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Methods of the theory complex functions and singular integral equations in problems of reinforcement of cracked anisotropic plates

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The fracture problem of the materials and constructions constitutes of the contemporary topics of the mechanics of deformable bodies. Although the fracture mechanism cannot be only referred to a phenomenon of crack propagation, however the examination of the conditions under which the body starts creating a crack or a system of cracks which later propagate in it, constitutes one of the main and interesting sides of the fracture problem. This side of the problem is referred to the study of the stress-strain state of the body in the region of the imperfections (defects) and the singular points.

The study of the problem of reinforced plates and shells becomes more complex when in the body imperfections of the form holes, crack, notch, inclusion etc exist. In this case the determination of the interaction of the two opposite factors, such as the reinforcement of the body and its weakening have a great importance.

1. Let us have an infinite anisotropic (orthotropic) body S with a crack l which is loaded in infinite with the stresses $\sigma_{xx}^{\infty}, \sigma_{yy}^{\infty}, \sigma_{xy}^{\infty}$. On a certain region γ of the plate, S where the crack l exists, an elastic isotropic inclusion S_1 it is put which has as aim the reinforcement of the plate because the crack starts propagating.

The boundary conditions of the above problem are as follows.

a) On the contact boundary γ between the body S and the inclusion S_1 we have equal displacements, as well as equal and opposite stresses i.e.

$$\begin{cases} \dfrac{d}{dt}[u_1(t) + iv_1(t)] = \dfrac{d}{dt}[u(t) + iv(t)] \\ \sigma_n^{\,1} + i\sigma_t^{\,1} = -(\sigma_n + i\sigma_t) = f^*(t). \end{cases} \tag{1}$$

b) On the two lips of the crack l the loadings are generally given:

$$\left. \sigma_n^{\pm} - i\sigma_1^{\pm} \right|_t . \tag{2}$$

The complex potentials $\Phi_0(z_1)$ and $\psi_0(z_2)$, on the basis of which the stress-strain state of the cracked plate, which is loaded in the infinite and on which concentrated force X + iY on the point z_0 acts can be studied, are given in the form [1-4]:

$$\Phi(z_1) = \frac{1}{2\pi i} \int_{l_1} \frac{\varphi(\tau_1)}{\tau_1 - z_1} d\tau_1 + \Gamma + \frac{c_{11}X + c_{12}Y}{z_1 - \xi_1} \tag{3}$$

$$\psi(z_2) = \frac{1}{2\pi i}\int_{l_2} \frac{\varphi(\tau_2)}{\tau_2 - z_2} d\tau_2 + \Gamma' + \frac{c_{21}X - c_{22}Y}{z_2 - \xi_2}$$

where $\varphi(t_k)$, $\psi(t_k)$-densities on the l_k ($k = 1,2$) and

$$z_k = x + \mu_k y, \quad \xi_k = x_0 + \mu_k y_0; \quad z = x + iy, \quad z_0 = x_0 + iy_0$$

$$c_{11} = \frac{1}{2\pi i}\left(\mu_2 + \bar{\mu}_2 + \bar{\mu}_1 + \bar{\mu}_1\mu_2\bar{\mu}_2\frac{\nu_x E_y}{E_x}\right) \bigg/ \left[(\mu_1 - \mu_2)(\mu_1 - \bar{\mu}_1)\left(1 - \frac{\bar{\mu}_2}{\mu_1}\right)\right]$$

$$c_{12} = \frac{1}{2\pi i}\left(\mu_2\bar{\mu}_2 + \bar{\mu}_1\mu_2 + \bar{\mu}_1\bar{\mu}_2 + \nu_x\right) \bigg/ \left[(\mu_1 - \mu_2)(\mu_1 - \bar{\mu}_1)\left(1 - \frac{\bar{\mu}_2}{\mu_1}\right)\right]$$

$$c_{21} = \frac{1}{2\pi i}\left(\mu_1 + \bar{\mu}_1 + \bar{\mu}_2 + \mu_1\bar{\mu}_1\bar{\mu}_2\frac{\nu_x E_y}{E_x}\right) \bigg/ \left[(\mu_2 - \mu_1)(\mu_2 - \bar{\mu}_1)\left(1 - \frac{\bar{\mu}_1}{\mu_2}\right)\right]$$

