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Oscillations of a Piezoceramic Space with Tunnel Opennings and Rigid Stringer (Antiplane Deformation)

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1. Introdaction. Many actual scientific and technological problems of modern engineering are connected with the investigatins of processes of propagation of waves in piezoelectrics and with the definition of dynamic strength in the vicinity of heterogeneities of various types. Solution of appearing in this case of complicated problems requires the usage of modern mathematical means and, in particular, methods and approaches of the dynamic theory of elasticity. Development of these methods is reflected in monographs [1-5] which appeared during the last decades. The procedure of application of the method of boundary integral equations to investigations of diffraction problems of electroelastic waves is developed in [6].

In the given article there is constructed an analitical algorithm for investigation of coupled fields in a piezoceramic medium, weakened by heterogenities of tunnel types along the axis of the material symmetry of the opening and rigid linear inclusions (stringer). The excitation of oscillations in the medium takes place due to the harmonically changing with time shear stresses acting on the surfaces of the cavities.

2. Statement of the problem. Consider the referring to the Cartesian system of coordinates $x_1x_2x_3$ piezoceramic space, containing tunnel in the direction of axis x_3 opening $\Gamma_j(j = 1,2,\dots,n_1)$, strengthened by rigid curvilinear stringer $L_m(m = 1,2,\dots,n_2)$. Excitation of an electroelastic field in the medium takes place under the influence of the prescribed on the surface of the openings harmonically changing with time, not depending on coordinate x_3 shear forces $X_{3n} = \mathrm{Re}(X_3e^{-iwt})$ (t is the time, is the circular frequency). Assuming that the vector of preliminarily polarization of piezoceramics is directed along axis x_3, considering two variants of the electric boundary condition: the surfaces of the openings are electrodized and grounded (variant A); the surfece of the openings are bounded with vacuum (variant B). We will also assume that functions X_3 and the curves of contours Γ_j and L_m satisfy the Holder condition [7].

Under the given conditions in a piecewise-homogeneous space we have an electroelastic field corresponding to the state of antiplane deformation. The full system of the equations in a quasistatic approximation includes the following relations [5]:

equation of motion
$$\partial_1\sigma_{13} + \partial_2\sigma_{23} = \rho\frac{\partial^2 u_3}{\partial_t^{\,2}}, \quad \partial_i = \frac{\partial}{\partial_{x_i}} \tag{2.1}$$

material equations of a medium

$$\sigma_{m3} = c^E_{44}\partial_m u_3 - e_{15}E_m, \quad D_m = e_{15}\partial_m u_3 + \nu^{\varepsilon}_{11} E_m \quad (m = 1,2) \tag{2.2}$$

equations of electrostatics
$$\operatorname{div} \vec{D} = 0, \quad \vec{E} = -\operatorname{grad}\phi. \tag{2.3}$$

In (2.1) - (2.3) σ_{m3} are the components of the stress tensor, u_3 is the component of the elastic displacement vector in the direction of axis x_3; $\vec{E}$ and $\vec{D}$ are the vectors of strenght and induction of an electric field; ϕ is the electric potential; c^E_{44}, e_{15} and ν^{ε}_{11} are the shear modulus, measured at constant value of an electric field, the piezoelectric constant and dielectric permeability, measured at fixed deformation, respectively; ρ is the mass density of the material.

The system of equations (2.1) - (2.3) will be brought to differential equations with respect to displacement u_3 and electric potential ϕ:

$$c^E_{44}\nabla^2 u_3 + e_{15}\nabla^2\phi = \rho \frac{\partial^2 u_3}{\partial t^2}, \quad e_{15}\nabla^2 u_3 - \nu^{\varepsilon}_{11}\nabla^2\phi = 0. \tag{2.4}$$

From (2.4) we have the following relations

$$\nabla^2 u_3 - c^{-2}\frac{\partial^2 u_3}{\partial t^2} = 0, \quad \nabla^2 F = 0,$$

$$\phi = \frac{e_{15}}{\nu^{\varepsilon}_{11}} u_3 + F, \qquad c = \sqrt{\frac{c^E_{44}(1+k^2_{15})}{\rho}}, \qquad k_{15} = \frac{e_{15}}{\sqrt{c^E_{44}\nu^{\varepsilon}_{11}}}, \tag{2.5}$$

where c is the velocity of a shear wave in the piezoceramic medium, k_{15} is the factor of a mechanical coupling [5].

