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A NOTE ON TIME - INDEPENDENT ELECTRIC FIELD IN SUPERCONDUCTORS

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We derive a set of coupled partial differential equations for the determination of the electric field and of the order parameter of the superconducting electrons. For this purpose, we propose an expression for the free energy of the superconducting electrons in the presence of an electric field, the minimization of which yields the above-mentioned equations. It is shown that for a superconductor at zero temperature the electric field of a test charge Ze decreases exponentially with distance from the charge and the London penetration depth plays the role of the Debye length.

Key words: *plasmas:electric field:conduction*

1. Introduction. As it is well known, in the presence of a magnetic field the constitutive relation expressing the connection of the electric current density $\bar{j}$ with the vector - potential of electromagnetic field $\bar{A}$, proposed by London in his phenomenological theory of superconductivity, is [1,2]

$$\bar{j} = -\frac{e^2 n}{mc}\bar{A}, \tag{1}$$

where e, m and n are respectively the charge, effective mass and density of Cooper pairs. Eq. (1) is valid in the interior of superconductors where n is constant. If the density of Cooper pairs is a function of space coordinates (e.g. at the interface between superconductor and normal matter or in the neighborhood of vortices), then the generalization of Eq. (1), proposed by Ginzburg-Landau, has the following form [3]:

$$\bar{j} = -\frac{e^2|\psi|^2}{mc}\bar{A} + \frac{\hbar}{2mi}\left(\psi\bar{\nabla}\psi^* - \psi^*\bar{\nabla}\psi\right), \tag{2}$$

where $|\psi|^2$ expresses the density n of Cooper pairs; the equation for ψ being

$$\left(\frac{\hbar}{i}\bar{\nabla} + \frac{e}{c}\bar{A}\right)^2\psi + 2m\left[a(T) + b(T)|\psi|^2\right]\psi = 0, \tag{3}$$

the density of Cooper pairs in the interior of the superconductor being equal to the ratio $-a(T)/b(T)$.

The aim of this paper is to derive the equations describing the behavior of superconducting electrons in the presence of a time-independent electric field. Let us note that in the interior of a superconductor the electric field may differ from zero, as it is the case in fully ionized plasma.

It is well known that in plasma an electric field exists in the neighborhood of an ion, which decreases exponentially over a characteristic distance λ_D, known as the Debye length. For the gaseous plasma λ_D^2 is proportional to the temperature T, whereas in metals it is proportional to the Fermi energy of the electrons. In this paper we shall see that in superconductors λ_D^2 is proportional to the rest mass of the Cooper pair. The equations, which we shall derive, can be used for investigating the distribution of the electric field at the interface between superconductor and vacuum. In Sec.2 we obtain the equation satisfied by the scalar potential φ under the assumption that the Cooper pairs density n is constant. In Sec.3 we derive the analogue of Ginzburg-Landau equations in the presence of an electric field.

2. *The equation for the scalar potential* φ *in regions of constant* n. Let us consider a superconductor at zero temperature. The continuity equation for charge and current densities is

$$\frac{\partial\rho}{\partial t} + div\,\bar{j} = 0. \tag{4}$$

Substitution of Eq. (1) in Eq. (4) gives

$$\frac{\partial\rho}{\partial t} - \frac{e^2 n}{mc} div\,\bar{A} = 0. \tag{5}$$

On the other hand, using the Lorenz condition for the potentials [3]

$$\frac{1}{c}\frac{\partial\varphi}{\partial t} + div\,\bar{A} = 0, \tag{6}$$

Eq. (5) may be written as

$$\frac{\partial}{\partial t}\left\{\varphi + \frac{mc^2}{e^2 n}\rho\right\} = 0. \tag{7}$$

