810 (2003).
3. A.Volokitin and N.Persson, Usp. Fiz. Nauk, **177**, 921 (2007).
4. C.Henkel, K.Joulain, J.Mulet, and J.Greffet, J. Opt. A, **4**, S109 (2002).
5. M.Antezza, L.Pitaevskii, S.Stringeri, and V.Svetovoy, Phys. Rev. A, **77,** 022901 (2008).
6. S.Buhmann and S.Scheel, Prog. Quant. Elect., **31**, 51 (2007).
7. H.C. van der Hulst, *Light Scattering by Small Particles*, Dover, New York, 1981.
8. J.Gordon and A.Ashkin, Phys. Rev. A, **21**, 1606 (1980).
9. L.Keldysh, Sov. Phys.-JETP, **20**, 1018 (1965).
10. J.Rammer and H.Smith, Rev. Mod. Phys., **58**, 323 (1986).
11. L.Novotny and C.Henkel, Opt. Lett., **33**, 1029 (2008).
12. S.Buhmann and S.Scheel, Phys. Rev. Lett., **100**, 253201(2008).
13. A.Maradudin and D.Mills, Phys. Rev. B, **11**, 1392 (1975).
14. V.Parsegian, *Van der Waals Forces,* Cambridge University Press, New York, 2006.
15. A.Einstein, Mitt. Phys. Ges., **18**, 47 (1916).
16. E.Lifshitz and L.Pitaevskii, *Statistical Physics, part 2,* Pergamon, New York, 1981.
17. V.Mkrtchian, V.Parsegian, R.Podgornik, and W.Saslow, Phys. Rev. Lett., **91**, 220801 (2003).
18. J.Zurita-Sanchez, J.Greffet, and L.Novotny, Phys. Rev. A, 69, 022902 (2004).