Mechanical and electric quantities taking into account (2.2), (2.3) and (2.5) may be expressed as functions u_3 and F over formula

$$\sigma_{13} - i\sigma_{23} = 2\frac{\partial}{\partial z}\left[c^E_{44}\left(1 + k^2_{15}\right)u_3 + e_{15}F \right], \tag{2.6}$$

$$D_1 - iD_2 = -2\nu^{\varepsilon}_{11}\frac{\partial F}{\partial z}, \quad E_1 - iE_2 = -2\frac{\partial}{\partial z}\left(F + \frac{e_{15}}{\nu^{\varepsilon}_{11}}\right), \quad z = x_1 + ix_2.$$

Assuming $u_3 = \mathrm{Re}(U_3 e^{-iwt})$, $\phi = \mathrm{Re}(\phi_* e^{-wt})$ and $F = \mathrm{Re}(F^* e^{-iwt})$ we will write down equations (2.5) with respect to amplitude quantities (where γ is the wave number).

$$\nabla^2 U_3 + \gamma^2 U_3 = 0, \quad \nabla^2 F^* = 0, \quad \phi_* = \frac{e_{15}}{\nu^{\varepsilon}_{11}} U_3 + F^*, \quad \gamma = \frac{w}{c}, \tag{2.7}$$

Assuming that the insert is fixed, let us represent the mechanical and electric boundary conditions on contour $L = \cup L_m$ as follows

$$U_3^{\pm} = 0, \tag{2.8}$$

$$E_s^{+} = E_s^{-}, \quad D_n^{+} = D_n^{-}. \tag{2.9}$$

Here E_s and D_n are tangential component of an electric strength vector and the normal component of an electric induction vector, respectively sign "plus" and "minus" refer to the left and right edges of inclusion L_m the moment from its beginning a_m to end b_m (Fig.1).

To obtain an efficient, in the sense of numerical realization of the system of integral equetions, boundary condition (2.8) it is recommended to differentiate over arc coordinates

$$\left(\frac{\partial U_3^{\pm}}{\partial s}\right) = 0. \tag{2.10}$$

The mathematical record of the boundary conditions on contour $\Gamma = \cup\Gamma_j$ for the considered variants of boundary conditions has the form

$$\frac{\partial}{\partial n}\{ c^{E}_{44}(1 + k^2_{15})U_3 + e_{15}F^*\} = X_3, \tag{2.11}$$

$$\phi_* = 0, \tag{2.12}$$

$$D_n = -\nu^{\varepsilon}_{11}\frac{\partial F^*}{\partial n} = 0. \tag{2.13}$$

Boundary equalities (2.12) and (2.13) satisfy variants A and B, respectively. Below instead of (2.12) we will use condition

$$\frac{\partial}{\partial s}\left(F^{*} + \frac{e_{15}}{\nu^{\varepsilon}_{11}} U_3 \right) = 0. \tag{2.14}$$

Thus, the problem consist of the determining of functions U_3 and F^* from differential equations (2.7) and boundary conditions (2.9) - (2.11), and also (2.13) or (2.14).

