ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱ
НАЦИОНАЛЬНАЯ АКАДЕМИЯ НАУК АРМЕНИИ
NATIONAL ACADEMY OF SCIENCES OF ARMENIA
ДОКЛАДЫ ԶԵԿՈՒՅՑՆԵՐ REPORTS

Հատոր Том Volume 116 2016 № 3

MECHANICS

УДК 539.3

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Application of Gauss Quadrature Formulas to the Solution of Integral Equations of One Class of Contact Problems of Elasticity Theory

(Submitted 1/II 2016)

Keywords: *contact problems*, *contact pressure*, *a beam on an elastic base, integral equations*, *Gauss quadrature formulas*, *systems of linear equations.*

The quadrature formulas, especially Gauss quadrature formulas for common integrals, are widely used in solving Fredholm integral equations of the second kind [1-3], and analogous Gauss formulas for singular integrals with Cauchy kernel are used in solving singular integral equations [4-6]. The both types of the integral equations are usually met in boundary value problems of mathematical elasticity theory, hydrodynamics, mathematical physics, and in many other problems of applied analysis.

In the present paper which is the continuation of the author's previous paper [7], Gauss quadrature formulas on Chebyshev nodes and on the nodes, coinciding with Legendre polynomials roots, are applied to the solution of Fredholm integral equations of the second kind with symmetric kernels of certain structures. These kernels are represented by the sums of their principle parts in the form of a logarithmic function and regular parts in the form of different continuous functions. By such integral equations a sufficiently large class of contact problems of the theory of elasticity on bending of a beam of finite length on an elastic foundation in the form of a half-plane, strip, wedge in the framework of I.Ja. Shtaerman contact model [8] as well as other similar problems are described. Ultimately, the solutions of these problems are reduced to the solutions of the systems of linear algebraic equations (SLAE).

1. In [7], in a dimensionless form the following governing integral equation (GIE) of a contact problem on bending of a beam of finite length on an elastic half-plane, taking into account the factor of the surface structure of the foundation, is derived by I.Ja. Shtaerman contact model [8]:

$$p_0(\xi)+\int_{-1}^{1}\left[\vartheta_0 \ln\frac{1}{|\xi-\eta|}+\lambda_0 G_0(\xi,\eta)\right]p_0(\eta)\,d\eta = h_0(\xi)-\gamma_0\xi-\alpha_0$$
$$G_0(\xi,\eta)=|\xi-\eta|^3;\quad h_0(\xi)=\lambda_0\int_{-1}^{1}G_0(\xi,\eta)q_0(\eta)\,d\eta. \qquad (-1\le\xi\le1)\ (1.1)$$

Here $p_0(\xi)$ is the unknown dimensionless contact pressure of the beam on the foundation, $q_0(\xi)$ is the dimensionless intensity of the given vertical forces, acting on the upper face of the beam, ϑ_0 and λ_0 are some combinations of the elastic constants, and the parameters γ_0 and α_0 characterize, correspondingly, the rigid rotation angle and the settlement of the beam. The GIE (1.1) is considered under the conditions of the beam equilibrium:

$$\int_{-1}^{1}p_0(\eta)\,d\eta=P_0,\qquad \int_{-1}^{1}\eta p_0(\eta)\,d\eta=M_0. \qquad (1.2)$$

To solve the GIE (1.1) under the conditions (1.2), let us represent its solution, as in [7], in the form of

$$p_0(\xi)=A_0\xi+B_0+\sqrt{1-\xi^2}\chi_0(\xi)\quad(-1\le\xi\le1) \qquad (1.3)$$

where A_0 and B_0 are unknown coefficients, and $\chi_0(\xi)$ is an unknown function continuous over the segment [-1,1]. Substituting $\xi=\pm1$ into (1.3), we find that after having determined A_0 and B_0, the values of the contact pressure at the endpoints of the segment $-1\le\xi\le1$ will be determined by the formulas $p(\pm1)=\pm A_0+B_0$.

