ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Մեխանիկա 56, №3, 2003 Механика

УДК 539.3

Stability of Bending Form of Piezoactive Bimorphfic Plates

M.Belubekyan, M.Karapetyan, M.Sarkisyan

The devices, which function based on direct and converse piezoeffects, are widely spreaded in contemporary technics. The interest in estimating the piezoceramical actuators is conditioned by their wide application in practical and scientific activity of human and permanently extending application area.

Due to wide application of layered piezoactuators, and in particular bimorphs, in technical devices, the effective model developing problems and the problems of estimating electric and magnetic fields generated in piezoelectric, are very actual.

Bimorphic elastic plates are being bended on application the reverse directed electric potential to the facing surface. But at the same time the compression stresses occur in piezoelectric, which may lead to instability of the estimated bending condition. In particular, by means of the electric field, it is possible a state of the plate when the cylindricity of bending takes place. In this connection the compression stresses occur along the generatrix of the cylindrical surface. The critical electric intensity which leads to losing the stability of cylindrical form of bending of the plate is being estimated.

The problem of losing the stability of cylindrical form of bending of the bimorphfic piezoceramical plate is being solved analytically within the limits of Kirchhoff's plate theory.

Устойчивость изгибной формы пьезоэлектрической биморфной пластинки
Белубекян М. В., Карапетян М.Э., Саркисян М.Г.

Биморфная пьезоэлектрическая пластина изгибается под действием электрических потенциалов разных знаков, приложенных на лицевых поверхностях пластины и на поверхности раздела. Показывается, что при этом появляются сжимающие напряжения, которые могут привести к неустойчивости пластинки. Определяются критические значения напряженности электрического поля в зависимости от физико-механических и геометрических характеристик пластинки.

Պյեզոէլեկտրիկ բիմորֆ սալի ծռման ձևի կայունությունը
Մ.Վ. Բելուբեկյան, Մ.Է. Կարապետյան, Մ.Գ. Սարգիսյան

Պյեզոէլեկտրիկ բիմորֆ սալը ծռվում է դիմային և բաժանման մակերևույթների վրա ազդող տարբեր նշանի պոտենցիալների հետևանքով: Ցույց է տրվում, որ այդ դեպքում առաջանում է սեղմող լարում, որը կարող է բերել սալի անկայունության: Որոշվում են էլեկտրական դաշտի լարվածության կրիտիկական արժեքները, կախված սալի ֆիզիկա-մեխանիկական և երկրաչափական պարամետրերից:

INTRODUCTION

Two thin plates of a different thickness made of piezoelectric material (class 6mm) are glued along the front-face area. The plates are polarized by the thickness. The front-face areas are electrified to establish the potential difference between the front-face areas with the help of the electric field. It is necessary to determine the strain-stress state of a thin bimorphfic plate, conditioned by the electric field. It is considered that the electric field is given and the influence of the converse piezoelectric effect is neglected. The influence of the thickness of the gluing material is also neglected.

Many authors investigated the case, when the thickness of the plates is the same. The review of the mentioned manuals is given in [1].

1. (X0Y) coordinate space coincides with the contact plane (Pic. 1). The electric field in the plate is set up with the help of the applied potential difference between the front-face areas $z = h_1$, $z = -h_2$ and the interface surface $z = 0$. Only the direct piezoeffect is taken into consideration and the electric field is given in the following way (Pic. 2):

$$E_3 = \begin{cases} -E_0(x,y), & 0 < z < h_1 \\ E_0(x,y), & -h_2 < z < 0 \end{cases}$$

$$\overline{E} = E_3\vec{k}, \quad E_3 = \begin{cases} -E_0(x,y) & when \quad 0 < z \le h_1 \\ E_0(x,y) & when \quad -h_2 \le z < 0 \end{cases} \tag{1.1}$$