3. Solvable system of singular integral equatins of boundary problems of electroelastisity. Constructing the integral representations of functions U_3 and F^* we will use the fundamental solution of the system of equations (2.4) in case, when the dependence on time has harmonical character. In this case we proceed from the system of equatons [6]:

$$\begin{aligned} &c^{E}_{44}\nabla^2 U_3 + e_{15}\nabla^2\phi_* + pw^2 U_3 = -P_0\delta(x_1 - x_{10}, x_2 - x_{20}), \\ &e_{15}\nabla^2 U_3 - \nu^{\varepsilon}_{11}\nabla\phi_* = Q_0\delta(x_1 - x_{10}, x_2 - x_{20}). \end{aligned} \tag{3.1}$$

Here P_0 and Q_0 are linear densities of concentrated shear conditions and charges, acting at point $z_0 = x_{10} + ix_{20}$ of the medium; $\delta(x,y) = \delta(x)\delta(y)$ is the Dirac δ-function. The solution of equations (3.1) is found simply and is determined by formulas

$$\begin{aligned} &U_3(x_1,x_2) = \left(\frac{k^2_{15}Q_0}{4ie_{15}(1 + k^2_{15})} - \frac{P_0}{4ic^{E}_{44}(1 + k^2_{15})} \right) H_0^{(1)}(\gamma r), \\ &\phi_*(x_1,x_2) = -\frac{Q_0}{2\pi\nu^{\varepsilon}_{11}} \ln r + \frac{i}{4(1 + k^2_{15})\nu^{\varepsilon}_{11}} \left[\frac{e_{15}P_0}{c^{E}_{44}} - k^2_{15}Q_0 \right] H_0^{(1)}(\gamma r), \\ &r = |z - z_0|, \qquad z = x_1 + ix_2 \end{aligned} \tag{3.2}$$

According to (3.2) we will write the representations of the solution in the form

$$\begin{aligned} U_3(x_1,x_2) = &\frac{i}{4c^{E}_{44}(1 + k^2_{15})} \left\{ \int_L q(\zeta)H_0^{(1)}(\gamma r)ds + \int_L p(\zeta^*)H_0^{(1)}(\gamma r_1)ds \right\} + \\ &+ \frac{k_{15}{}^2}{4ie_{15}(1 + k^2_{15})} \int_\Gamma f(\zeta^*)H_0^{(1)}(\gamma r_1)ds, \end{aligned} \tag{3.3}$$

$$F^*(x_1,x_2) = -\frac{1}{2\pi\nu^{\varepsilon}_{11}}\int_{\Gamma} f(\zeta^*)\ln r_1)ds, \quad r = |\zeta - z|, \quad r_1 = |\zeta^* - z|, \quad \zeta \in L, \quad \zeta^* \in \Gamma.$$

Here $H_\nu^{(1)}(x)$ is the Hankel's function of the first kind of order ν, ds is the element of arc length of the contour, over which the integration is carried out. It is easy to become convinced that the determined in (3.3) functions U_3 and F^* automatically satisfy electric conditions (2.9) on L and radiation conditions at infinity, and also provide the carrying out of equality $[U_3] = U_3^+ - U_3^- = 0$ in (2.8). Unknown "densities" $q(\zeta)$, $p(\zeta^*)$ and $f(\zeta^*)$ are determined from the complex system of three integral equations, which are obtained as a result of substitution of limiting values corresponding to derivative functions (3.3) at $z \to \zeta \in L$ and $z \to \zeta^* \in \Gamma$ in boundary conditions (2.10), (2.11), and also (2.13) or (2.14). The given system will be represented in the form:

$$\int_L q(\zeta)G_1(\zeta, \zeta_0)ds + \int_\Gamma p(\zeta^*)G_2(\zeta^*, \zeta_0)ds + \int_\Gamma f(\zeta^*)G_3(\zeta^*, \zeta_0)ds = 0, \tag{3.4}$$

$$-\frac{1}{2}p(\zeta_0^*) + \int_L q(\zeta)G_4(\zeta, \zeta_0)ds + \int_\Gamma p(\zeta^*)G_5(\zeta^*, \zeta_0^*)ds + \int_\Gamma f(\zeta^*)G_6(\zeta^*, \zeta_0^*)ds = X_3(\zeta_0^*),$$

