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New Inner Bounds of the Rate-reliability Regions for Certain Multiple-access Channels

(Submitted by academician Yu. H. Shoukourian 28/XI 2001)

Several models of discrete memoryless multiple-access channels are investigated. New inner bounds for Incapacity regions for average error probability are obtained.

1. Introduction

The multiple-access channel (MAC) $W=\{W:\mathcal{X}_1\times\mathcal{X}_2\rightarrow\mathcal{Y}\}$ with two encoders and one decoder is defined by a matrix of transition probabilities

$$W=\{W(y|x_1,x_2),\ x_1\in\mathcal{X}_1,\ x_2\in\mathcal{X}_2, y\in\mathcal{Y}\},$$

where $\mathcal{X}_1$ and $\mathcal{X}_2$ are the finite alphabets of, respectively, the first and the second inputs of the channel and $\mathcal{Y}$ is the finite output alphabet. The channel is memoryless, that is for N-length sequences

$$\mathbf{x}_1=(x_{11},x_{12},\cdots,x_{1N})\in\mathcal{X}_1^N,\quad \mathbf{x}_2=(x_{21},x_{22},\cdots,x_{2N})\in\mathcal{X}_2^N,$$

$$\mathbf{y}=(y_1,y_2,\cdots,Y_N)\in\mathcal{Y}^N,$$

the transition probabilities are given in the following way

$$W^N(\mathbf{y}|\mathbf{x}_1,\mathbf{x}_2)=\prod_{n=1}^{N}W\,(y|x_1,x_2).$$

The first model of the MAC we investigate is the most general one: the MAC with correlated encoders, it was first studied by Slepian and Wolf [1]. Three independent sources (Fig. 1) create messages to be transmitted by two encoders. One of the sources is connected with both encoders and each of the other two is connected with only one of the encoders.

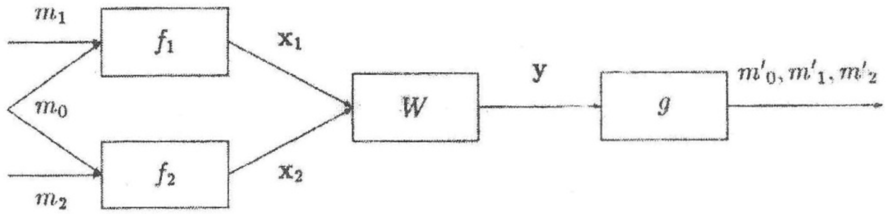

Fig. 1. MAC with correlated encoder inputs

Let $\mathcal{M}_0 = \{1,2,\ldots,M_0\}$, $\mathcal{M}_1 = \{1,2,\ldots,M_1\}$ and $\mathcal{M}_2 = \{1,2,\ldots,M_2\}$ be the message sets of corresponding sources. The code of length N for this model is a collection of mappings (f_1, f_2, g) where $f_1 : \mathcal{M}_0 \times \mathcal{M}_1 \to \mathcal{X}_1^N, f_2 : \mathcal{M}_0 \times \mathcal{M}_2 \to \mathcal{X}_2^N$ are encodings and $g : \mathcal{Y}^N \to \mathcal{M}_0 \times \mathcal{M}_1 \times \mathcal{M}_2$ is decoding. The numbers

$$\frac{1}{N} log\, M_i, \ \ i = 0,1,2,$$

are called code rates. We use the logarithmic and exponential functions to the base 2. Denote

$$f_1(m_0, m_1) = \mathbf{x}_1(m_0, m_1), \ \ f_2(m_0, m_2) = \mathbf{x}_2(m_0, m_2),$$

$$f(m_0, m_1, m_2) = (f_1(m_0, m_1),\ f_2(m_0, m_2)),$$

$$g^{-1}(m_0, m_1, m_2) = \{\mathbf{y} : g(\mathbf{y}) = (m_0, m_1, m_2)\},$$

then

$$e(m_0, m_1, m_2) = W^N\{\mathcal{Y}^N - g^{-1}(m_0, m_1, m_2) | f(m_0, m_1, m_2)\},$$

is the error probability of messages m_0, m_1 and m_2. We study the average error probability of the cod

$$\bar{e}(f, g, N, W) = \frac{1}{M_0 M_1 M_2} \sum_{m_0, m_1, m_2} e(m_0, m_1, m_2).$$