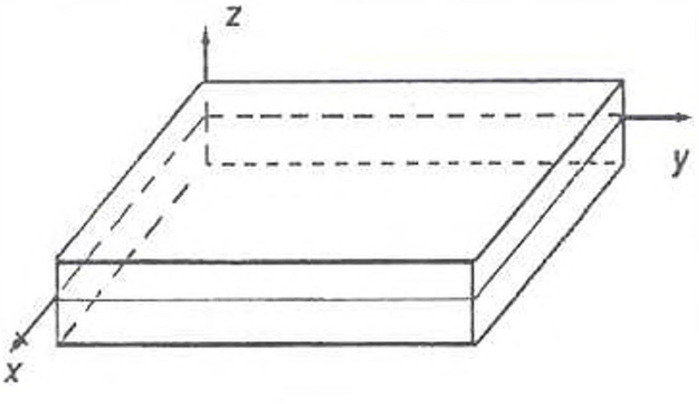
Taking into account the assumptions of the Kirchoff's theory for the package in general, the constitutive equations for the principal stresses of the piezoelectric material (class 6mm) have the following appearance [2]

$$\sigma_{11} = a_{11}\varepsilon_{11} + a_{12}\varepsilon_{22} - sE_3, \sigma_{22} = a_{12}\varepsilon_{11} + a_{11}\varepsilon_{22} - sE_3$$

$$\sigma_{12} = \frac{1}{2}(c_{11} - c_{12})\varepsilon_{12} \tag{1.2}$$

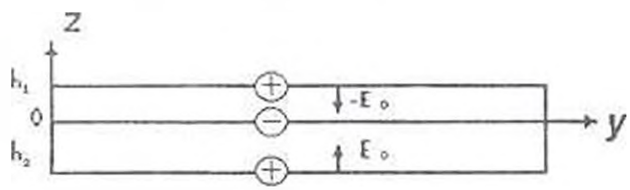
where

$$a_{11} = c_{11} - \frac{c_{13}^2}{c_{33}}, \; a_{12} = c_{12} - \frac{c_{13}^2}{c_{33}}, \; s = e_{31} - \frac{e_{33}c_{13}}{c_{33}} \tag{1.3}$$



Pic. 1

$0 \le x \le a, \; 0 \le y \le b, \; -h_2 \le z \le h_1$



Pic.2

Taking into consideration the assumptions concerning the character of displacements' modification by plate thickness the (1.2) equations take the following forms:

$$\sigma_{11} = a_{11}\frac{\partial u}{\partial x} + a_{12}\frac{\partial v}{\partial y} - z\left(a_{11}\frac{\partial^2 w}{\partial x^2} + a_{12}\frac{\partial^2 w}{\partial y^2}\right) - sE_3$$

$$\sigma_{22} = a_{12}\frac{\partial u}{\partial x} + a_{11}\frac{\partial v}{\partial y} - z\left(a_{12}\frac{\partial^2 w}{\partial x^2} + a_{11}\frac{\partial^2 w}{\partial y^2}\right) - sE_3 \qquad (1.4)$$

$$\sigma_{12} = \frac{c_{11} - c_{12}}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - 2z\frac{\partial^2 w}{\partial x \partial y}\right)$$

where u, v, w are the appropriate displacements of plate's $z = 0$ plane

In averaged equations of balance

$$\int_{-h_2}^{h_1}\frac{\partial \sigma_{ij}}{\partial x_j}dz = 0 \ (i = 1,2,3), \ \int_{-h_2}^{h_1} z\frac{\partial \sigma_{ij}}{\partial x_j}dz = 0 \ (i = 1,2) \qquad (1.5)$$

Strains and moments are defined in the following way

$$T_1 = \int_{-h_2}^{h_1}\sigma_{11}dz = 2h\left(a_{11}\frac{\partial u}{\partial x} + a_{12}\frac{\partial v}{\partial y}\right) - \frac{h_1^2 - h_2^2}{2}\left(a_{11}\frac{\partial^2 w}{\partial x^2} + a_{12}\frac{\partial^2 w}{\partial y^2}\right) - s(h_2 - h_1)E_0$$

