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GRAVITATIONAL RADIATION FROM PULSATING MAGNETIC WHITE DWARFS

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Rotating white dwarfs undergoing quasi-radial oscillations can emit gravitational radiation in a frequency range from 0.1-0.3 Hz. Assuming that the energy source for the gravitational radiation comes from the oblateness of the white dwarf induced by the rotation, the strain amplitude is found to be 10^{-25} for a white dwarf at ~50 pc. We had calculated thermal energy loses through magneto-hydrodynamic mechanism and found it smaller than estimated before. The galactic population of these sources is estimated to be 10^7 and may produce a confusion-limited foreground for proposed advanced detectors in the frequency band between space-based and ground-based interferometers. Nearby oscillating white dwarfs may provide a clear enough signal to investigate white dwarf interiors through gravitational wave astroseismology.

Key words: *(stars:)white dwarfs: gravitational radiation*

1. *Introduction.* There are a number of gravitational radiation detectors, planned, under construction and operational covering a wide frequency spectrum from $\sim 10^{-9}$ Hz all the way up to $\sim 10^{4}$ Hz. Expected sources of gravitational radiation include numerous astrophysical sources such as compact binaries, supermassive black holes, and binary coalescence. In addition, there is an expected cosmological background of gravitational radiation arising from the very earliest times of the universe. The coverage of the spectrum is not complete and the gap between space-based interferometers (such as LISA) and ground-based interferometers (such as LIGO and VIRGO) has been proposed as a possible "clean window", devoid of continuous foreground sources, through which the cosmological background of gravitational radiation could be seen [1]. Most binary white dwarf systems will coalesce before their gravitational wave frequency rises above 0.1 Hz, while more massive binaries such as double neutron star or black hole binaries will be sweeping through this band on the way to their eventual coalescence in the ground-based frequency band. As it was mentioned in [2] magnetized rotating white dwarfs undergoing quasi-radial oscillations will emit gravitational radiation in frequency band above 0.1 Hz. In that paper the fraction of energy gone to gravitational radiation was estimated requiring that the largest Doppler broadening of spectral lines due to pulsations be less than thermal Doppler broadening, mentioning that it would be difficult to find exact values because of dearth in observational data. Here

we will give another approach: we estimate energy dissipation and compare power losses with gravitational radiation energy. We assume that deformation energy of white dwarf will dissipate through two mechanisms: a) Joule losses: energy losses coming from motion of plasma in magnetic field and b) viscous friction of matter during self-similar oscillations of white dwarfs. Viscous friction will be zero because during self-similar oscillations there is no relative motion of matter. Magneto-hydrodynamic losses are calculated for different configurations of white dwarfs. For all considered configurations energy losses are much smaller than estimated before in [2]. So the gravitational wave amplitudes and gravitational wave fluxes from rotating and oscillating white dwarfs now are reconsidered. These sources will be essentially monochromatic and long-lived. Given the number of white dwarfs in the galaxy, it is quite possible that this population will produce a confusion-limited foreground of sources in this frequency band that will mask the cosmological background. In section 2 quasiradial oscillations of white dwarf will be considered, expressions for gravitational radiation power and gravitational wave amplitude will be obtained. In section 3 thermal losses are considered. In section 4 deformation energy of spin-down white dwarf is suggested as the source of energy for oscillations and from energy balance equation expression for relative amplitude of oscillations was obtained. The results of numerical calculations are presented in section 5 with the emphasis on the strain amplitudes and fluxes of gravitational waves from a population of white dwarfs. Our conclusions are summarized in section 6.

2. *Quasiradial Oscillations.* Quasiradial oscillations of rotating white dwarfs were investigated in the early 1970's [3,4] where the frequency spectrum of the fundamental oscillation mode for maximally rotating white dwarfs was determined. These stars are oblate due to their rotation and consequently they have a non-zero quadrupole moment. The oscillations add a time dependence to the quadrupole moment [5]. The oscillation is described by assigning each mass element a time dependent coordinate given by $x_\alpha = x_\alpha^0(1+\eta\sin\omega t)$ where $\eta \ll 1$ and is a constant. Thus, the reduced quadrupole moment is given by:

$$Q_{\alpha\beta} = \int \rho\left(x_\alpha x_\beta - \frac{1}{3}\delta_{\alpha\beta} x^2\right) d^3x \approx Q_{\alpha\beta}^0(1+2\eta\sin\omega t), \tag{1}$$

where $Q_{\alpha\beta}$ are the components of the quadrupole moment of the rotating oblate white dwarf in equilibrium and we have neglected terms of order η^2. Taking the axis of rotation to lie along the z-axis, the non-zero components of the quadrupole moment obey:

$$Q^0 = -Q_{zz}^0 = 2Q_{xx}^0 = 2Q_{yy}^0 \,. \tag{2}$$

The power emitted in gravitational radiation is given by:

$$J = \frac{G}{5c^5}\left|\frac{d^3}{dt^3}Q_{\alpha\beta}\right|^2 , \tag{3}$$

and consequently one obtains:

$$J = \frac{6G}{5c^5}\eta^2\omega^6\left|Q^0\right|^2\cos^2\omega t' = J_0\cos^2\omega t' , \tag{4}$$

where the retarded time is $t' = t - r/c$ for a source at distance r. To determine the waveform and the angular distribution of the radiation we rotate to coordinates in which the wave vector lies along the z-axis and use the transverse-traceless gauge. Consequently,

$$h_+ = \frac{1}{2}\left(h_{xx} - h_{yy}\right) = \frac{3GQ^0\eta\omega^2}{c^4 r}\sin^2\theta\sin\omega t' , \tag{5}$$

$$h_\times = h_{xy} = 0 , \tag{6}$$

where θ is the angle between the wave vector and the axis of rotation of the white dwarf. We can express the strain amplitude in terms of the power by combining (4) with (5) to obtain:

$$h_+ = \sqrt{\frac{15GJ_0}{2c^3}}\frac{1}{r\omega}\sin^2\theta\sin\omega t' = h_0\sin^2\theta\sin\omega t' . \tag{7}$$

If an energy source can be found to drive the pulsations, the rate at which power is put into the vibrations can be combined with the lifetime of the energy source to estimate the strain amplitude from an individual white dwarf and thus from the galactic population as a whole. We discuss a possible mechanism in the section 4. But first we shall calculate energy dissipation.

3. *Thermal losses.* During the self-similar oscillations there will be following dissipative processes in viscous conducting plasma of rotating oblate magnetic white dwarf. These processes can be taken into account by the equation of the heat transfer [6]

$$\rho T\left(\frac{\partial S}{\partial t} + \left(\bar{V}\nabla\right)S\right) = \frac{c^2}{16\pi^2\sigma_e}\left(\mathrm{rot}\,\bar{b}\right)^2 + \frac{\xi}{2}\left(\sigma_{ij} - \frac{2}{3}\delta_{ij}\mathrm{div}\,\bar{V}\right)^2 + \zeta\left(\mathrm{div}\,\bar{V}\right)^2 + \kappa\Delta T . \tag{8}$$

The first term in right hand side is dissipation through Joule thermal losses, σ_e is coefficient of electric conductivity that for white dwarfs equals to $\sigma_e = 5.34\cdot 10^{21}\sqrt{1+\rho_6^{2/3}}/Z$ [7], where Z is atomic number of matter. Second and third terms are dissipation through viscosity (ξ and ζ are first and second coefficients of viscosity), forth term is dissipation through thermal conductivity, and

$$\sigma_{ij} = \frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \tag{9}$$

is stress tensor. The self-similar oscillations of magnetized plasma could be considered as particular case of the standing magneto-hydrodynamic waves when $\lambda >> R$, where λ is wavelength, R - radius of the white dwarf. In this case, according to [8], the amplitude b_0 of the induced magnetic field b is connected to velocity of oscillating plasma V by following equation:

$$b_0 = \frac{BV}{c_n}, \tag{10}$$

where B is the constant magnetic field of white dwarf, c_n is the normal velocity of magneto-hydrodynamic wave, which is given by [9]

$$c_n = \sqrt{c_s^2 + c_A^2}, \tag{11}$$

where $c_s = \sqrt{(\partial P/\partial\rho)_s}$ is the velocity of sound waves and $c_A = B/\sqrt{4\pi\rho}$ is the velocity of Alfven waves. For the velocity of plasma during self-similar oscillations one can write

$$\vec{V} = \vec{r}_0\, \eta\omega\cos\omega t, \tag{12}$$

so the first term of (8), which is the dissipating energy in unit volume in unit time, is equal to:

$$J_1 = \frac{c^2}{32\pi^2\sigma_e} \frac{\omega^4}{c_n^4} B^2 r_0^2 \eta^2. \tag{13}$$