Let $E > 0$. Nonnegative numbers R_0, R_1, R_2 are called E-achievable rates triple for MAC W, if for any $\delta_i > 0,\ i = 0,1,2$, for sufficiently large N there exists a code such that

$$\frac{1}{N} \log M_i \geq R_i - \delta_i, \ \ i = 0, 1, 2,$$

and the average error probability satisfies the condition

$$\bar{e}(f, g, N, W) \leq \exp\{-NE\}.$$

The region of all E-achievable rates triples is called the E-capacity region for average error probability and denoted $\overline{C}\,(E, W)$. When $E \to 0$ we obtain the capacity region $\overline{C}\,(W)$ of the channel W for average probability of error.

Dueck [2] has shown that in general the maximal error capacity region of MAC is smaller than the corresponding average error capacity region. The determination of the maximal error capacity region of the MAC in various communication situations is still an open problem.

In [1] the achievable rates region of MAC with correlated coders was found, the random coding bound for reliability function was constructed and in [3] the sphere packing bound was obtained.

In the case with $M_0 = 1$ (Fig. 2) we have the classical MAC, first introduced by Shannon [4] and studied by Ahlswede [5], [6] and Van der Meulen [7].

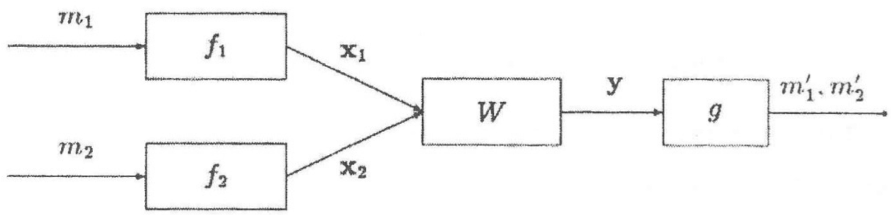

Fig. 2. Regular MAC

Shannon described the capacity region of this channel, Ahlswede obtained a simple characterization of the capacity region.

The model, when $M_1 = 1$ (Fig. 3), is called asymmetric MAC, it was considered by Haroutunian [3]. Here one of two messages have access only to one encoder, whereas the other message have access to both encoders.

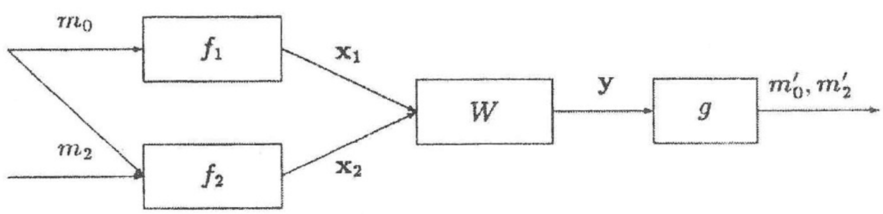

Fig. 3. Asymmetric MAC

Willems and Van der Meuien [12], [13] investigated MAC with cribbing encoders in various communication situations and established the corresponding capacity regions. In this paper we shall consider only one of this configurations (Fig. 4), investigated by van der Meuien [14], when the first coder encodes only after learning the entire codeword produced by the second encoder, which encodes his message m_2 in the usual way.

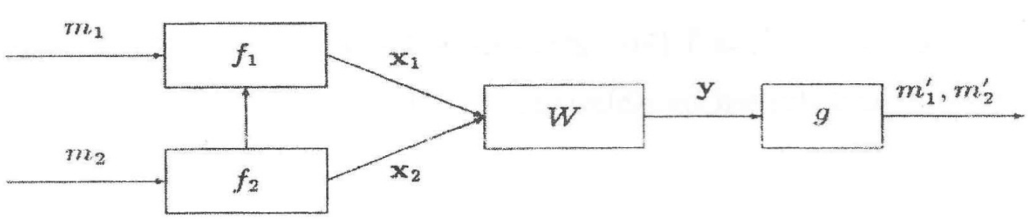

Fig. 4. MAC with cribbing encoders

For above mentioned models the E-apacity regions were investigated in [15], where corresponding sphere packing bounds were obtained, but there was a mistake in the construction of random coding bounds. Here we derive the new inner bounds of E-capacity regions for the same cases.