$$\lambda f(\zeta_0^*) + \int_L q(\zeta)G_7(\zeta, \zeta_0^*)ds + \int_\Gamma p(\zeta^*)G_8(\zeta^*, \zeta_0^*)ds + \int_\Gamma f(\zeta^*)G_9(\zeta^*, \zeta_0^*)ds = 0,$$

$$G_1(\zeta, \zeta_0) = \frac{1}{c^E_{44}}\left\{ \gamma H_1(\gamma r_0)\sin(\psi_0 - a_0) - \frac{2i}{\pi}\,\mathrm{Im}\,\frac{e^{i\psi_0}}{\zeta - \zeta_0} \right\},$$

$$G_2(\zeta^*, \zeta_0) = \frac{\gamma}{c^E_{44}} H_1^{(1)}(\gamma r_0)\sin(\psi_0 - a_{10}),\ G_3(\zeta^*, \zeta_0) = -\frac{k^2_{15}\gamma}{e_{15}} H_1^{(1)}(\gamma r_0)\sin(\psi_0 - a_{10}),$$

$$G_4(\zeta, \zeta_0^*) = \frac{i\gamma}{4} H_1^{(1)}(\gamma r_{20})\cos(\psi_{10} - a_{20}),$$

$$G_5(\zeta^*, \zeta_0^*) = \frac{1}{2\pi}\,\mathrm{Re}\,\frac{e^{i\psi_{10}}}{\zeta^* - \zeta_0^*} + \frac{i\gamma}{4} H_1^{(1)}(\gamma r_{30})\cos(\psi_{10} - a_{30}),$$

$$G_6(\zeta^*, \zeta_0^*) = \frac{c^E{}_{44}k^2{}_{15}}{4ie_{15}}\gamma H_1(\gamma r_{30})\cos(\psi_{10} - a_{30}), \quad H_1(x) = \frac{2i}{\pi x} + H_1{}^{(1)}(x),$$

$$r_0 = |\zeta - \zeta_0|,\ r_{10} = |\zeta^* - \zeta_0|,\ r_{20} = |\zeta - \zeta_0^*|,\ r_{30} = |\zeta^* - \zeta_0^*|,\ k = \frac{k^2{}_{15}}{1 + k^2{}_{15}},$$

$$a_0 = \arg(\zeta - \zeta_0),\ a_{10} = \arg(\zeta^* - \zeta_0),\ a_{20} = \arg(\zeta - \zeta_0^*),\ a_{30} = \arg(\zeta^* - \zeta_0^*),$$

$$\psi_0 = \psi(\zeta_0),\ \psi_{10} = \psi_1(\xi)_0^*, \quad \zeta^*, \zeta_0^* \in \Gamma.$$

In case when $\phi = 0$ on contour Γ (variant A), we have

$$\lambda = 0,\ G_7(\zeta, \zeta_0^*) = \frac{k\gamma}{4ie_{15}} H_1{}^{(1)}(\gamma r_{20})\sin(\psi_{10} - a_{20}),$$

$$G_8(\zeta^*, \zeta_0^*) = \frac{k}{4ie_{15}}\left\{\gamma H_1(\gamma r_{30})\sin(\psi_{10} - a_{30}) - \frac{2i}{\pi}\operatorname{Im}\frac{e^{i\psi_{10}}}{\zeta^* - \zeta_0^*}\right\}, \tag{3.5}$$

$$G_9(\zeta^*, \zeta_0) = -\frac{1}{2\pi\nu^\varepsilon{}_{11}(1 + k^2{}_{15})}\operatorname{Im}\frac{e^{i\psi_{10}}}{\zeta^* - \zeta_0^*} - \frac{k\gamma}{4i\nu^\varepsilon{}_{11}} H_1(\gamma r_{30})\sin(\psi_{10} - a_{30}).$$

Satisfying boundary conditions $D_n = 0$ on contour (variant B) in (3.4) it is necessary to put

$$\lambda = -\frac{1}{2}, \quad G_7(\zeta, \zeta_0^*) = G_8(\zeta^*, \zeta_0^*) = 0, \quad G_9(\zeta^*, \zeta_0^*) = \frac{1}{2\pi}\operatorname{Re}\frac{e^{i\psi_{10}}}{\zeta^* - \zeta_0^*}. \tag{3.6}$$

In (3.4) - (3.6) by quantities $\psi = \psi(\zeta)$ and $\psi_1 = \psi_1(\zeta^*)$ are designated the angles between axis x_1 and normals to contour L and Γ, respectively.

