

# ON THE THEORY OF MULTI-PHOTON RESONANT INTERACTION OF STRONG ELECTROMAGNETIC WAVE WITH SEMICONDUCTORS

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## 1. Introduction

Owing to unique properties of the new generation of lasers the research interest in the nonlinear phenomena is renewed recently. It becomes possible now to broaden essentially the list of objects under investigation and simultaneously to observe a number of new original nonlinear effects which have conceptual character and, undoubtedly, represent certain practical interest (see, e.g., [1,2]).

In such conditions, for direct comparison of the theory and experiment, it is necessary to consider precisely, whenever possible, interactions of a target with electromagnetic wave. Otherwise, when it is impossible, the perturbation theory technique should be applied, considering as higher orders of approximation as possible. It is also necessary for the chosen model to describe adequately all physical properties and characteristics of the target (e.g., the band structure and effective masses in the bands of the semiconductor).

In [3] an attempt was undertaken to investigate the influence of interband accelerations of charge carriers on the resonant reorganization of the quasi-energy spectrum of a direct-band semiconductor. However, later in [4] the technique of work [3] was critically analyzed and it was shown that in the specified approximation the square-law term of interaction of a particle with the field of laser radiation is absent. At the same time the square-law term arises if the influence of non-resonant bands is also taken into account. As a result it was proved that the high-frequency square-law Stark shifts of semiconductor's quasi-energy are defined by the preformed effective masses.

Note that in the specified works [3,4] the unperturbed Bloch functions are chosen as basic states:  $\Psi_n(\vec{r}, t) = U_{n,p}(\vec{r}) \exp -\frac{i}{\hbar}(E_n t - \vec{p}\vec{r})$ , where  $U_{n,p}(\vec{r})$  is a periodic modulating multiplier,  $E_n$  and  $\vec{p}$  are the electron's energy and quasi-momentum, respectively, and the index  $n$  indicates the resonant bands ( $n = c$  corresponds to the conductivity band and  $n = v$  to the valence band).

In the present work, the resonant interaction of direct-band semiconductor is investigated, using basic wave functions of the following type:

$$\Psi_{n,p(t)}(\vec{r}, t) = U_{n,p(t)}(\vec{r}, t) \exp\left(\frac{i}{\hbar} \vec{p} \vec{r}\right) \exp\left(-\frac{i}{\hbar} \int_0^t E_n(p(\tau)) d\tau\right). \quad (1)$$

These wave functions initially precisely describe the interband acceleration in the band under the perturbation  $V(t) = \frac{e\vec{A}\hat{p}}{m_0c} + \frac{e^2\vec{A}^2}{2m_0c^2}$ , where  $\vec{A}(t) = \vec{A}_0 \cos\Omega t$  is the vector-potential of the laser radiation,  $m_0$  is the mass of a free electron, and  $\vec{p}(t) = \vec{p} + \frac{e}{c}\vec{A}(t)$  (see, e.g., [2]).

## 2. Theory

In order to take into account the influence of non-resonant bands on the reorganization of quasi-energy in the field of a wave we should spread out the wave function of a particle in the field of a wave by means of basis functions (1):

$$\Psi(\vec{r}, t) = \sum_n C_n(t) U_{n,p(t)}(\vec{r}, t) \exp\left(\frac{i}{\hbar} \vec{p} \vec{r}\right) \exp\left(-\frac{i}{\hbar} \int_0^t E_n(p(\tau)) d\tau\right). \quad (2)$$