Finally, the full dissipating energy in unit time equals to

$$J_T = \eta^2 \frac{c^2\omega^4}{8\pi} B^2 \int_0^R \frac{r^4}{c_n^4 \sigma_e} dr. \tag{14}$$

Let's consider now the second term at right hand side of (8) describing energy losses through viscous friction. For self-similar oscillations the components of plasma velocity in spherical coordinates are:

$$V_r = \dot{r} = \frac{r\eta\omega\cos\omega t}{1+\eta\sin\omega t}, \quad V_\theta = 0, \quad V_\varphi = \Omega r \sin\theta. \tag{15}$$

For stress tensor we obtain the following components:

$$\begin{aligned}
\sigma_{rr} &= 2\frac{\partial V_r}{\partial r} = \frac{2\eta\omega\cos\omega t}{1+\eta\sin\omega t}, \\
\sigma_{\varphi\varphi} &= 2\left(\frac{1}{r\sin\theta}\frac{\partial V_\varphi}{\partial\varphi} + \frac{V_r}{r} + \frac{V_\theta \operatorname{ctg}\theta}{r}\right) = \frac{2\eta\omega\cos\omega t}{1+\eta\sin\omega t}, \\
\sigma_{\theta\theta} &= 2\left(\frac{1}{r}\frac{\partial V_\theta}{\partial\theta} + \frac{V_r}{r}\right) = \frac{2\eta\omega\cos\omega t}{1+\eta\sin\omega t}, \\
\sigma_{r\theta} &= \left(\frac{1}{r}\frac{\partial V_r}{\partial\theta} + \frac{\partial V_\theta}{\partial r} - \frac{V_\theta}{r}\right) = 0, \\
\sigma_{\theta\varphi} &= \left(\frac{1}{r\sin\theta}\frac{\partial V_\theta}{\partial\varphi} + \frac{1}{r}\frac{\partial V_\varphi}{\partial\theta} - \frac{V_\varphi \operatorname{ctg}\theta}{r}\right) = 0, \\
\sigma_{\varphi r} &= \left(\frac{\partial V_\varphi}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial V_r}{\partial\varphi} - \frac{V_\varphi}{r}\right) = 0.
\end{aligned} \tag{16}$$

In spherical coordinates

$$\frac{2}{3}\operatorname{div}\vec{V} = \frac{2}{3}\left(\frac{1}{r^2}\frac{\partial\left(r^2V_r\right)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta V_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial V_\varphi}{\partial\varphi}\right) = \frac{2\eta\omega\cos\omega t}{1+\eta\sin\omega t},$$

so one can easily see that $\sigma_{ij} - \frac{2}{3}\delta_{ij}\operatorname{div}\vec{V} = 0$. This result is coming from combining rigid rotation with self-similar oscillations: during this kind of oscillations the matter do not have relative motion in the configuration. We neglect the third and forth terms in (8) because they are too small in comparison to the first term.

In order to calculate thermal losses (14), we need to know the radial dependence of conductivity of plasma σ_e and velocity c_n. For that purpose we integrate stellar configurations using following parametric equation of state for white dwarfs:

$$P = \frac{4}{3}\left(\frac{m_e}{m_n}\right)^4 K_n\left[x\left(2x^3-3\right)\sqrt{1+x^2} + 3\ln\left(x+\sqrt{1+x^2}\right)\right], \tag{17}$$

$$\rho = \frac{32}{3}\left(\frac{m_e}{m_n}\right)^3 \frac{K_n}{c^2}\frac{A}{Z}x^3, \tag{18}$$

here $K_n = m_n^4c^5/32\pi^2\hbar^3$, $x = p_e/m_ec$, where p_e is Fermi momentum of electron, A and Z are atomic mass and number of nuclei. For white dwarfs usually $A/Z = 2$, but in the case of full degenerate plasma more accurate formula is [10]

$$\frac{A}{Z} = 2 + 1.255\cdot10^{-2}\,x + 1.755\cdot10^{-5}\,x^2 + 1.376\cdot10^{-6}\,x^3. \tag{19}$$

Here dimensionless parameter x is varying from x_{cent} to $x=0.03$ which corresponds to density about $50\,\mathrm{gr/cm^3}$.