Further we put (1.3) in GIE (1.1) and calculate elementary integrals of the logarithmic function. We shall have

$$\left[\xi+\vartheta_0 I_2(\xi)+\lambda_0\int_{-1}^{1}G_0(\xi,\eta)\eta\,d\eta\right]A_0+\left[1+\vartheta_0 I_1(\xi)+\lambda_0\int_{-1}^{1}G_0(\xi,\eta)\,d\eta\right]B_0+$$
$$+\sqrt{1-\xi^2}\chi_0(\xi)+\int_{-1}^{1}\left[\vartheta_0\ln\frac{1}{|\xi-\eta|}+\lambda_0 G_0(\xi,\eta)\right]\sqrt{1-\eta^2}\chi_0(\eta)\,d\eta=h_0(\xi)-\gamma_0\xi-\alpha_0 \qquad (1.4)$$

$$I_1(\xi)=\int_{-1}^{1}\ln\frac{1}{|\xi-\eta|}\,d\eta=2-(1-\xi)\ln(1-\xi)-(1+\xi)\ln(1+\xi); \qquad (1.5)$$

$$I_2(\xi)=\int_{-1}^{1}\eta\ln\frac{1}{|\xi-\eta|}\,d\eta=\frac{1}{2}\left[(1+\xi)^2\ln(1+\xi)-(1-\xi)^2\ln(1-\xi)-2\xi\right]+\xi I_1(\xi),\quad -1\le\xi\le1.$$

We put the representation (1.3) into the conditions (1.2).As a result we come to the equations

$$2B_0+\int_{-1}^{1}\sqrt{1-\eta^2}\chi_0(\eta)\,d\eta=P_0,\qquad \frac{2}{3}A_0+\int_{-1}^{1}\eta\sqrt{1-\eta^2}\chi_0(\eta)\,d\eta=M_0. \quad (1.6)$$

Now in order to calculate the integrals in (1.4) and (1.6), we use Gauss quadrature formulas on Chebyshev nodes [9] and simultaneously choose inner and outer nodes. As the inner nodes we take roots of Chebyshev polynomials of the second kind $U_{N-1}(\eta)$, i. e. $\eta_r=\cos(\pi r/N)\quad\left(r=\overline{1,N-1}\right)$, but as the outer nodes we take roots of Chebyshev polynomials of the first kind $T_N(\xi)$, i.e. $\xi_m=\cos\left[(2m-1)\pi/2N\right]$ $\left(m=\overline{1,N}\right)$, where N is an arbitrary natural number. It is evident, that $\eta_r<\xi_r<\eta_{r-1}$ $\left(r=\overline{1,N};\ \eta_0=1,\ \ r_N=-1\right)$.

As a result, the equations (1.4) and (1.6) will have, correspondingly, the form of

$$\sqrt{1-\xi_n^2}\chi_0(\xi_n)+\left[\xi_n+\vartheta_0 I_2(\xi_n)+\frac{\pi\lambda_0}{N}\sum_{m=1}^{N}G_0(\xi_n,\xi_m)\xi_m\sqrt{1-\xi_m^2}\right]A_0+$$
$$+\left[1+\vartheta_0 I_1(\xi_n)+\frac{\pi\lambda_0}{N}\sum_{m=1}^{N}G_0(\xi_n,\xi_m)\sqrt{1-\xi_N^2}\right]B_0+\vartheta_0\sum_{r=1}^{N-1}\ln\frac{1}{|\xi_n-\eta_r|}a_r\chi_0(\eta_r)+ \quad (1.7)$$
$$+\lambda_0\sum_{r=1}^{N-1}G_0(\xi_n,\eta_r)a_r\chi_0(\eta_r)=h_0(\xi_n)-\gamma_0\xi_n-\alpha_0 \qquad \left(n=\overline{1,N}\right);$$

$$2B_0+\sum_{r=1}^{N-1}a_r\chi_0(\eta_r)=P_0;\ \frac{2}{3}A_0+\sum_{r=1}^{N-1}a_r\eta_r\chi_0(\eta_r)=M_0;\ a_r=\frac{\pi}{N}\sin^2\left(\frac{\pi r}{N}\right). \quad (1.8)$$

Then we take $\chi_0(\eta_r)=P_{N-1}(\eta_r)$ $\left(r=\overline{1,N-1}\right)$, where

$$P_{N-1}(\eta)=\frac{2}{N}\sum_{m=1}^{N}\left[\frac{1}{2}+\sum_{k=1}^{N-1}T_k(\eta)T_k(\xi_m)\right]\chi_0(\xi_m)$$

is the Lagrange interpolation polynomial of the function $\chi_0(\eta)$ on Chebyshev nodes [10]. It is evident, that

$$\chi_0(\eta_r)=\frac{1}{N}\sum_{m=1}^{N}S_{rm}^{(N)}\chi_0(\xi_m);\ S_{rm}^{(N)}=1+2\sum_{k=1}^{N-1}\cos\left(\frac{\pi kr}{N}\right)\cos\left[\frac{\pi k(2m-1)}{2N}\right]. \quad (1.9)$$