2. Formulation of results

Let us introduce an auxiliary random variable U with values in a finite set $\mathcal{U}$. Let random variables U, X_1, X_2, Y with values in alphabets $\mathcal{U}, \mathcal{X}_1, \mathcal{X}_2, \mathcal{Y}$ respectively, form Markov chain $U \ominus X_1\, X_2 \ominus Y$ and are given by the following probability distributions :

$$P_0 = \{P_0(u),\ u \in \mathcal{U}\},$$

$$P_i^* = \{P_i^*(x_i|u), x_i \in \mathcal{X}_i,\ i = 1,2,$$

$$P^* = \{P_0(u)P_1^*(x_1|u)\, P_2^*(x_2|u), x_1 \in \mathcal{X}_1, x_2 \in \mathcal{X}_2,$$

$$P = \{P(u, x_1, x_2) = P_0(u)\, P(x_1, x_2|u), x_1 \in \mathcal{X}_1, x_2 \in \mathcal{X}_2,$$

with $\sum\limits_{x_{3-i}} P(x_1, x_2|u) = P_i^*(x_i|u),\ i = 1,2,$

and $P \text{ o } V = \{P_0(u)P(x_1, x_2|u)V(y|x_1, x_2), x_1 \in \mathcal{X}_1,$

where $V = \{V(y|x_1, x_2), x_1 \in \mathcal{X}_1, x_2 \in \mathcal{X}_2, y \in \mathcal{Y}\}$ is some probability matrix.

For our notations of entropies, mutual informations, divergences as well as for the notions of types, conditional types and well known relations we refere to [16], [17], [18]. In the proofs we use the following representation: for $(\mathbf{x}_1, \mathbf{x}_2) \in \mathcal{T}_P^N(X_1, X_2),\ \mathbf{y} \in \mathcal{T}_{P,V}^N(Y|\mathbf{x}_1, \mathbf{x}_2)$

$$W^N(\mathbf{y}|\mathbf{x}_1, \mathbf{x}_2) = exp\{-N(H_{P,V}(Y|(X_1, X_2) + D(V||W|P))\}.$$

Note also that $D(P \text{ o } V||P^* \text{ o } W) = D(P||P^*) + D(V||W|P)$

and $D(P||P^*) = I_p(X_1 \wedge X_2 U)$.

To introduce the random coding region $\mathcal{R}_r(E, W)$ we shall use the definition of conditional mutual information among three random variables introduced by Liu and Hughes in [11]:

$$I_{P,V}(X_1 \wedge X_2 \wedge Y|U) = H_{P_1^*}(X_1|U) + H_{P_2^*}(X_2|U) + H_{P,V}(Y|U -$$

$$-H_{P,V}(Y, X_1, X_2|U) = I_{P,V}(Y \wedge X_1, X_2|U) + I_P(X_1 \wedge X_2|U).$$

Then the *random coding* region is

$\mathcal{R}_r(P^*, E, W) = \{(R_0,\ R_1,\ R_2) :$

$$0 \leq R_i \leq \min_{P,V: D(P \circ V \| P^* \circ W) \leq E} |I_{P,V}(X_i \wedge X_{3-i}, Y|U) +$$

$$+ D(P \circ V \| P^* \circ W) - E^+,\ i = 1,2,$$

$$0 \leq R_1 + R_2 \leq \min_{P,V: D(P \circ V \| P^* \circ W) \leq E} |I_{P,V}(X_1 \wedge X_2 \wedge Y|U) +$$

$$+ D(P \circ V \| P^* \circ W) - E^+,$$

$$R_0 + R_1 + R_2 \leq \min_{P,V: D(P \circ V \| P^* \circ W) \leq E} |I_{P,V}(Y \wedge X_1, X_2) + I_P(X_1 \wedge X_2|U) +$$

$$+D(P\circ V\|P^*\circ W)-E|^+\},$$

and

$$\mathcal{R}_r(E,W)=co\{\bigcup_{P^*}\mathcal{R}_r(P^*,E,W)\},$$

where $co\{\mathcal{R}\}$ is the convex hull of the region $\mathcal{R}$.