The results of numerical calculations are discussed in section 5.

4. *Energy Source.* If there is no permanent source of energy to feed the gravitational radiation, the oscillation energy will quickly radiate away in about 1000 years [5]. Since the ultimate source of the gravitational radiation from the white dwarf is the oblateness arising from the rotation, we propose that the deformation energy of the white dwarf provides the energy to drive the oscillations. In this scenario, as the white dwarf spins down, it will transition from oblate to spherical shape. This transition will trigger starquakes, which will feed the oscillations, which drive the gravitational radiation. Thus, we assume that the power fed into the oscillations is equal to rate at which energy is lost in the deformation energy subtracting dissipation energy. We use the technique of [11] to calculate the deformation energy.

From numerical results, the dependence of the mass of the rotating and non-rotating configurations is found to be linear with the baryon number, so $M = kN$ and $M_0 = k_0 N$, where the subscript "0" indicates the non-rotating configuration. The mass difference between the rotating and non-rotating configurations with the same baryon number is a result of the additional energy of rotation $W_r(\Omega)$ as well as the potential energy due to crustal deformation $W_g(\Omega)$, thus:

$$W_g(\Omega) = \Delta Mc^2 - W_r(\Omega), \tag{20}$$

where Ω is the angular velocity of the white dwarf, I is its moment of inertia, and the mass difference in grams is given by

$$\Delta M = (k - k_0)N = 8.96 \cdot 10^{-29}\, N\,. \tag{21}$$

The appropriate parameters for maximally rotating white dwarfs were calculated in [11,12]. These parameters are presented in Table 1. To obtain the

Table 1

STRUCTURAL PARAMETERS OF MAXIMALLY ROTATING WHITE DWARFS

$\rho_{c(6)}$	$N_{(57)}$	$M / M_\odot$	Ω_{max}	$I_{(48)}$	$Q^0_{(48)}$	$R_{e(8)}$	$\Delta Mc^2_{(49)}$	$W_{r(49)}$	$W_{g(49)}$	ω
2.403	0.4997	0.5946	0.196	128	20.48	10.93	4.799	0.246	4.55	0.758
19.38	0.8398	0.9993	0.476	88.6	14.27	7.342	8.066	1.00	7.06	0.794
157.7	1.0695	1.2731	1.06	39.5	4.766	4.625	1.027	2.23	8.05	1.51
866.1	1.134	1.3502	2.04	15.9	1.554	3.044	1.090	3.32	7.58	1.99
2586	1.1261	1.3412	3.11	8.17	0.673	2.287	1.082	3.94	6.89	0.967

Comments:

ρ_c is the central density of the configuration in units of 10^6 g/cm^3, N - baryon number of configuration in units of 10^{57} particles, I - moment of inertia and Q^0 - quadrupole moment in units of 10^{48} g cm^2, R_e - equatorial radius in units of 10^8cm. ΔMc^2, W_r, W_g are given in units of 10^{49} erg.

deformation energy for rotation rates less than Ω_{max}, we use the fact that the above-mentioned results were obtained using a linear expansion in the small dimensionless parameter $\beta_0 = \Omega^2/8\pi G\rho_c$, where ρ_c is the central density. Consequently, we can write:

$$W_g(\Omega) = \left(\frac{\Omega}{\Omega_{max}}\right)^2 W_g(\Omega_{max}). \tag{22}$$