Using the well–known expression of the finite sum of cosines [11] (p. 44, formula 1.342.2), we have

$$S_{rm}^{(N)}=\frac{\cos\left[\pi(2r+2m-1)(N-1)/4N\right]\sin\left[\pi(2r+2m-1)/4\right]}{\sin\left[\pi(2r+2m-1)/4N\right]}+$$
$$+\frac{\cos\left[\pi(2r-2m+1)/4\right]\sin\left[\pi(2r-2m+1)(N-1)/4N\right]}{\sin\left[\pi(2r-2m+1)/4N\right]} \quad \left(r=\overline{1,N-1};\ m=\overline{1,N}\right). \quad (1.10)$$

Now, with the help of (1.9)–(1.10) it is easy to see that the equations (1.7) – (1.8) form a SLAEof N+2 equations in N+4 unknowns $A_0,B_0,\gamma_0,\alpha_0,\chi_0(\xi_m)$ $\left(m=\overline{1,N}\right)$. The missing two equations will be obtained from (1.4), requiring that it was also satisfied at the endpoints $\xi=\pm1$ of the segment $-1\le\xi\le1$. In this way we get the following two equations:

$$\left[1\mp\vartheta_0 I_2(\mp1)\mp\frac{\pi\lambda_0}{N}\sum_{m=1}^{N}G_0(\mp1,\xi_m)\xi_m\sqrt{1-\xi_m^2}\right]A_0\mp$$
$$\mp\left[1+\vartheta_0 I_1(\mp1)+\frac{\pi\lambda_0}{N}\sum_{m=1}^{N}G_0(\mp1,\xi_m)\sqrt{1-\xi_m^2}\right]B_0+\gamma_0\mp\alpha_0\pm \quad (1.11)$$
$$\pm\vartheta_0\sum_{r=1}^{N-1}\ln(1\pm\eta_r)a_r\chi_0(\eta_r)\mp\lambda_0\sum_{r=1}^{N-1}G_0(\mp1,\eta_r)a_r\chi_0(\eta_r)=\mp h_0(\mp1).$$

Here, according to (1.5) $I_1(-1)=I_1(1)=2-\ln 4,\quad I_2(-1)=-I_2(1)=-1$. Further we substitute the expressions $\chi_0(\eta_r)$ from (1.9)–(1.10) into the equations (1.7), (1.8) and (1.11), and introduce the following new unknowns and notations $\left(\xi_m=\cos\left[\pi(2m-1)/2N\right];\ m=\overline{1,N}\right)$

$$Y_1=A_0,\ \ Y_2=B_0,\ \ Y_3=\gamma_0,\ \ Y_4=\alpha_0,\ \ Y_{k+4}=X_k=\chi_0(\xi_k)\quad \left(k=\overline{1,N}\right);$$

$$H_0^{\pm}=\frac{\pi}{N}\sum_{m=1}^{N}G_0(\pm1,\xi_m)\sqrt{1-\xi_m^2};\ \ L_0^{\pm}=\frac{\pi}{N}\sum_{m=1}^{N}G_0(\pm1,\xi_m)\xi_m\sqrt{1-\xi_m^2}.$$

As a result, we come to the following SLAE:

$$Y_n+\sum_{m=1}^{N+4}K_{nm}Y_m=c_n\quad \left(n=\overline{1,N+4}\right), \tag{1.12}$$

$$K_{1m}=\begin{cases}0\ \ \left(m=\overline{1,4}\right);\ \ m^*=m-4;\ c_1=\dfrac{3}{2}M_0;\\ \dfrac{3}{2N}\displaystyle\sum_{r=1}^{N-1}a_r\eta_r S_{rm^*}^{(N)}\ \ \left(m=\overline{5,N+4}\right);\end{cases}\quad K_{2m}=\begin{cases}0\ \ \left(m=\overline{1,4}\right);\ m^*=m-4;\ c_2=\dfrac{1}{2}P_0;\\ \dfrac{1}{2N}\displaystyle\sum_{r=1}^{N-1}a_r S_{rm^*}^{(N)}\ \ \left(m=\overline{5,N+4}\right);\end{cases}$$