$$T_2 = 2h\left(a_{12}\frac{\partial u}{\partial x} + a_{11}\frac{\partial v}{\partial y}\right) - \frac{h_1^2 - h_2^2}{2}\left(a_{12}\frac{\partial^2 w}{\partial x^2} + a_{11}\frac{\partial^2 w}{\partial y^2}\right) - s(h_2 - h_1)E_0$$

$$S = h(c_{11} - c_{12})\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - \frac{h_1^2 - h_2^2}{2h}\frac{\partial^2 w}{\partial x \partial y}\right)$$

$$(1.6)$$

$$M_1 = \frac{h_1^2 - h_2^2}{2}\left(a_{11}\frac{\partial u}{\partial x} + a_{12}\frac{\partial v}{\partial y}\right) - \frac{h_1^3 + h_2^3}{3}\left(a_{11}\frac{\partial^2 w}{\partial x^2} + a_{12}\frac{\partial^2 w}{\partial y^2}\right) + s\frac{h_1^2 + h_2^2}{2}E_0$$

$$M_2 = \frac{h_1^2 - h_2^2}{2}\left(a_{12}\frac{\partial u}{\partial x} + a_{11}\frac{\partial v}{\partial y}\right) - \frac{h_1^3 + h_2^3}{3}\left(a_{12}\frac{\partial^2 w}{\partial x^2} + a_{11}\frac{\partial^2 w}{\partial y^2}\right) + s\frac{h_1^2 + h_2^2}{2}E_0$$

$$H = \frac{h_1^2 - h_2^2}{4}(c_{11} - c_{12})\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - \frac{4}{3}\frac{h_1^3 + h_2^3}{h_1^2 - h_2^2}\frac{\partial^2 w}{\partial x \partial y}\right)$$

The substitution of (1.6) to the averaged equations (1.5) brings to the following equations concerning the planar displacements u, v and w flexure:

$$\Delta u + \theta\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \frac{(h_1 - h_2)a_{11}}{c_{11} - c_{12}}\frac{\partial}{\partial x}\Delta w - \frac{s(h_1 - h_2)}{h(c_{11} - c_{12})}\frac{\partial E_0}{\partial x}$$

$$\Delta v + \theta\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \frac{(h_1 - h_2)a_{11}}{c_{11} - c_{12}}\frac{\partial}{\partial y}\Delta w - \frac{s(h_1 - h_2)}{h(c_{11} - c_{12})}\frac{\partial E_0}{\partial y} \qquad (1.7)$$

$$D\Delta^2 w = h_1 h_2 s \Delta E_0 \qquad (1.8)$$

where

$$D = \frac{2h^3}{3}a_{11},\ 2h = h_1 + h_2,\ \theta = \frac{c_{11} + c_{12}}{c_{11} - c_{12}} - \frac{2c_{13}^2}{(c_{11} - c_{12})c_{33}} \tag{1.9}$$

Hereby the expressions for the intersecting strains look like

$$\begin{aligned} N_1 &= -D\frac{\partial}{\partial x}\Delta w + h_1 h_2 s\frac{\partial E_0}{\partial x} \\ N_2 &= -D\frac{\partial}{\partial y}\Delta w + h_1 h_2 s\frac{\partial E_0}{\partial y} \end{aligned} \tag{1.10}$$

It is necessary to mention that the equation (1.8), which defines the bending, is autonomous. In particular, when $h_1 = h_2$ the equations defining the planar displacements are also being separated.

2. The boundary conditions of the problem are determined in accordance with Kirchhoff's theory of plate. Hereinafter are the versions of known conditions for the plate's $x = \text{const}$ edge.