Having determined functions $q(\zeta)$, $p(\zeta^*)$ and $f(\zeta^*)$ over formulas (2.6) taking into account integral representations (3.3) we may calculate all the components of the electroelastic field in the field. At e_{15}=0 system (3.4) will correspond to a piezopassive (isotropic) space.

4. Determination of the concentration of stresses in a piecewise-homogeneous space. Calculate shear stress $\sigma_s = \sigma_{23}\cos\psi_1 - \sigma_{13}\sin\psi_1$ on the surface of an opening. Taking into account (2.2) we find

$$\sigma_s = \operatorname{Re}(T_s e^{-iwt}), \quad T_s(\zeta^*) = c_{44}(1 + k^2{}_{15})\frac{\partial U_3}{\partial s} + e_{15}\frac{\partial F^*}{\partial s}, \quad \zeta^* \in \Gamma. \tag{4.1}$$

Substituting into (4.1) the limiting values of derivatives $\partial U_3/\partial s$, $\partial F^*/\partial s$ at $z \to \zeta_0^* \in \Gamma$, calculated with the help of representations (3.3) we will obtain the expression for amplitude of shear stress T_s

$$T_s(\zeta_0^*) = \frac{e_{15}}{k}\left\{\int_\Gamma p(\zeta^*)G_8(\zeta^*, \zeta_0^*)ds + \int_\Gamma q(\zeta)G_7(\zeta, \zeta_0^*)ds\right\} - \\ - \frac{e_{15}\gamma}{4i\nu^{\varepsilon}_{11}}\int_\Gamma f(\zeta^*)\sin(\psi_{10} - a_{30})H_1(\gamma r_{30})ds \tag{4.2}$$

Appearing in (4.2) function $G_7(\zeta, \zeta_0^*)$, $G_8(\zeta^*, \zeta_0^*)$ are determined in (3.5).

Formula (4.2) permits to investigate the concentration of stresses in the space according to the frequency of excitation, position and configuration of heterogenities. Here we also should mention the circumstance concerning the behaviour of electroelastic quantities in the vicinity of the inclusion. From the integral representations of the displacement amplitude in (3.3) we obtain equality

$$q(\zeta) = c^E_{44}(1 + k^2_{15})\left[\frac{\partial U_3}{\partial n}\right], \tag{4.3}$$

where the square brackets designate the jump of the corresponding quantity on L. From relations (2.2), (2.7) and (2.9) it follows that

$$\sigma_n = \mathrm{Re}(T_n e^{-iwt}),\ [T_n] = c^E_{44}\left[\frac{\partial U_3}{\partial n}\right] + e_{15}\left[\frac{\partial \Phi}{\partial n}\right],\ \left[\frac{\partial \Phi}{\partial n}\right] = \frac{e_{15}}{\nu^{\varepsilon}_{11}}\left[\frac{\partial U_3}{\partial n}\right]. \tag{4.4}$$

From expressions (4.3), (4.4) we obtain equality

$$q(\zeta) = [T_n]. \tag{4.5}$$

Thuse, on the basis of (4.5) function $q(\zeta)$ may be interpreted as intensity of contact forces of interchange of the rigid inclusion and medium. From here it follows that for equilibrium of the inclusion there should be performed equality

$$\int_L q(\zeta)ds = 0. \tag{4.6}$$

Condition (4.6) should be considered as an additional one when solving the system of singular integral equations (3.4) in the class of functions, not restrained on tips L [5]. Due to (2.2), (2.9) and (2.10) we have

$$[D_s^*] = \nu^{\varepsilon}_{11}[E_s^*] = 0, \ \ [E_n^*] = -\frac{e_{15}}{\nu^{\varepsilon}_{11}}\left[\frac{\partial U_3}{\partial n}\right] = -\frac{k}{e_{15}}q(\zeta). \tag{4.7}$$

