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On a Problem of the Located Instability of Compound Plate

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The problem of instability of two rectangular plates connected by the hinge, when one of them is compressed, is investigated.

Let two semi-infinite plates-strips connected by the hinge at $x = 0$, and fixed by the hinge on edges $y = 0, b$, when one of them is compressed towards 0y by the load of intensity P_0.

The equations of instability problem with regard to flexure w_i, selected according to the theory of thin plates and based on Kirchhoff hypothesis [1], are following:

$$D_2\Delta^2 w_2 + P_0 \frac{\partial^2 w_2}{\partial y^2} = 0 \quad (x < 0), \quad \Delta^2 w_1 = 0 \quad (x > 0), \tag{1}$$

$$D_i = \frac{2E_i h_i^3}{3(1-\nu_i^2)}, \quad i = 1,2,$$

where D_i - flexural rigidities, w_i - flexures, E_i - modules of elasticity, ν_i - Poisson's ratios, Δ- two-dimensional operator of Laplace.

As conditions of contact are used well-known in structure mechanics conditions, when on a line of contact $x = 0$, plates are connected by the hinge [2], i.e. bending moments are equal to zero.

$$w_1 = w_2 \tag{2}$$

$$\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2} = 0, \tag{3}$$

$$\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2} = 0, \tag{4}$$

$$D_1 \frac{\partial}{\partial x}\left[\frac{\partial^2 w_1}{\partial x^2} + (2-\nu_1)\frac{\partial^2 w_1}{\partial y^2}\right] = D_2 \frac{\partial}{\partial x}\left[\frac{\partial^2 w_2}{\partial x^2} + (2-\nu_2)\frac{\partial^2 w_2}{\partial y^2}\right]. \quad (5)$$

The conditions (3), (4) represent the equality of bending moments ($M_1^{(1)} = M_1^{(2)}$) to zero, condition (5) the equality of generalized over cutting forces ($\tilde{N}_1^{(1)} = \tilde{N}_1^{(2)}$) of contacting plates.

It is required to find the solutions of the equations (1) which are satisfying boundary conditions (2)-(5) and conditions of damping on infinity.

$$\lim_{x\to+\infty} w_1 = 0, \qquad \lim_{x\to-\infty} w_2 = 0. \quad (6)$$

If the equations of instability problem have not trivial solutions which are satisfying boundary conditions, when x = 0, y = 0, b and the conditions of damping on infinity, we can call that solutions located by the edge x = 0. If such kinds of solutions exist, we can say that the located instability of a plate takes place.

Let's present the solutions (1) which are satisfying the conditions at y = 0, b, ($w_i = 0$, $[(\partial^2 w_2)/(\partial y^2)]$) as follows:

$$w_i = W_{in}(x)\sin\lambda_n y, \quad \lambda_n = n\pi/b, \quad n = 1,2,... \quad (7)$$

Substituting (7) into (1) we obtain the ordinary differential equations of the fourth order for definition $W_{in}(x)$:

$$W_{1n}^{IV} - 2\lambda_n^{\ 2} W_{1n}^{II} - \lambda_n^{\ 4} W_{1n} = 0, \quad (8)$$

$$W_{2n}^{IV} - 2\lambda_n^{\ 2} W_{2n}^{II} - \lambda_n^{\ 4}(1-\alpha_{2n}^{\ 2}) W_{2n} = 0,$$

where

$$\alpha_{in}^{\ 2} = P_0/D_i\lambda_n^{\ 2}. \quad (9)$$

From the common solutions of the equations (8) we shall choose just satisfying to a condition (6).

$$W_{1n} = Ae^{-\lambda_n X} + Bxe^{-\lambda_n X}, \quad (10)$$

$$W_{1n} = Ce^{\lambda_n P_3 X} + Ee^{\lambda_n P_4 X},$$

where

$$P_3 = \sqrt{1+\alpha_{1n}}, \quad P_4 = \sqrt{1+\alpha_{2n}}. \quad (11)$$

Also the realization of condition $0 \le \alpha_{2n}^2 \le 1$ of existence of the localized solution is necessary. Substitution (10), (with the account (7)) in the boundary conditions (2)-(5), results in system of the homogeneous algebraic equations concerning any constants A, B, C, E.
where