$$K_{3m}=\begin{cases}1-\vartheta_0 I_2(-1)-\lambda_0 L_0^- & (m=1);\\ -\left[1+\vartheta_0 I_1(-1)+\lambda_0 H_0^-\right] & (m=2);\\ 0 \qquad (m=3);\ \ -1\ \ (m=4);\quad c_3=-h_0(-1) & \\ \dfrac{1}{N}\left[\vartheta_0\displaystyle\sum_{r=1}^{N-1}a_r\ln(1+\eta_r)S_{rm^*}^{(N)}-\lambda_0\sum_{r=1}^{N-1}a_r G_0(-1,\eta_r)S_{rm^*}^{(N)}\right. & \left(m^*=m-4;\ m=\overline{5,N+4}\right);\end{cases}$$

$$K_{4m}=\begin{cases}1+\vartheta_0 I_2(1)+\lambda_0 L_0^+ & (m=1);\\ 1+\vartheta_0 I_1(1)+\lambda_0 H_0^+ & (m=2);\\ 1\quad (m=3);\qquad 0\quad (m=4);\qquad c_4=h(1); & \end{cases}$$

$$K_{4m}=-\frac{1}{N}\left[\vartheta_0\sum_{r=1}^{N-1}a_r\ln(1-\eta_r)S_{rm^*}^{(N)}-\lambda_0\sum_{r=1}^{N-1}a_r G_0(1,\eta_r)S_{rm^*}^{(N)}\right.\ \left(m^*=m-4;\ \ m=\overline{5,N+4}\right);$$

$$K_{nm}=\begin{cases}\xi_n^*+\vartheta_0 I_2(\xi_n^*)+\dfrac{\pi\lambda_0}{N}\displaystyle\sum_{p=1}^{N}G_0(\xi_n^*,\xi_p)\xi_p\sqrt{1-\xi_p^2}\qquad (m=1);\\ 1+\vartheta_0 I_1(\xi_n^*)+\dfrac{\pi\lambda_0}{N}\displaystyle\sum_{p=1}^{N}G_0(\xi_n^*,\xi_p)\sqrt{1-\xi_p^2}\qquad (m=2);\\ \qquad \xi_n^*\qquad (m=3);\qquad 1\qquad (m=4);\qquad c_n=h_0(\xi_n^*)\\ \dfrac{1}{N}\left[\vartheta_0\displaystyle\sum_{r=1}^{N-1}a_r\ln\frac{1}{\left|\xi_n^*-\eta_r\right|}S_{rm^*}^{(N)}+\right.\qquad \left(\xi_n^*=\xi_{n-4}\right)\\ \left.+\lambda_0\displaystyle\sum_{r=1}^{N-1}G_0(\xi_n^*,\eta_r)a_r S_{rm^*}^{(N)}\right]+\left(\sqrt{1-\xi_n^2}-1\right)\delta_{mn}\\ \left(m^*=m-4;\ \ m=\overline{5,N+4}\right);\ \ \delta_{mn}=\begin{cases}1\ \ (m=n);\\ 0\ \ (m\neq n).\end{cases}\end{cases}$$

The main characteristics of the considered problem: dimensionless contact pressure $p_0(\xi)$, dimensionless bending moments $M_0(\xi)$ and transversal forces $Q_0(\xi)$ will be expressed through the solution of SLAE (1.12). According to (1.3) in nodes ξ_m we have

$$p_0(\xi_m) = Y_1\xi_m + Y_2 + \sqrt{1-\xi_m^2}\,Y_{m+4}; \quad \left(m=\overline{1,N}\right) \tag{1.13}$$

and the formulas for the bending moments and transversal forces in [7] for the given case will give

$$\begin{aligned} M_0(\xi) &= \frac{Y_1}{2}\left[\xi\left(\xi^2-1\right)+\frac{2}{3}\left(1-\xi^3\right)\right]+\frac{Y_2}{2}\left(1+\xi^2\right)+ \\ &+\frac{1}{2}\sum_{r=1}^{N-1}\left|\xi-\eta_r\right| a_r\chi_0(\eta_r) - f_0(\xi); \end{aligned} \tag{1.14}$$

$$Q_0(\xi) = \frac{Y_1}{2}\left(\xi^2-1\right)+Y_2\xi+\frac{1}{2}\sum_{r=1}^{N-1}\operatorname{sign}(\xi-\eta_r)\,a_r\chi_0(\eta_r) - g_0(\xi), \tag{1.15}$$

where the values $\chi_0(\eta_r)$ in (1.9)–(1.10) should be expressed through $\chi_0(\xi_m) = X_m = Y_{m+4}$.