It now remains to determine the time scale τ, for a spin-down mechanism so that we can relate the power in gravitational radiation to the decrease in deformation energy by:

$$J_0 = \beta \frac{W_g}{\tau}, \tag{23}$$

β is "branching ratio" that quantifies the fraction of deformation energy that goes into gravitational radiation from fundamental mode. For calculating coefficient β we can write the emitted power in gravitational radiation from (4) and energy losses from (14) as:

$$J_0 = U_g \eta^2 , \tag{24}$$

$$J_T = U_t \eta^2 . \tag{25}$$

The deformation energy released in unit time goes to gravitational radiation and thermal losses:

$$\frac{W_g}{\tau} = U_g \eta^2 + U_t \eta^2 , \tag{26}$$

Finally we come out with expression for "branching ratio"

$$\beta = \frac{U_g}{U_g + U_t} , \tag{27}$$

while for amplitude of white dwarfs oscillations we get

$$\eta = \sqrt{\frac{W_g}{U_g + U_t} \frac{1}{\tau}} . \tag{28}$$

Now let's turn to determination of spin-down time τ. We assume that white dwarf spins-down due to magneto-dipole radiation torque, which occurs if the magnetic field is oblique [13]. Observational data for 65 isolated white dwarfs indicates the magnetic field strength on the surface of these stars lies in the range $\sim 3 \cdot 10^4$ to $\sim 10^9$ G [14]. If we define α to be the angle between the magnetic and rotation axes, the spin-down rate of the white dwarf is given by:

$$\dot{\Omega} = -\frac{2\mu^2 \Omega^3}{3 I c^3} \sin^2 \alpha , \tag{29}$$

where $\mu = BR^3$ is the magnetic moment, B is the magnetic field strength. Then the characteristic time scale will be:

$$\tau = \frac{\Omega}{2 |\dot{\Omega}|} . \tag{30}$$

As one can see, energy radiated through gravitational waves is greater than energy lost on magneto-dipole radiation. Gravitational radiation cannot impact spin-down rate because white dwarf remains axial symmetric during oscillations so gravitational wave would not take angular momentum. In our model there are two sources of energy: rotational energy of the star that is taken by magneto-dipole radiation and deformation energy that is feeding on gravitational radiation and thermal losses.

5. *Strain Amplitudes and Luminosities.* The rotation rates for white dwarfs are difficult to measure since these objects have little or no surface blemishes and gravitational broadening of their spectral lines overwhelms the expected rotational broadening [15]. Fortunately, some magnetic white dwarfs show time variability in their magnetic features, which allows for their rotation rates to be inferred [14,15]. Magnetic white dwarfs are thought to make up roughly 1% of the white dwarf population and isolated magnetic

white dwarfs tend to have an average mass ($\sim 0.95\, M_{\odot}$) that is well above that of nonmagnetic white dwarfs ($\sim 0.57\, M_{\odot}$). There is evidence that the population of magnetic white dwarfs is bimodal with high mass stars (near the Chandrasekhar limit) having rotational periods between ~700 s and several hours [14]. This may be evidence that such objects are the result of double degenerate mergers [14,16], as has been proposed for EUVE J0317-855 [17].

The population of merged magnetic white dwarfs may be the most promising source of gravitational radiation from the mechanism of quasi-radial oscillations. To estimate the expected strain amplitude from both individual sources and the population as a whole, we identify eight candidates from the 65 known isolated magnetic white dwarfs from [14] as those with rotational periods less than one day. The properties of the candidates are listed in Table 2. We estimate the local space density of merged magnetic white dwarfs by taking the local space density of white dwarfs to be 0.003 pc^{-3} [18], the fraction of isolated magnetic white dwarfs to be 5.1% [14], and the fraction of isolated magnetic white dwarfs

Table 2

ISOLATED MAGNETIC WHITE DWARFS

Name	r (pc)	B (MG)	P_{rot} (s)	$M / M_{\odot}$
PG 1031+234	142[a]	500[b]	12240	0.93[c]
EUVE J0317-855	35[a]	450	725	1.35
PG 1015+015	66[a]	90	5940	1.15[c]
Feige 7	49[a]	35	7920	0.6
G99-47	8[c]	25	3600?	0.71
KPD 0253+5052	81[d]	17	13644	...
PG 1312+098	...	10	19548	...
G217-037	11[c]	<0.2	7200-72000?[d]	0.89

Comments:

Numbers marked [a)] were taken from paper [22], [c)] are taken from [23], [d)] is taken from [24]. [b)] estimated magnetic field is in the range 500-1000 MG, we took 500 MG for calculations. For G217-037 magnetic field $B = 0.2$ MG and period $P_{rot} = 7200$ s was taken.