$$c_{22} = \frac{1}{2\pi i}\left(\mu_1\bar{\mu}_1 + \bar{\mu}_2\mu_1 + \bar{\mu}_2\bar{\mu}_1 + \nu_x\right) \bigg/ \left[(\mu_2 - \mu_1)(\mu_2 - \bar{\mu}_2)\left(1 - \frac{\bar{\mu}_1}{\mu_2}\right)\right]$$

and μ_k ($k = 1,2$) are the roots of the equation:

$$\mu^4 + \left(\frac{E_x}{G_{xy}} - 2\nu_x\right)\mu^2 + \frac{E_x}{E_y} = 0 \quad (\mathrm{Im}\,\mu_k > 0)$$

with E_x, E_y, ν_x, ν_y being elastic constants of the plate in directions x and y respectively, G_{xy} is the shear modulus. Apart this, for the constants Γ, Γ' we know that:

$$\Gamma - \bar{\Gamma} = 0, \qquad \mu_1^2\Gamma + \bar{\mu}_1^2\bar{\Gamma} + \mu_2^2\Gamma' + \bar{\mu}_2^2\bar{\Gamma}' = \sigma_{xx}^{\infty}$$

$$\Gamma + \bar{\Gamma} + \Gamma' + \bar{\Gamma}' = \sigma_{yy}^{\infty}, \qquad \mu_1\Gamma + \bar{\mu}_1\bar{\Gamma} + \mu_2\Gamma' + \bar{\mu}_2\bar{\Gamma}' = -\sigma_{xy}^{\infty}$$

Now, we shall proceed to the determination of the complex potentials of the composite cracked plate (S and S_1).

For the finite elastic isotropic inclusion S_1 the complex potential is given in the form:

$$\Phi(z) = \begin{cases} \Phi^+(z) = \dfrac{1}{2\pi i}\displaystyle\int_{\gamma_1} \frac{G(\tau)}{\tau - z} d\tau, & z \in S_1 \\ 0, & z \notin S_1 \end{cases} \tag{4}$$

The stresses and the displacements on the boundary γ of the inclusion are given by the relationships [1]:

$$\Phi^+(t) + \overline{\Phi^+(t)} + \frac{d}{dt}\left[\bar{t}\Phi'^+(t) + \psi^+(t)\right] = \sigma_n^1 - i\sigma_t^1 = f^*(t) \tag{5}$$

$$\Phi^{+}(t) - k_1 \overline{\Phi^{+}(t)} + \frac{d}{dt}\left[\bar{t}\Phi'^{+}(t) + \psi^{+}(t)\right] = -2\mu_1 \frac{d}{dt}\left[u_1(t) - i\nu_1(t)\right] \tag{6}$$

where $k_1 = [(3 - v_1)/(1 + v_1)]$, $\mu_1 = [(E_1)/(2(1 + v_1))]$ and E_1, v_1 being the elastic constants of the indusion S_1.

Taking into consideration the Plemelj formulae for the function $\Phi(z)$ when $z \to t$, $t \in \gamma$ [1,5]

$$\begin{cases} \Phi^{+}(t) = \frac{1}{2}G(t) + \frac{1}{2\pi i}\oint_{\gamma} \frac{G(\tau)}{\tau - t}d\tau \\ \Phi^{-}(t) = -\frac{1}{2}G(t) + \frac{1}{2\pi i}\oint_{\gamma} \frac{G(\tau)}{\tau - t}d\tau \end{cases} \tag{7}$$

and solving Eq. (5) to obtain $\psi^{+}(t)$, the expression of the other complex potential $\psi(z)$ takes the following form:

$$\psi(z) = \begin{cases} \psi^{+}(z) = \frac{1}{2\pi i}\oint_{\gamma} \frac{\overline{f^{*}(\tau)}}{\tau - z}\overline{d\tau} - \frac{1}{2\pi i}\oint_{\gamma} \frac{\overline{G(\tau)}}{\tau - t}\overline{d\tau} - \frac{1}{2\pi i}\oint_{\gamma} \frac{\bar{\tau}G(\tau)}{(\tau - z)^{2}}d\tau, & z \in S_1 \\ 0, & z \notin S_1. \end{cases} \tag{8}$$