On the basis (4.7) we may conclude, that electric induction vector $\vec{D}$ is continuous in the area of a cylinder, and electric stress vector $\vec{E}$ undergoes on the inclusion. If we consider a crack contour (mathematical cut) as L, in case when the prescribed on its edges stresses are self-balancing, vector $\vec{D}$ undergoes a jump on L, and $\vec{E}$ is continuous [6].

5. Results of calculations. As an example consider a space with circular opening and linear inclusions, orientated under angle ϑ to axis Ox_1 (material is ceramics PZT-4 [8]). Parametric equations of contour L has the form

$$\mathrm{Re}\zeta = g\delta\cos\vartheta, \ \ \mathrm{Im}\zeta = g\delta\sin\vartheta + h \quad (-1 \le \delta \le 1) \tag{5.1}$$

Solution of system (3.4) together with additional condition (4.6) taking into account (5.1) was carried out numerically by the method of quadratures [9, 10].

In Fig.2 there is shown the change of quantity $\mu = |T_s/Z|$ at point of the contour of opening $\beta = \pi$ in the function of normalized wave number $\gamma^*R = \gamma R\sqrt{1+k_{15}^2}$ at $\vartheta = 0$, $h/R = 3$, $g/R = 1.5$ (β is the polar angle, R is the radius of the opening). The curve with number m is given for loading $X_3 = Z\sin(m\beta)$ (m = 1,2,3). The full lines conform to variant A, the dashed ones to variant B. It is seen that by increasing parameters in peak values γ^*R dispeace to the right.

Concluding remarks. The represented approach to the solution of the stationary dynamic problem of electroelasticity permits to investigate the influence of the inertial effect on the behaviour of the components of the electric field in a piezoceramic space with tunnel heterogenities of a rather arbitrary configuration. As it follow from Fig. 2 under dynamic loading quantity μ may exceed its static analogue almost by 2.5 times (curve 3).

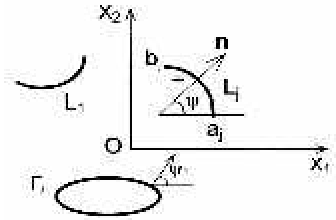

Fig. 1

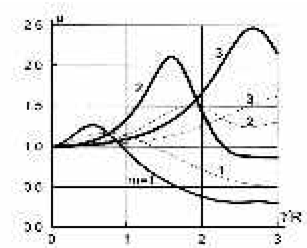

Fig. 2

From the represented result of the calculations it follows that the behaviour of the electric and mechanical quantities considerably depend on the frequency of the harmonic loading, mutual position and configuration of heterogenities.

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Թունելային անցքերով և կոշտ ստրինգերներով պյեզոկերամիկ տարածության տատանումներ (հակահարթ դեֆորմացիա)

Հոդվածում կառուցված է թունելային անցքերի և կոշտ գծային ստրինգերների տիպի անհամասեռություններով թուլացված պյեզոկերամիկ միջավայրում լծորդված դաշտերի հետազոտման վերլուծական ալգորիթմ: Տատանումների գրգռումը տեղի է ունենում անցքերի մակերևույթների վրա ազդող և ժամանակի ընթացքում ներդաշնակորեն փոփոխվող սահքի լարումների հաշվին:

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Колебания пьезокерамического пространства с туннельными отверстиями и жесткими стрингерами (антиплоская деформация)

В статье построен аналитический алгоритм для исследования сопряженных полей в пьезокерамической среде, ослабленной неоднородностями типа туннельных отверстий и жестких линейных стрингеров. Возбуждение колебаний в среде происходит за счет гармонически изменяющихся со временем напряжений сдвига, действующих на поверхностях полостей.