that are remnants of mergers to be 8/65. These results are in a local space density of $\rho_s = 1.9 \cdot 10^{-5}\,\mathrm{pc}^{-3}$. The expected distance to the nearest source is then found by:

$$r = 2\left(\frac{3}{4\pi\rho_s}\right)^{1/3} \approx 46\,\mathrm{pc}. \tag{31}$$

Assuming a galactic distribution of white dwarfs to follow the disk population, we assign a density distribution of:

$$\rho = \rho_0\, e^{-r/R_0}\, e^{-z/h} \tag{32}$$

in galacto-centric cylindrical coordinates, with $R_0 = 2.5$ kpc and $h = 200$ pc.

Taking the solar location as $r_s = 8.5\,\text{kpc}$ and $z_s = 0$, we obtain $\rho_0 = 5.5 \cdot 10^{-4}\,\text{pc}^{-3}$ and a total number of $N = 8.6 \cdot 10^6$ in the galaxy.

The expected values of U_t and U_g, "branching ratio" β, strain amplitude h_0, energy flux F, spin-down time τ and relative amplitude of oscillations η for the eight isolated magnetic white dwarfs are presented in Table 3. We start calculations by first determining τ from (29) and (30), where we have chosen $\sin^2\alpha = 0.5$. This is a reasonable assumption since a distribution in α that is either uniform or spiked at $\alpha = \pi/4$ is supported by observation [19]. U_g is calculated comparing (4) and (24), and U_t - using (25) and (14). Numerical integration for U_t is made using equation of state (17) and (18), central density ρ_c is taken from Table 1 accord to observed mass of configuration from Table 2. The luminosity J_0 is calculated from (23) and used with (31) and (7) to determine the strain amplitude where we have averaged over all orientations. We have used the average mass of $0.95\,M_\odot$

Table 3

STRAIN AMPLITUDES AND ENERGY FLUXES FOR ISOLATED WHITE DWARFS

	U_t	U_g	β	h_0	F	τ (Gyr)	η
PG 1031+234	$1.23 \cdot 10^{29}$	$2.27 \cdot 10^{26}$	0.00184	$2.58 \cdot 10^{-28}$	$1.13 \cdot 10^{-15}$	11.0	$4.37 \cdot 10^{-2}$
EUVE J0317-855	$1.61 \cdot 10^{28}$	$1.58 \cdot 10^{29}$	0.90727	$9.69 \cdot 10^{-26}$	$6.04 \cdot 10^{-11}$	1.7	$3.84 \cdot 10^{-1}$
PG 1015+015	$4.34 \cdot 10^{27}$	$8.80 \cdot 10^{26}$	0.16853	$3.81 \cdot 10^{-28}$	$1.93 \cdot 10^{-15}$	571.9	$2.91 \cdot 10^{-2}$
Feige 7	$1.68 \cdot 10^{28}$	$7.04 \cdot 10^{28}$	0.80759	$1.47 \cdot 10^{-26}$	$3.96 \cdot 10^{-13}$	125.1	$4.66 \cdot 10^{-2}$
G99-47	$8.55 \cdot 10^{27}$	$1.65 \cdot 10^{30}$	0.99484	$3.45 \cdot 10^{-25}$	$5.84 \cdot 10^{-12}$	50.6	$3.69 \cdot 10^{-2}$
KPD 0253+5052	$1.42 \cdot 10^{26}$	$1.47 \cdot 10^{26}$	0.50833	$2.06 \cdot 10^{-28}$	$2.33 \cdot 10^{-16}$	11852.8	$2.47 \cdot 10^{-2}$
PG 1312+098	$4.93 \cdot 10^{25}$	$3.50 \cdot 10^{25}$	0.41490	$9.38 \cdot 10^{-29}$	$1.56 \cdot 10^{-17}$	70313.8	$1.31 \cdot 10^{-2}$
G 217-037	$1.97 \cdot 10^{22}$	$1.90 \cdot 10^{27}$	0.99999	$8.97 \cdot 10^{-29}$	$8.19 \cdot 10^{-19}$	$2.4 \cdot 10^7$	$4.08 \cdot 10^{-4}$