By applying a standard technique the following system of equations for the resonant bands  $n = c, v$  is obtained:

$$\frac{dC_1}{dt} + C_2 \frac{e}{c} \frac{d\vec{A}}{dt} \vec{u}_{c,v} \exp\left(\frac{i}{\hbar} \int_0^t E_{cv} dt + \alpha_{cv}(t)\right) = 0, \quad \frac{dC_2}{dt} + C_1 \frac{e}{c} \frac{d\vec{A}}{dt} \vec{u}_{v,c}^* \exp\left(-\frac{i}{\hbar} \int_0^t E_{cv} dt + \alpha_{cv}(t)\right) = 0, \quad (3)$$

and for non-resonant bands

$$\frac{dC_m}{dt} + C_1 \frac{e}{c} \frac{d\vec{A}}{dt} \vec{u}_{m,c} \exp\left(\frac{i}{\hbar} \int_0^t E_{m,c} dt + \alpha_{m,c}(t)\right) + C_2 \frac{e}{c} \frac{d\vec{A}}{dt} \vec{u}_{m,v} \exp\left(\frac{i}{\hbar} \int_0^t E_{m,v} dt + \alpha_{m,v}(t)\right) = 0, \quad (4)$$

where the following notations are made:  $C_{1,2,m}(t) = C_{c,v,m}(t) \exp\left(\frac{i}{\hbar} \alpha_{c,v,m}(t)\right)$  (here the indices

$m = 1, 2$ , correspond to the resonant bands  $c, v$ , and the others to the non-resonant bands)

$E_{ij} = E_i - E_j$ ,  $\alpha_{ij}(t) = \alpha_i(t) - \alpha_j(t)$ ,  $u_{i,j} = \int U_i^* \frac{\partial}{\partial \vec{p}} U_j d\vec{r}$ , and the parameters  $\alpha_i(t)$  and  $\alpha_j(t)$

consider the influence of non-resonant bands and are defined by the equations

$$C_i(t) \frac{d\alpha_i}{dt} + i\hbar \frac{e}{c} \frac{dA}{dt} \sum_{k \neq c,v} u_{mk} C_k(t) \exp\left(\frac{i}{\hbar} (E_{m,k} + \alpha_{mk}(t))\right) \quad i = 1, 2, m. \quad (5)$$

Let us choose the law of dispersion in bands as square-law and isotropic:

$E_{c,v} = \pm \frac{\Delta}{2} + \frac{p^2}{2m_{c,v}}$ , where effective masses are defined in an ordinary way:

$\frac{1}{m_n} = \frac{1}{m_0} + \frac{2}{m_0^2} \sum_{n' \neq n} \frac{\langle U_{n,0} | \vec{e}_p \vec{p} | U_{n',0} \rangle}{E_n(0) - E_{n'}(0)}$ ,  $\vec{e}_p$  is the unit vector in the direction of electron's momentum,  $\Delta$  is the width of the forbidden band.

We assume also that the Keldysh adiabatic parameter satisfies to the condition  $\gamma = \frac{\hbar\Omega}{eE_0 a_0} \gg 1$  ( $E_0 = \frac{c}{\Omega} A_0$  is the intensity of the laser radiation,  $a_0$  is the crystal lattice constant); in this case the multiphoton processes predominate over processes of tunnel photoionization (see, e.g., [1, 2]).

**1. Resonant approximation.** In this case, if one neglects the influence of non-resonant bands, supposing that  $\alpha_i(t), \alpha_j(t) = 0$ , at the  $n$ -photon resonance ( $n\hbar\Omega = \Delta + \frac{p_0^2}{2\mu} + \frac{e^2 E_0^2}{4\mu\Omega^2}$ ,  $p_0$  is the resonant momentum), from equations (2-5) the specified results of work [3] turn out:

$$E_c^{1,2} = \frac{\Delta}{2} + \frac{p_0^2}{2\mu} + \frac{e^2 E_0^2}{4\mu\Omega^2} \mp \sqrt{\left(\frac{p^2 - p_0^2}{2\mu}\right)^2 + \hbar^2 |\Lambda|^2},$$

$$E_v^{1,2} = -\frac{\Delta}{2} - \frac{p_0^2}{2\mu} - \frac{e^2 E_0^2}{4\mu\Omega^2} \mp \sqrt{\left(\frac{p^2 - p_0^2}{2\mu}\right)^2 + \hbar^2 |\Lambda|^2}.$$