Substituting, next, the limit values of $\Phi(z)$, $\psi(z)$ $z \to t \in \gamma_1$, on the basis of Plemelj formulae, in Eq. (6) we obtain the following singular integral equation

$$\frac{1}{\pi i}\oint_{\gamma} \frac{G(\tau)}{\tau - t}d\tau + \frac{k_1}{\pi i}\oint_{\gamma} \frac{\overline{G(\tau)}}{\bar{\tau} - \bar{t}}\overline{d\tau} + \frac{d}{dt}\left[\frac{1}{\pi i}\oint_{\gamma} \frac{\overline{f^{*}(\tau)}}{\tau - t}\overline{d\tau} - \frac{1}{\pi i}\oint_{\gamma} \frac{\overline{G(\tau)}}{\tau - t}\overline{d\tau} - \frac{1}{\pi i}\oint_{\gamma} \frac{\bar{\tau} - \bar{t}}{(\tau - t)^{2}}G(\tau)d\tau\right] = -2\mu_1[u_1(t) - i\nu_1(t)] \tag{9}$$

where $\lambda(t) = 2\mu_1[d/dt](u_1(t) + i\nu_1(t))$, $G(t) = [(f^{*}(t))/(1 + k_1)] + [(\lambda(t))/(1 + k_1)]$ - density of Cauchy integral.

On the anisotropic (orthotropic) cracked plate where along γ the effect of the inclusion is transmitted, it is created such a stress-strain field that can be described by the complex potentials $\Phi_0(z_1)$ and $\psi_0(z_1)$

$$\Phi_0(z_1) = \Phi(z_1) - \frac{i(c_{11} - ic_{12})}{2}\oint_{\gamma} \frac{f^{*}(\tau)}{\tau_1^{*} - z_1}d\tau_1 + \frac{i(c_{11} + ic_{12})}{2}\oint_{\gamma} \frac{\overline{f^{*}(\tau)}}{\tau_1^{*} - z_1}\overline{d\tau} + \Gamma \tag{10}$$

$$\psi_0(z_2) = \psi(z_1) - \frac{i(c_{21} - ic_{22})}{2}\oint_{\gamma} \frac{f^{*}(\tau)}{\tau_2^{*} - z_2}d\tau_1 + \frac{i(c_{21} - ic_{22})}{2}\oint_{\gamma} \frac{\overline{f^{*}(\tau)}}{\tau_2^{*} - z_2}\overline{d\tau} + \Gamma' \tag{11}$$

where $\Phi(z_1) = [1/(2\pi i)]\oint g(\tau_1)/(\tau_1 - z_1)]d\tau_1$, $\psi(z_2) = [1/(2\pi i)]\oint y(\tau_2)/(\tau_2 - z_2)]d\tau_2$, $g(t_1)$, $y(t_2)$- unknown densities along L_1, L_2 and $\tau_k^{*} = x + \mu_k y$, with $x + iy = z$ $(k = 1,2)$.

Now, we shall describe the boundary conditions (1) and (2) by the aid of the boundary values of complex potentials (10) and (11) on the basis of Plemelj formulae.

The stress components σ_{xx}, σ_{yy}, τ_{xy} and the displacements for any point of the anisotropic plate are given by and of complex potentials by the following expressions [3]:

$$\begin{cases} \sigma_{xx} = 2\mathrm{Re}[\ \mu_1{}^2\Phi_0(z_1) + \mu_2{}^2\psi_0(z_2)] \\ \sigma_{yy} = 2\mathrm{Re}[\ \Phi_0(z_1) + \psi_0(z_2)] \\ \tau_{xy} = -2\mathrm{Re}\{\ \mu_1\Phi_0(z_1) + \mu_2\psi_0(z_2)\} \end{cases} \tag{12}$$

and

$$\begin{cases} u(z) = 2\mathrm{Re}[\ p_1\varphi_0(z_1) + p_2\psi_0(z_2)] \\ \nu(z) = 2\mathrm{Re}[\ q_1\varphi_0(z_1) + q_2\psi_0(z_2)] \end{cases} \tag{13}$$

where $p_j = [(\mu_j{}^2)/(E_x)] - [(v_y)/(E_x)]$, $q_j = [1/(E_y\mu_j)] - [(v_x)/(E_y)]\mu_j$ $(j = 1,2)$.