Integration of Eq. (7) yields:

$$\varphi + \frac{mc^2}{e^2 n}\rho = f(\bar{r}), \tag{8}$$

where $f(\bar{r})$ is an arbitrary function of coordinates. Following London, the physically meaningful solution corresponds to $f(\bar{r}) \equiv 0$. Accordingly, we have

$$\rho = -\frac{e^2 n}{mc^2}\varphi. \tag{9}$$

Poisson's equation for the potential φ, when a test charge Ze is placed at the origin, reads

$$\Delta\varphi = -4\pi\rho - 4\pi Ze\,\delta(\bar{r}). \tag{10}$$

Inserting Eq. (9) into Eq. (10) we obtain the following partial differential equation for φ:

$$\Delta\varphi - \frac{1}{\lambda^2}\varphi = -4\pi Ze\,\delta(\bar{r}), \tag{11}$$

where

$$\lambda = \left(\frac{mc^2}{4\pi e^2 n} \right)^{1/2} \tag{12}$$

is the London penetration depth. Eq. (11) has the well known spherical symmetric solution

$$\varphi(r) = \frac{Ze}{r} e^{-r/\lambda} , \tag{13}$$

in which the London penetration depth plays the same role as the Debye length in the case of normal electrons.

3. *The Ginzburg-Landau equations in the presence of an electric field.* In Sec.1 we investigated the distribution of the electric field in the regions of constant electron density n and obtained the analogue of the London equation. We now consider the generalization of that equation for regions where n is a function of coordinates. In such a situation we must also derive the equation for the order parameter ψ, a complex function, which is connected to n by the formula $n = |\psi|^2$. Therefore, the problem under consideration contains two unknown functions ψ and φ which must satisfy a system of coupled partial differential equations obtained by minimizing the free energy of the superconducting electrons. Following Ginzburg and Landau, the free energy F in the presence of an electric field may be written as

$$F = \int \frac{1}{2m} \left| \frac{\hbar}{i} \bar{\nabla} \psi \right|^2 dV + \int \left[a(T) + \frac{e^2 \varphi^2}{2mc^2} \right] |\psi|^2 dV + \\ + \frac{1}{2} \int b(T) |\psi|^4 dV + \int \frac{E^2}{8\pi} dV . \tag{14}$$

Let us now vary F with respect to φ and ψ^*, taking into account that $\bar{E} = -\mathrm{grad}\, \varphi$, and requiring the vanishing of δF, we get

$$\rho = -\frac{e^2 |\psi|^2}{mc^2} \varphi , \tag{15}$$

$$\left(\Delta - \frac{e^2}{\hbar^2 c^2} \varphi^2 \right) \psi - \frac{2m}{\hbar^2} \left[a(T) + b(T) |\psi|^2 \right] \psi = 0 . \tag{16}$$

As we know, Poisson's equation for φ reads

$$\Delta\varphi = -4\pi\rho - 4\pi\rho_s , \tag{17}$$

where ρ_s is the charge density of the sources. Inserting Eq. (15) into Eq. (17), we get the equation for the unknown function φ:

$$\Delta\varphi - \frac{4\pi e^2 |\psi|^2}{mc^2} \varphi = -4\pi\rho_s . \tag{18}$$

Eqs. (16) and (18) are a set of two coupled nonlinear equations for φ and

ψ. To solve the above equations, we have to know either the charge density distribution of the sources or the distribution of the applied electric fields. The boundary conditions for these equations are specified by the problem under consideration. For example, for the problem discussed in Sec.2 we have to require that ψ must be zero at the origin of the coordinate system. The solutions of these equations will be examined in a near future.

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О НЕЗАВИСЯЩЕМ ОТ ВРЕМЕНИ ЭЛЕКТРИЧЕСКОМ ПОЛЕ В СВЕРХПРОВОДНИКАХ

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Получена система дифференциальных уравнений для определения электрического поля и параметра порядка для сверхпроводящих электронов. Для этого предложено выражение для свободной энергии сверхпроводящих электронов в присутствии электрического поля, минимизация которой приводит к вышеуказанным уравнениям. Показано, что для сверхпроводников при абсолютном нуле электрическое поле пробного заряда Ze уменьшается экспоненциально в зависимости расстояния от заряда, и лондоновская длина проникновения играет роль дебаевской длины экранирования.

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