$$\chi = D_1 \cdot D_2^{-1}, \tag{12}$$

$$\begin{cases} A - C - E = 0 \\ A \cdot \lambda_n \cdot (1 - \nu_1) - 2 \cdot B = 0 \\ C \cdot (P_3^2 - \nu_2) + E \cdot (P_4^2 - \nu_2) = 0 \\ A \cdot \chi \cdot \lambda_n \cdot (1 - \nu_1) + B \cdot \chi \cdot (1 + \nu_1) - \\ -C \cdot \lambda_n (P_3^3 - (2 - \nu_2) P_3) - E \cdot \lambda_n (P_4^3 - (2 - \nu_2) P_4) = 0. \end{cases} \tag{13}$$

Equating to zero a determinant of a system and after lines of transformations we come to the following dispersive equation:

$$L(\alpha_{2n}) \equiv (P_4 - P_3) \cdot L_1(\alpha_{2n}) = 0, \tag{14}$$

where

$$L_1(\alpha_{2n}) = \left\{ 2 \cdot (P_3^2 P_4^2 + 2(1 - \nu_2) P_3 P_4 - \nu_2^2) + \chi (P_4 + P_3)(3 - 2 \cdot \nu_1 - \nu_1^2) \right\}. \tag{15}$$

As $P_3 = P_4$ only at $\alpha_{2n} = 0$, and taking into the consideration the fact that the root of the equation (14) at $\alpha_{2n} = 0$ is trivial, in the given task in stead of (14) we shall consider the following equation:

$$L_1(\alpha_{2n}) = 0. \tag{16}$$

This function $L_1(\alpha_{2n}) = 0$ has the following properties:

$$L_1(0) = 2 \cdot (1 + 2(1 - \nu_2) - \nu_2^2) + 2 \cdot \chi \cdot (3 - 2\nu_1^2 - \nu_1^2) > 0. \tag{17}$$

In this case for the existence of the located instability it is necessary that:

$$L_1(1) = -2\nu_2^2 + \sqrt{2} \cdot \chi \cdot (3 - 2\nu_1^2 - \nu_1^2) < 0. \tag{18}$$

Hence, we can say that the located instability takes place at the observance of the following condition:

$$\chi < \frac{\sqrt{2} \cdot \nu_2^{\,2}}{3 - (2 + \nu_1) \cdot \nu_1} = \chi_*$$

The numerical results are given in the table.

ν_1	ν_2	χ_*
0.1	0.1	0.005
0.1	0.2	0.1
0.2	0.2	0.022
0.2	0.1	0.005
0.1	0.3	0.1
0.3	0.3	0.1
0.3	0.3	0.055
0.4	0.4	0.111
0.5	0.5	0.020

In this table are given the values of the parameter χ_* for various Poisson's ratios, which are satisfying to condition (19).

From the data resulted in the table it is visible, that the located instability exists since the value of $\chi < 0.202$ for the natural conditions ($0 \le \nu_1 \le 0.5$, $0 \le \nu_2 \le 0.5$) and substantially depends on Poisson's ratios: ν_1 and ν_2.

Similar problems for one plate and for two compressed plates connected by the hinge, have been investigated in [3, 4] accordingly.

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Բաղադրյալ սալի տեղայնացված անկայունության մասին

Դիտարկվում է երկու ուղղանկյուն հոդակապով ամրակցված սալերի անկայունության խնդիրը, երբ նրանցից մեկը սեղմված է:

Դուրս է բերված տեղայնացված անկայունության պայմանը, որի հիման վրա հաշված է կրիտիկական պարամետրի այն արժեքը, որից սկսած գոյություն ունի տեղայնացված անկայունությունը:

Ցույց է տրված, որ տեղայնացված անկայունությունը այս խնդրում էապես կախված է Պուասոնի գործակիցներից:

М. В. Белубекян, Э. О. Чил-Акопян

О задаче локализованной неустойчивости составной пластинки

Исследуется задача неустойчивости двух прямоугольных пластинок, соединенных шарниром, когда одна из них сжата. Выведено условие существования локализованной неустойчивости. На его основе рассчитано значение критического параметра, начиная с которого имеет место локализованная неустойчивость. Показано, что локализованная неустойчивость в данной задаче в значительной мере зависит от коэффициентов Пуассона.

Литература

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