By derivating Eq. (13) and taking into account the relationship

$$\sigma_n + i\sigma_i = \frac{1}{2}(\sigma_{xx} + \sigma_{yy}) - \frac{e^{-2i\theta}}{2}(\sigma_{xx} - \sigma_{yy} + 2i\sigma_{xy}) \tag{14}$$

we obtain the following relationships on the boundaries considered:

$$\frac{d}{dt}(u(t) + i\nu(t)) = (p_1 + iq_1)\frac{dt_1}{dt}\Phi_0(t_1^*) + (\overline{p}_1 + i\overline{q}_1)\overline{\frac{dt_1}{dt}}\,\overline{\Phi_0(t_1^*)} + + (p_2 + iq_2)\frac{dt_2}{dt}\psi_0(t_2^*) + (\overline{p}_2 + i\overline{q}_2)\overline{\frac{dt_2}{dt}}\,\overline{\psi_0(t_2^*)},\quad t_k^* \in \gamma_k^*,\quad (k = 1,2) \tag{15}$$

$$\sigma_n^{\pm} + i\sigma_t^{\pm} = \mathrm{Re}\left\{(1+\mu_1^2)\Phi_0^{\pm}(t_1) + (1+\mu_2^2)\psi_0^{\pm}(t_2)\right\} + \frac{\overline{d}}{dt}\left\{\mathrm{Re}\left[(1-\mu_1^2)\Phi_0^{\pm}(t_1) + + (1-\mu_2^2)\psi_0^{\pm}(t_2)\right] + 2i\mathrm{Re}\left[\mu_1\Phi_0^{\pm}(t_1) + \mu_2\psi_0^{\pm}(t_2)\right]\right\},\quad (t_k \in l_k,\quad k = 1,2) \tag{16}$$

Solving for $\psi^+(t_2) - \psi\_(t_2)$ after some algebra and taking into account the Plemelj formulae we obtain:

$$\psi(z_2) = \frac{1}{2\pi i}\int_{t_2}\frac{y(t_2)}{\tau_2 - z_2} = \frac{1}{2\pi i\,(\mu_2\overline{\mu}_2)}\int_{l}\frac{q^{**}}{\tau_2 - z_2}d\tau - - \frac{\mu_1 - \overline{\mu}_2}{2\pi i\,(\mu_2 - \overline{\mu}_2)}\int_{l_1}\frac{g(\tau_1)}{\tau_2 - z_2}d\tau_1 - \frac{\overline{\mu}_1 - \overline{\mu}_2}{2\pi i\,(\mu_2 - \overline{\mu}_2)}\int_{l_1}\frac{\overline{g(\tau_1)}}{\tau_2 - z_2}\overline{d\tau_1} \tag{17}$$

$$2q^{**}(t) = q^+(t) - q^-(t),\quad q^{\pm}(t) = -i(1 - i\overline{\mu}_2)f^{\pm}(t) + i\frac{\overline{d}}{dt}(1 + i\overline{\mu}_2)\overline{f^{\pm}(t)}$$

where

$$f^{\pm}(t) = \sigma_n^{\pm} + i\sigma_t^{\pm} - \mathrm{Re}\left[(1+\mu_1^2)\Gamma + (1+\mu_2^2)\Gamma'\right] - \frac{\overline{d}}{dt}\mathrm{Re}\left[(1+i\mu_1)^2\Gamma + (1+i\mu_2)^2\Gamma'\right].$$

2. By substraction of the Eq. (16) and taking into account the Plemelj formulae for the complex potentials $\Phi_0(z_1)$ and $\psi_0(z_2)$, when $z_k \to t_k \in l_k$ $k = 1, 2$ after some algebra the following singular integral equation results:

$$
\begin{aligned}
&(\mu_1-\bar{\mu}_2)\frac{dt_1}{dt}\left[\frac{1}{\pi i}\int_{l_1}\frac{g(\tau_1)}{\tau_1-t_1}d\tau_1 - i(c_{11}-ic_{12})\oint_{\gamma}\frac{f^*(\tau)}{\tau_1^*-t_1}d\tau + i(c_{11}+ic_{12})\oint_{\gamma}\frac{\overline{f^*(\tau)}}{\tau_1^*-t_1}\overline{d\tau}\right]- \\
&-(\bar{\mu}_1-\bar{\mu}_2)\overline{\frac{dt_1}{dt}}\left[\frac{1}{\pi i}\int_{l_1}\frac{\overline{g(\tau_1)}}{\bar{\tau}_1-\bar{t}_1}\overline{d\tau_1} - i(c_{11}+ic_{12})\oint_{\gamma}\frac{\overline{f^*(\tau)}}{\overline{\tau_1^*}-\bar{t}_1}d\tau + i(c_{11}-ic_{22})\oint_{\gamma}\frac{f^*(\tau)}{\bar{\tau}_1^*-\bar{t}_1}d\tau\right]+ \\
&+\frac{dt_2}{dt}\left[\frac{1}{\pi i}\int_{l}\frac{q^{**}(\tau)}{\tau_2-t_2}d\tau - \frac{\mu_1-\bar{\mu}_2}{\pi i}\int_{l_1}\frac{g(\tau_1)}{\tau_2-t_2}d\tau_1 - (\bar{\mu}_1-\bar{\mu}_2)\int_{l_1}\frac{\overline{g(\tau_1)}}{\tau_2-t_2}\overline{d\tau_1}-\right. \\
&\left.-i(c_{21}+ic_{22})\oint_{\gamma}\frac{f^*(\tau)}{\tau_2^*-t_2}d\tau + i(c_{21}-ic_{22})\oint_{\gamma}\frac{\overline{f^*(\tau)}}{\tau_2^*-t_2}\overline{d\tau}\right] = q^*(t) \quad (t\in l)
\end{aligned}
\tag{18}
$$