Thus, the calculation formulas of the discussed problem will be the formulas (1.13)–(1.15).

2. Due to the fact that the logarithmic term of GIE (1.1) on the diagonal $\eta=\xi$ of the square $-1\le\xi,\eta\le1$ turns into infinity, it was necessary above to choose the inner and outer node points differently. But this difficulty is easy to overcome and then we can take identical inner and outer node points, as it is done when applying Fredholm well known method [1, 2]. Namely, following [1], we transform the GIE (1) to the following form:

$$\begin{aligned} &\left[1+\vartheta_0 I_1(\xi)\right]p_0(\xi)+\vartheta_0\int_{-1}^{1}\ln\frac{1}{|\xi-\eta|}\left[p_0(\eta)-p_0(\xi)\right]d\eta+ \\ &+\lambda_0\int_{-1}^{1}G_0(\xi,\eta)\,p_0(\eta)\,d\eta = h_0(\xi)-\gamma_0\xi-\alpha_0 \qquad (-1\le\xi\le1). \end{aligned} \tag{2.1}$$

It is evident, that the intergrand in the first integral at $\xi=\eta$ remains bounded.

GIE (2.1) under the conditions (1.2) is again reduced to SLAE, for the calculation of the integrals this time we usethe Gauss quadrature formula on nodes, coinciding with the roots ξ_k of Legendre polynomials $P_N(\xi)$: $P_N(\xi_k)=0$. As a result, we come to the following SLAE:

$$X_m + \sum_{k=1}^{N} L_{mk}X_k = b_m \quad \left(m=\overline{1,N}\right); \tag{2.2}$$

$$L_{mk} = \begin{cases} \vartheta_0\left[I_1(\xi_m) - {\sum_{k=1}^{N}}' A_k \ln\dfrac{1}{|\xi_m-\xi_k|}\right]+\lambda_0 A_m G_0(\xi_m,\xi_m) & (k=m); \\ \vartheta_0 A_k\ln\dfrac{1}{|\xi_m-\xi_k|}+\lambda_0 A_k G_0(\xi_m,\xi_k) & (k=1,2,\ldots,\ m-1,\ m+2,\ldots,\ N); \end{cases}$$

$$A_k = \frac{2\left(1-\xi_k^2\right)}{N^2 P_{N-1}^2\left(\xi_k\right)} \quad \left(k=\overline{1,N}\right); \;\; b_m = h_0\left(\xi_m\right) - \gamma_0 \xi_m - \alpha_0 \quad \left(m=\overline{1,N}\right); \;\; X_k = p_0\left(\xi_k\right).$$

Here a prime mark on the summation symbol means a pass of the member with the number $k=m$. Note, that in case of the considered contact problem $G_0\left(\xi_m, \xi_m\right)=0$.

Then the conditions (1.2) lead to the equations

$$\sum_{k=1}^{N} A_k X_k = P_0; \quad \sum_{k=1}^{N} A_k \xi_k X_k = M_0. \tag{2.3}$$

Now let the solution of SLAE (2.2) with the right-hand side $h_0\left(\xi_m\right)$ be denoted by $X_m^{(1)}$, with the right-hand side ξ_m by $X_m^{(2)}$ and with the right-hand side 1 by $X_m^{(3)}$. Then

$$X_m = X_m^{(1)} - \gamma_0 X_m^{(2)} - \alpha_0 X_m^{(3)} \quad \left(m=\overline{1,N}\right). \tag{2.4}$$

Taking into account (2.4), we get from (2.3) the following system of linear equations for parameters γ_0 and α_0:

$$\begin{cases} a_{11}\gamma_0 + a_{12}\alpha_0 = d_1 \\ a_{21}\gamma_0 + a_{22}\alpha_0 = d_2 \end{cases}; \quad d_1 = -P_0 + \sum_{k=1}^{N} A_k \xi_k^{(1)}; \quad d_2 = -M_0 + \sum_{k=1}^{N} \xi_k A_k X_k^{(1)}; \tag{2.5}$$

$$a_{11} = \sum_{k=1}^{N} A_k X_k^{(2)}; \;\; a_{12} = \sum_{k=1}^{N} A_k X_k^{(3)}; \;\; a_{21} = \sum_{k=1}^{N} \xi_k A_k X_k^{(2)}; \;\; a_{22} = \sum_{k=1}^{N} \xi_k A_k X_k^{(3)}.$$

Thus, the solution of GIE (2.1) under the conditions (1.2) is reduced to the successive solution of SLAEs (2.2) and (2.5).