Comments:

U_t and U_g are given in ergs, h_0 is the amplitude of gravitational wave.

whenever the mass was undetermined. The expected gravitational energy flux F on earth (in $\text{erg}\,\text{s}^{-1}\,\text{cm}^{-2}$) for a population made entirely of each type of white dwarf is also shown in Table 3. The flux F is calculated using formula

$$F = \frac{4\pi\rho_s h f(h) J_0}{4\pi\left(3 \cdot 10^{18}\right)^2}, \tag{33}$$

where $h = 200\,\text{pc}$ and $f(h) = 6.15$ is calculated using Appendix A of [20]. Finally, we note that a simple average of the strain amplitudes in Table 3 gives $h_0 = 5.7 \cdot 10^{-26}$ and an average flux of $F = 8.3 \cdot 10^{-12}\,\text{erg}\,\text{s}^{-1}\,\text{cm}^{-2}$. The flux is spread out over a frequency band of $\upsilon_1 = 0.12$ to $\upsilon_2 = 0.32\,\text{Hz}$, and we can estimate an average strain amplitude for the galactic population of pulsating white dwarfs by using the angle and polarization averaged expression of [21] and averaging over the frequency range $\Delta\upsilon = \upsilon_2 - \upsilon_1$ to obtain:

$$h_{0\,ave} = \frac{\ln \upsilon_2/\upsilon_1}{\Delta \upsilon} \sqrt{\frac{4GF}{\pi c^3}} \tag{34}$$

which gives $h_{0\,ave} = 7.94 \cdot 10^{-25}$.

6. *Conclusions.* We have shown that the galactic population of magnetic rotating oblate white dwarfs can produce a background of gravitational radiation in the frequency range of 0.12-0.32 Hz through quasiradial pulsations with average strain amplitude $h_{0\,ave} = 7.94 \cdot 10^{-25}$. The source of energy to drive these pulsations is found in the deformation energy releases of the white dwarf due to its rotation. This energy can be extracted from the white dwarf as it spins down because of magneto-dipole radiation.

Along with gravitational radiation deformation energy of white dwarf will dissipate through two mechanisms: a) energy losses coming from motion of plasma in magnetic field and b) viscous friction of matter during self-similar oscillations of white dwarfs.

We had proved that viscous friction will be zero. Detailed calculation of thermal losses during self-similar oscillations of conducting magnetized plasma in white dwarfs show that fraction of deformation energy radiated in gravitational waves is larger than dissipated energy.

We have proposed that a population of isolated magnetic white dwarfs, which are the remnants of merged double degenerate binaries, can be source of gravitational radiation. So the average strain amplitudes of gravitational waves from galactic population of white dwarfs are calculated taking into account dissipation processes in stars. It appeared that gravitational wave amplitudes are about 200 times bigger than it was calculated in [2]. These estimates of the signal strength over the frequency band of interest indicate that this population will be comparable in strength to the level of the stochastic cosmological background of gravitational radiation predicted by standard inflationary models.

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ГРАВИТАЦИОННОЕ ИЗЛУЧЕНИЕ ПУЛЬСИРУЮЩИХ МАГНИТНЫХ БЕЛЫХ КАРЛИКОВ

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Вращающиеся белые карлики, претерпевающие квазирадиальные осцилляции, могут излучать гравитационные волны в диапазоне частот 0.1-0.3 Гц. Принято, что источником энергии гравитационного излучения является деформационная энергия белого карлика, обусловленная вращением. Получено, что относительная амплитуда гравитационной волны белого карлика на расстоянии ~50 пк равна порядка 10^{-25}. Вычислены также джоулевые потери, обусловленные магнитогидродинамическим механизмом. Показано, что эти потери намного меньше, чем предполагалось в предыдущих работах. Галактическая популяция таких источников порядка 10^7, что может создать фон помех для планируемых усовершенствованных детекторов в диапазоне частот между орбитальными и наземными интерферометрами. Находящиеся поблизости осциллирующие белые карлики могут быть источником достаточно отчетливых гравитационных сигналов для изучения внутреннего строения белых карликов методами гравитационной астросейсмологии.

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