which describes the conditions (2) and where $q^*(t) = q^+(t) + q^-(t)$.

Finally, substituting in Eq. (15) the limit values of the complex potentials by the Plemelj formulae we have the following singular integral equation on the γ of the anisotropic plate.

$$
\begin{aligned}
&(p_1+iq_1)\left[\left[\frac{dt_1}{dt} - \frac{i(c_{11}-ic_{12})}{2}\oint_{\gamma}\frac{f^*(\tau)}{\tau_1^*-t_1^*}d\tau_1 + i\frac{c_{11}+ic_{12}}{2}\oint_{\gamma}\frac{\overline{f^*(\tau)}}{\tau_1^*-t_1}\overline{d\tau} + \frac{1}{2\pi i}\oint_{l_1}\frac{g(\tau_1)}{\tau_1-t_1^*}d\tau\right]+\right. \\
&+(\bar{p}_1+i\bar{q}_1)\overline{\frac{dt_1}{dt}}\left[\frac{i(c_{11}+ic_{12})}{2}\oint_{\gamma}\frac{\overline{f^*(\tau)}\overline{d\tau}}{\overline{\tau_1^*}-\bar{t}_1^*} - i\frac{c_{11}-ic_{12}}{2}\oint_{\gamma}\frac{f^*(\tau)}{\bar{\tau}_1^*-\bar{t}_1^*}d\tau - \frac{1}{2\pi i}\int_{l_1}\frac{\overline{g(\tau_1)}}{\bar{\tau}_1-\bar{t}_1^*}\overline{d\tau_1}\right]+ \\
&+(p_2+iq_2)\frac{dt_2}{dt} - \frac{i(c_{21}-ic_{22})}{2}\oint_{\gamma}\frac{f^*(\tau)}{\tau_2^*-t_2^*}d\tau_1 + i\frac{c_{21}-ic_{22}}{2}\oint_{\gamma}\frac{\overline{f^*(\tau)}}{\tau_2^*-t_2}+ \\
&+\frac{1}{2\pi i}(\mu_2-\bar{\mu}_2)\int_{l}\frac{q^{**}(\tau)d\tau}{\tau_2-t_2^*} - \frac{\mu_1-\bar{\mu}_2}{2\pi i(\mu_2-\bar{\mu}_2)}\int_{l_1}\frac{g(\tau_1)}{\tau_2-t_2^*}d\tau_1 - \frac{\bar{\mu}_1-\bar{\mu}_2}{2\pi i(\mu_2-\bar{\mu}_2)}\int_{l_1}\frac{\overline{g(\tau_1)}}{\tau_2-t_2^*}\overline{d\tau_1}\right]+ \\
&+(\bar{p}_2+iq_2)\overline{\frac{dt_2}{dt}}\left[\frac{i(c_{21}-ic_{22})}{2}\oint_{\gamma}\frac{\overline{f^*(\tau)}}{\bar{\tau}_2^*-\bar{t}_2^*}\overline{d\tau} - i\frac{c_{21}+ic_{22}}{2}\oint_{\gamma}\frac{f^*(\tau)d\tau}{\bar{\tau}_2^*-t_2^*} - (t_k^*\in\gamma_k^*,\ \ k=1,2)\right. \\
&\left.-\frac{1}{2\pi i(\bar{\mu}_2-\mu_2)}\int_{l}\frac{\overline{q^{**}(\tau)}\overline{d\tau}}{\bar{\tau}_2^*-\bar{t}_2^*} + \frac{\bar{\mu}_1-\mu_2}{2\pi i(\bar{\mu}_2-\mu_2)}\int_{l_1}\frac{\overline{g(\tau_1)}}{\bar{\tau}_2-\bar{t}_2^*}\overline{d\tau_1} + \frac{\mu_1-\mu_2}{2\pi i(\bar{\mu}_2-\mu_2)}\int_{l_1}\frac{g(\tau_1)d\tau_1}{\bar{\tau}_2-\bar{t}_2^*}\right]- \\
&-\frac{\lambda(t)}{2\mu_1} = -\left\{(p_1+iq_1)\frac{dt_1}{dt}\Gamma + (\bar{p}_1+\overline{iq_1})\overline{\frac{dt_1}{dt}}\bar{\Gamma} + (p_1+iq_2)\frac{dt_2}{dt}\Gamma' + (\bar{p}_2+\overline{iq_2})\overline{\frac{dt_2}{dt}}\bar{\Gamma}'\right\},
\end{aligned}
\tag{19}
$$