After solving these systems, the values of the dimensionless contact pressure at the node points ξ_m according to (2.4) will be determined by the formula

$$p_0\left(\xi_m\right) = X_m = X_m^{(1)} - \gamma_0 X_m^{(2)} - \alpha_0 X_m^{(3)} \quad \left(m=\overline{1,N}\right) \tag{2.6}$$

and the dimensionless bending moments $M_0(\xi)$ and transversal forces $Q_0(\xi)$ by the formulas

$$\begin{aligned} M_0(\xi) &= \frac{1}{2}\sum_{k=1}^{N} \left|\xi-\xi_k\right| A_k X_k - f_0(\xi); \\ Q_0(\xi) &= \frac{1}{2}\sum_{k=1}^{N} \operatorname{sign}\left(\xi-\xi_k\right) A_k X_k - g_0(\xi). \end{aligned} \quad (-1 \le \xi \le 1) \tag{2.7}$$

So, the main characteristics of the contact problem in the given case are expressed by the formulas (2.6)–(2.7). In the future the numerical analysis of these characteristics obtained by different methods will be carried out and the comparative analysis of obtained results will be conducted.

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Application of Gauss Quadrature Formulas to the Solution of Integral Equations of One Class of Contact Problems of Elasticity Theory

Gauss quadrature formulas on Chebyshev nodes and on the nodes, coinciding with Legendre polynomials roots, are applied to the solution of Fredholm integral equations of the second kind with symmetric kernels. These kernels are represented by the sums of their principle parts in the form of a logarithmic function and regular parts in the form of various continuous functions. A fairly wide class of contact problems on the bending of a beam of finite length on an elastic foundation in the form of half-plane, strip, wedge and other similar problems are described by such equations. Finally, the solutions of these problems are reduced to the solutions of the systems of linear algebraic equations.

ՀՀ ԳԱԱ թղթակից անդամ Ս. Մ. Մխիթարյան

Գաուսի քառակուսացման բանաձևերի կիրառությունը առաձգականության տեսության կոնտակտային խնդիրների մի դասի ինտեգրալ հավասարումների լուծմանը

Չեբիշևի և Լեժանդրի բազմանդամների արմատների հետ համընկնող հանգույցներով Գաուսի քառակուսացման բանաձևերը կիրառվում են սիմետրիկ կորիզներով, Ֆրեդհոլմի երկրորդ սեռի ինտեգրալ հավասարումների լուծմանը: Այդ կորիզները ներկայացվում են լոգարիթմական ֆունկցիայի տեսքով իրենց գլխավոր մասի և տարբեր անընդհատ ֆունկցիաների տեսքով իրենց ռեգուլյար մասերի գումարներով: Այդպիսի հավասարումներով նկարագրվում է կիսահարթության, շերտի, սեպի տեսքով առաձգական հիմքերի վրա վերջավոր երկարության հեծանի ծռման վերաբերյալ կոնտակտային խնդիրների բավականաչափ լայն դաս: Արդյունքում այդ խնդիրների լուծումները բերվում են գծային հանրահաշվական համակարգերի լուծումներին:

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Применение квадратурных формул Гаусса к решению интегральных уравнений одного класса контактных задач теории упругости

Квадратурные формулы Гаусса по чебышевским узлам и по узлам, совпадающим с корнями многочленов Лежандра, применяются к решению интегральных уравнений Фредгольма второго рода с симметрическими ядрами. Эти ядра представляются суммами своих главных частей в виде логарифмической функции и регулярных частей в виде различных непрерывных функций. Такими уравнениями описывается достаточно широкий класс контактных задач об изгибе балки конечной длины на упругом основании в форме полуплоскости, полосы, клина. В конечном итоге решения этих задач сводятся к решениям систем линейных алгебраических уравнений.

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