For the fixed edge these conditions are not changed and have the following appearance:

$$u = v = 0,\ w = 0,\ \partial w/\partial x = 0 \tag{2.1}$$

Boundary conditions of free supporting

$$T_1 = 0,\ v = 0,\ w = 0,\ M_1 = 0$$

after some transformations take the following form (with regard to (1.6) expressions)

$$\begin{aligned} &\frac{\partial u}{\partial x} = \frac{h_1 - h_2}{12D}h_1^2\zeta s E_0,\ v = 0\ \left(\zeta = -1 + 4\frac{h_2}{h_1} - \frac{h_2^2}{h_1^2}\right) \\ &w = 0,\ \frac{\partial^2 w}{\partial x^2} = \frac{h_1 h_2}{D}sE_0 \end{aligned} \tag{2.2}$$

Conditions of sliding contact

$$u = 0,\ S = 0,\ \frac{\partial w}{\partial x} = 0,\ N_1 = 0$$

are transformed to

$$u = 0,\ \frac{\partial v}{\partial x} = 0,\ \frac{\partial w}{\partial x} = 0,\ \frac{\partial^3 w}{\partial x^3} = \frac{h_1 h_2}{D}s\frac{\partial E_0}{\partial x} \tag{2.3}$$

At last the conditions of free edge

$$T_1 = 0,\ S = 0,\ M_1 = 0,\ \tilde{N}_1 = 0,\ \left(\tilde{N}_1 = N_1 + \frac{\partial H}{\partial y}\right)$$

take the following form

$$\begin{aligned} &a_{11}\frac{\partial u}{\partial x} + a_{12}\frac{\partial v}{\partial y} = \frac{h_1 - h_2}{2}\left(a_{11}\frac{\partial^2 w}{\partial x^2} + a_{12}\frac{\partial^2 w}{\partial y^2} - \frac{s}{h}E_0\right) \\ &\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = (h_1 - h_2)\frac{\partial^2 w}{\partial x \partial y} \end{aligned}$$

$$a_{11}\frac{\partial^2 w}{\partial x^2}+a_{12}\frac{\partial^2 w}{\partial y^2}=\frac{3h_1h_2}{2h^3}sE_0$$
$$\frac{\partial}{\partial x}\left[a_{11}\frac{\partial^2 w}{\partial x^2}+\left(a_{11}+c_{11}-c_{12}\right)\frac{\partial^2 w}{\partial y^2}\right]=\frac{3h_1h_2}{2h^3}s\frac{\partial E_0}{\partial x} \tag{2.4}$$

3. From the equation (1.8) and boundary conditions follow, that the electric field leads to bending of the plate. Hereby, in accordance with equations (1.7) the planar displacements appear in the plate's $z=0$ plane, which brings to the T_1, T_2 forces appearance. These forces, in general, can be compressive, which will bring to the plate stability losing [3].

As an example a rectangular plate occupying area $0 \le x \le a$, $0 \le y \le b$, $-h_2 \le z \le h_1$ is under consideration. It is assumed that

$$E_0=\text{const} \tag{3.1}$$

On the $y=0,b$ edges of plate the conditions of sliding contact are given which look like (according to (2.3) and condition (3.1))

$$\frac{\partial u_0}{\partial y}=0,\ v_0=0,\ \frac{\partial w_0}{\partial y}=0,\ \frac{\partial^3 w_0}{\partial y^3}=0 \tag{3.2}$$

In case when the plate's $x=0,a$ edges are freely supported (2.2), the plate is bending to a form of cylindrical surface.

$$w_0=-\frac{h_1h_2}{2D}sE_0ax\left(1-\frac{x}{a}\right) \tag{3.3}$$

For the planar strains the following expressions are received

$$T_{10}=0, T_{20}=\left(h_1-h_2\right)\left(c_{11}-c_{12}\right)s\,a_{11}^{-1}E_0,\ S_0=0 \tag{3.4}$$

In case when plate's $x=0$ edge is fixed (2.1), and the $x=a$ edge is free (2.4), cylindrical form of the plate's bending surface looks like

$$w_0=\frac{h_1h_2}{2D}sE_0x^2 \tag{3.5}$$

and the appropriate strains are determined in the following way:

$$T_{10}=0, T_{20}=\left(h_1-h_2\right)\left(1-\frac{a_{12}}{a_{11}}\right)sE_0,\ \ S_0=0 \tag{3.6}$$