Eqs. (9), (18) and (19) are completed with the single valuedness condition for the displacements on the crack l

$$
\int_{l}\frac{d}{dt}[(u^+-u^-)+i(v^+-v^-)]dt = 0 \tag{20}
$$

and the equilibrium condition of the stresses on γ

$$\int_{\gamma} f^{*}(t)ds = 0 \tag{21}$$

where Eq. (20), after substitution of the relationships for the displacements through the complex functions on the basis of Eq. (15) and using Plemelj formulae, will take the following form

$$\left[(p_1 - iq_1) - (p_2 + iq_2)\frac{\mu_1 - \bar{\mu}_2}{\mu_2 - \bar{\mu}_2} - (\bar{p}_2 + i\bar{q}_2)\frac{\mu_1 - \mu_2}{\bar{\mu}_2 - \mu_2}\right]\int_{l} g(t_1)\,dt_1 +$$
$$+\left[(\bar{p}_2 + i\bar{q}_2) - (p_2 + iq_2)\frac{\bar{\mu}_1 - \bar{\mu}_2}{\mu_2 - \bar{\mu}_2} - (\bar{p}_2 + i\bar{q}_2)\frac{\bar{\mu}_1 - \mu_2}{\bar{\mu}_2 - \mu_2}\right]\int_{l} \overline{g(t_1)}\,\overline{dt_1} = \tag{22}$$
$$-\frac{p_2 + iq_2}{\mu_2 - \bar{\mu}_2}\int_{\gamma} q^{**}(t)\,dt - \frac{\bar{p}_2 + i\overline{q_2}}{\bar{\mu}_2 - \mu_2}\int_{\gamma} \overline{q^{**}(t)}\,\overline{dt}$$

Eqs. (9), (18), (19), (21) and (22) give the possibility to describe the stress-strain state field of the plane cracked orthotropic plate which is reinforced with an elastic isotropic inclusion.

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Դ.Ի. Բարձոկաս

Կոմպլեքս փոփոխականի ֆունկցիաների և սինգուլյար ինտեգրալ հավասարումների տեսության մեթոդները ճաք պարունակող ուժեղացված անիզոտրոպ սալերի խնդիրներում

Դիտարկվում է ողորկ եզրագծով կամայական հարթ տիրույթի տեսքով իզոտրոպ առաձգական ներդրակով օրթոտրոպ անվերջ սալի լարվածային վիճակը, որը պարունակում է կորագիծ ճաք: Սալը անվերջում ենթարկված է հավասարաչափ բաշխված նորմալ և շոշափող ուժերի ազդեցության, իսկ ճաքի ափերին գործում են կամայական ուժեր: Կոմպլեքս պոտենցիալների օգնությամբ ստացված է դրված խնդրի որոշիչ սինգուլյար ինտեգրալ հավասարումների համակարգը, որի լուծելիությունը հայտնի պայմանների դեպքում ապահովված է:

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Методы теории функций комплексного переменного и сингулярных интегральных уравнений в задачах усиленных анизотропных пластин с трещиной

Рассматривается напряженное состояние ортотропной бесконечной пластины с упругим изотропным включением в форме произвольной плоской области с гладким контуром, содержащей криволинейную трещину. Пластина на бесконечности подвержена воздействию равномерно распределенных нормальных и касательных сил, а на берегах трещины действуют произвольные силы. При помощи комплексных потенциалов получены определяющая система сингулярных интегральных уравнений поставленной задачи, разрешимость которой при известных условиях обеспечена.