The *sphere packing* bound obtained in [15] is the follwoing:

$\mathcal{R}_{sp}(P,E,W)=\{(R_0,R_1,R_2):$

$$0\leq R_i\leq \min_{V:D(V\|W|P)\leq E} I_{P,V}(X_i\wedge Y|U,X_{3-i}),\ i=1,2,$$

$$0\leq R_1+R_2\leq \min_{V:D(V\|W|P)\leq E} I_{P,V}(Y\wedge X_1,X_2|U),$$

$$R_0+R_1+R_2\leq \min_{V:D(V\|W|P)\leq E} I_{P,V}(Y\wedge X_1,X_2)\},$$

and

$$\mathcal{R}_{sp}(E,W)=co\{\bigcup_{P}\mathcal{R}_{sp}(P,E,W)\}.$$

Theorem 1. *For all* $E>0$, *for MAC with correlated sources*

$$\mathcal{R}_r(E,W)\subset\overline{C}(E,W)\subset\mathcal{R}_{sp}(E,W).$$

Corollary. When $E\to 0$, we obtain the inner and outer estimates for the channel capacity region, the expressions of which are similar but differe with the PD P and P^*. The inner bound coincides with the capacity region:

$$R_1+R_2\leq I_{P^*,W}(Y\wedge X_1,X_2|U).$$

$$R_0+R_1+R_2\leq I_{P^*,W}(Y\wedge X_1,X_2)\}.$$

Corresponding results are obtained also for the regular MAC.

For the asymmetric MAC and MAC with cribbing encoders similar outer and inner bounds of E-capacity are obtained, which when $E\to 0$ are equal and coincide with the capacity region.

The proof of inner bound from theorem 1 is based on the random coding method.

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Արագություն-հուսալիություն տիրույթների նոր ներքին գնահատականներ որոշակի բազմամուտք կապուղիների համար

Ուսումնասիրվել են առանց հիշողության դիսկրետ բազմամուտք կապուղիների տարբեր մոդելներ: Ստացվել են E-ունակության տիրույթի նոր ներքին գնահատականներ միջին սխալի հավանականության դեպքում: Երբ հուսալիությունը ձգտում է զրոյի, ստացվում է կապուղու ունակության տիրույթի ներքին գնահատականը: Ընդհանուր բազմամուտք կապուղու դեպքում այդ գնահատականը նման է արդեն հայտնի արտաքին գնահատականին, բայց տարբերվում է հավանականային բաշխումներով: Ասիմետրիկ բազմամուտք կապուղու դեպքում, ինչպես նաև այն մասնավոր դեպքում, երբ կոդավորիչներից մեկը տեղեկություն է ստանում մյուս կոդավորիչից, ունակության տիրույթների ստացված ներքին գնահատականները հավասար են համապատասխան արտաքին գնահատականներին և համընկնում են արդեն հայտնի ունակության տիրույթների հետ:

М. Е. Арутюнян

Новые внутренние оценки области скорость-надежность для определенных каналов множественного доступа

Исследованы разные модели дискретных каналов множественного доступа без памяти. Получены новые внутренние оценки области E- пропускной способности в случае средней вероятности ошибки. Когда надежность стремится к нулю, получается оценка области пропускной способности канала. В случае общего канала множественного доступа эта оценка похожа на уже известную внешнюю оценку, но отличается вероятностными распределениями. В случае асимметричного канала множественного доступа, а также в том частном случае, когда один из кодеров получает информацию с другого кодера, полученные внутренние оценки областей пропускных способностей равны соответствующим внешним оценкам и совпадают с уже известными областями пропускных способностей.

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