From (3.4) and (3.5) follows that T_{20} strain will be compressive under the following condition:

$$\left(h_1-h_2\right)s<0 \tag{3.7}$$

4. The equation of the stability of the plate with given strains ((3.4) or (3.5)) take the following form:

$$D\Delta^2 w-T_{20}\frac{\partial^2 w}{\partial y^2}=0 \tag{4.1}$$

The solution of (4.1) equation with the appropriate boundary conditions has a following appearance:

$$w=w_0\left(x\right)+\overline{w}\left(x,y\right) \tag{4.2}$$

where w_0 is the initial form of the bending of plate, $\tilde{w}(x, y)$ – is arbitrary disturbance. The substitution (4.2) to (4.1) and boundary conditions brings to the homogeneous equation and homogeneous boundary conditions relative to $\tilde{w}$, i.e. brings to the known problems of the stability of plate.In case when the conditions of sliding contact are given on the plate's $y = 0, b$ edges and $x = 0, a$ edges are hinged, for the critical values of the intensity of electric field E_0 the following expression is received:

$$E_{mn} = \frac{\left(\mu_m^2 + \lambda_n^2\right)^2 a_{11} D}{\lambda_n^2 (h_2 - h_1) s (c_{11} - c_{12})}, \quad \mu_m = \frac{m\pi}{a}, \lambda_n = \frac{n\pi}{b} \tag{4.3}$$

It is obvious that the minimal critical value will be received when $m = 1$. The value of n for the minimal critical value depends on relative dimensions of the plate.

For the square plate $(b = a)$ $n = 1$ the value of the minimal intensity of the electric field has the following appearance:

$$E_* = \frac{4\pi^2 a_{11} D}{(h_2 - h_1) s (c_{11} - c_{12}) a^2} \tag{4.4}$$

An appropriate expression for the plate with dimensions $b = 2a$ $(n = 2)$ looks like

$$E_* = \frac{25\pi^2 a_{11} D}{(h_2 - h_1) s (c_{11} - c_{12}) a^2} \tag{4.5}$$

The typical values of a_{11} and s (in SI system) calculated on the base of electro elastic modules from [2] are listed in the table 1.

Table 1

Material	$10^{10}\ N/m^2$				c/m^2			$10^{-11}\ F/m$		$10^{10}\ N/m^2$	c/m^2
	c_{11}	c_{12}	c_{13}	c_{33}	e_{15}	e_{31}	e_{33}	ε_{11}	ε_{33}	a_{11}	s
$PZT-4$	13,9	7,8	7,4	11,5	12,7	-5,2	15,1	650	560	9,14	-14,92
ZnO	20,97	12,11	10,51	21,09	-0,59	-0,61	1,14	7,38	7,83	15,73	-1,18
CdS	8,56	5,32	4,62	9,36	-0,21	-0,24	0,44	7,99	8,44	6,28	-0,46

In a particular case of bimorphic construction $h_2 = 2h_1$, the expression (4.4) defining the critical value of the intensity of electric field, can be expressed in a more suitable way:

$$E_* = k\left(\frac{2h}{a}\right)^2 \cdot 10^{10} \quad v/m \tag{4.6}$$

For the materials listed in Table 1 the k coefficient has the following values:

$$\alpha = 0.92\ (PZT-4),\ \alpha = 23.65\ (ZnO),\ \alpha = 26.47\ (CdS)$$

From the expression (4.6) it follows that the critical value of the intensity of electric field is essentially dependent on the relative thickness of the plate.

REFERENCES

1. Saravanos D.A, Heyliger P.R. Mechanics and computational models for laminated piezoelectric beams, plates and shells.// Appl. Mech. Rev 1999, vol 52, N 10, p.305-320.
2. Royer D, Dieulesaint E. Elastic Waves in Solids. Masson 1999, v1. 446p.
3. Ambartsumyan S.A., Belubekyan M.V. On an approach to defining effective moduli of nonsymmetrically assembled multilayered plates. In "Problems of Strength and Plasticity". Higher School Collection. Nizhni Novgorod state University Publishers. 2000, p.26-30. (In Russian)

Institute of Mechanics NAS Armenia
Yerevan State Engineering University

Received
24.08.2003