Here  $E_v^{1,2}$  and  $E_c^{1,2}$  are the quasi-energies of the semiconductor with allowance for the interband acceleration of particles,  $\Lambda_n(p) \equiv \Lambda = (-1)^{n+1} 2\lambda_p z_1^{-1} \sum_l (n+2l) \cdot J_{n+2l}(z_1) \cdot J_l(z_2)$  and  $\lambda_p = \frac{eE_0}{2\hbar\omega m_0} \int U_{c,p} \hat{p} U_{v,p} d\vec{r}$  are the sizes of energy gaps, taking into account and without taking into account the interband acceleration of particles, respectively,  $j_n(z)$  is the real-argument Bessel function,  $\Lambda$  is the detuning of resonance,  $z_1 = \frac{e\vec{p}\vec{E}_0}{\mu\hbar\Omega^2}$ ,  $z_2 = \frac{e^2 E_0^2}{8\mu\hbar\Omega^3}$  are parameters considering the interband acceleration, and  $\mu^{-1} = m_c^{-1} - m_v^{-1}$  is the effective mass of the transition.

**2. Consideration of influence of non-resonant bands.** Following the technique of work [4], we consider the role of non-resonant bands, using the perturbation theory. Then from formulas (5) and (6), taking into account the condition  $\Omega \ll \Omega_{v,k}, k \neq c$  ( $\Omega_{v,k}$  is the frequency of the non-resonant transitions  $v \rightarrow k$ ), for the parameters  $\alpha_n(t)$  we have

$$\alpha_c(t) = \frac{e^2 E_0^2}{2\hbar\Omega^2 \tilde{m}_c} t - \frac{e^2 E_0^2}{4\hbar\Omega^3 \tilde{m}_c} \sin 2\Omega t, \quad \alpha_v(t) = \frac{e^2 E_0^2}{2\hbar\Omega^2 \tilde{m}_v} t - \frac{e^2 E_0^2}{4\hbar\Omega^3 \tilde{m}_v} \sin 2\Omega t,$$

where the quantities  $\tilde{m}_c$  and  $\tilde{m}_v$  with the dimensionality of mass are defined by the expressions

$$\frac{1}{\tilde{m}_c} = \sum_{k \neq c} \left( \frac{\Omega}{\Omega_{c,k}} \right)^3 \frac{|\vec{e}_0 \vec{p}_{ck}|^2}{\hbar \Omega_{c,k}}, \quad \frac{1}{\tilde{m}_v} = \sum_{k \neq v} \left( \frac{\Omega}{\Omega_{v,k}} \right)^3 \frac{|\vec{e}_0 \vec{p}_{v,k}|^2}{\hbar \Omega_{v,k}}.$$

Here  $\vec{e}_0$  is the unit vector in the direction of polarization of the laser radiation.

### 3. Conclusions

Comparing the obtained results with results of works [3, 4], the following specific features could be noted:

1. Unlike [3], in the matrix element of resonant transition the mass of a free electron is present and therefore the width of the gap considerably decreases (this was first noted in [4]).

2. Basis functions of type (1) allow considering precisely the interband acceleration of electrons within the framework of resonant approximation. Therefore in the parameter  $z_2$ , which considers the influence of the square-law term of the perturbation  $V(t)$  on interband acceleration, the effective weight of the transition  $\mu^{-1} = m_c^{-1} - m_v^{-1}$  appears instead of the renormalized effective mass as in [4].

3. Consideration of the non-resonant transitions leads to the fact that in the parameter  $z_2$  the effective mass of transition becomes  $\tilde{\mu}^{-1} = \mu^{-1} + \Delta\mu^{-1}$ , where  $\Delta\mu^{-1} = \frac{1}{\tilde{m}_c} - \frac{1}{\tilde{m}_v}$  is defined by

means of formulas (7) and is the small amendment due to the factor  $\left( \frac{\Omega}{\Omega_{n,k}} \right)^3 \ll 1$  ( $n = c, v$ ).

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