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Coherent Accumulation of Modulated Probability Amplitudes: Scattering of the Atoms in the Field of Counterpropagating Light Pulses:

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Scattering of an atomic beam in the field of resonant standing wave, composed by the reflection of nanosecond scale laser pulses from the immovable mirror, shows anomalous regularities, particularly, high asymmetric angular distribution ([1-3]). The obtained regularities didn't pack up into the established at that time theoretical picture about the scattering of atoms in the field of standing wave ([4-8]). It impeled a new rise of interest to the problem of coherent multiphoton scattering of atoms in the laser radiation field. After the persistent quest it has been shown that one can explain qualitevly the observed asymmetries, if admit the atom to be initially in the mixture state of ground and excited energy levels, which, in addition, has different values of translational motion momentum ([8]). Just resembled situation is implemented for experimental conditions ([1-3]),

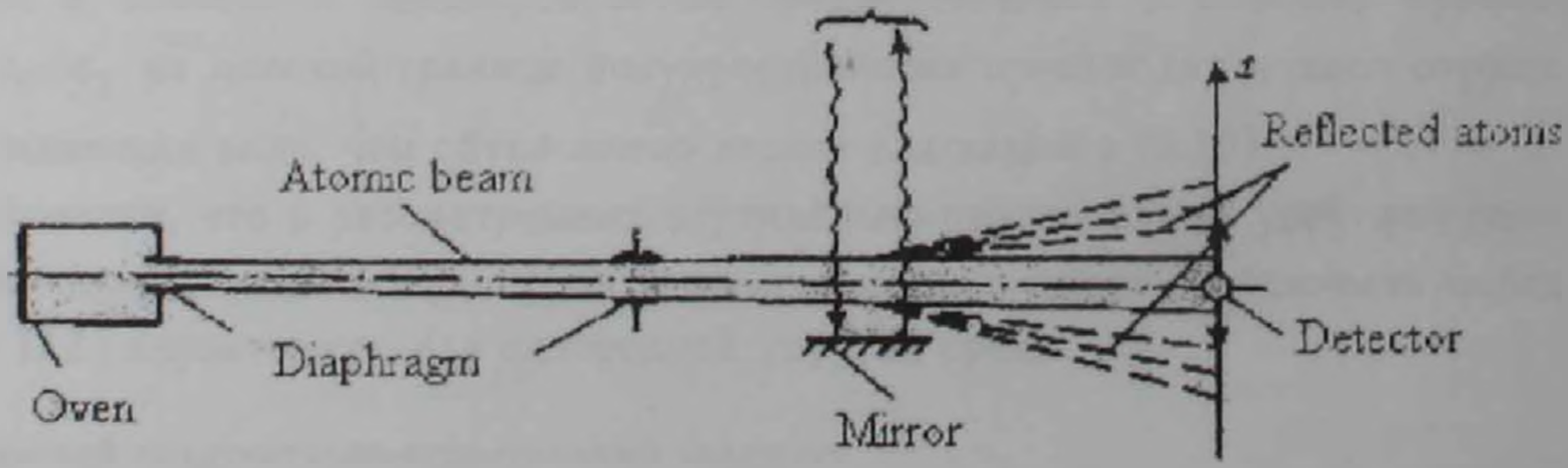

Fig.1. The scheme of experiment

where an atomic beam propagates on some distance L from the surface of a flat mirror (Fig.1), and, because of this distance, the standing wave is formed only after the time (from the initial moment of falling of the first wavefront on the atom). During this time the atom interacts only with a travelling wave, which creates the coherent connection between atomic energy levels (mixture state). And the values of momentum for these energy levels are distinguished by one photon momentum $\hbar k$ (k is the wave number).

But actually the third period comes in the conditions of experiments([1-3]) after standing wave formation, when there is again one travelling wave in the atom's place. It is the ''tail'' wave, which reflects from the mirror and propagates along the direction, opposite to the initial one. The mentioned time period has not been taken into account formerly, probably supposing that one -photon scattering process can't noticeably have an influence on the multi-photon scattering of the atoms in the standing wave field. In the present paper on the base of the simplest model, we show that in the third final stage of interaction a new type of quantum-mechanical coherent phenomenon takes place which we call Coherent Accumulation of Modulated Probability Amplitudes(CAMPA) and which entirely changes the angular distribution of the atoms compared to the end of the interaction with the standing wave. The matter of phenomenon is the following: after the interaction with the standing wave the translational motion wave function, as it is well known ([4-8]), can be presented as a family of definite momentum states with $\langle P \rangle =$ step, both, for ground and excited atomic energy levels. The probability amplitudes for neighborhood states have different signs, and it is the first principle cause for the presenting phenomenon. Moreover, a displacement $\hbar k$ exists between level's momentum distributions. So, the probability amplitudes with the different signs are intercovered with the new $\hbar k$ displacement in the momentum distribution at the one-photon transitions between the energy levels in travelling wave. This different sign amplitude superposition strongly suppresses the amplitudes in the inner region of momentum distribution. Essentially, the amplitudes become different from zero at the boundary values of these distributions. As a result, due to the interaction with the travelling wave the quantum mechanical translational states collect around the certain values of momentum for ground and excited state atoms separately.

Let us consider the quantum-mechanical behavior of an atom in the resonant field of the coherent wave, which falls and reflects from a mirror at a rest ([1-3]). The laser radiation and also the translational motion of atomic beam will be represented as a one dimensional flat wave propagating near the intercrossed direction. The Hamiltonian of the atom in dipole approximation

$$\hat{H} = \hat{H}_0 - \hat{d}E(t,z) \tag{1}$$

comprises the Hamiltonian for the free atom $\hat{H}_0$ and the interaction part $dE(t,z)$. The center of mass kinetic energy can be omitted in Hamiltonian due to the small interaction time, when the atom velocity variations are small in comparison with the thermal velocities, and the distance passed by atom along the wave direction is less than the wavelength. The dipole moment operator $\hat{d}$ and the electric field $E(t,z)$ are presented in scalar form. Exact atom-field resonance also is assumed. The atom momentum is conserved in laser wavefront plane and so the corresponding part of atomic wave function will not be written:

$$\Psi = \sum_{m=-1}^{1} A_m(t,z)\varphi_m(\vec{r},t) + B_m(t,z)\psi_m(\vec{r},t) \tag{2}$$

where φ_m and ψ_m are the stationary wave functions of free atom in the ground and excited levels correspondingly, $\vec{r}$ is the coordinate of the atom center, $A(t,z)$ and $B(t,z)$ are unknown coefficients to be found(they are the probability amplitudes of the atom on the ground or excited states independent of the atom's translational motion along the z axis). In the first stage of interaction ($0 \le t \le 2L/c$) the atom interacts with the travelling wave $E(t,z) = E\exp(ikz - \omega t) + c.c$. Substituting the Hamiltonian (1) with the mentioned field form and the wave function (2) into Schrodinger equation we get after standard transformations (in the rotating wave approximation) the following equations.

$$\begin{aligned} i\hbar\frac{\partial A(t,z)}{\partial t} &= -U\exp(-ikz)B(t,z), \\ i\hbar\frac{\partial B(t,z)}{\partial t} &= -U^{*}\exp(ikz)A(t,z), \end{aligned} \tag{3}$$

where $U = dE$, $d = \left\langle \varphi_a \middle| \hat{d} \middle| \psi_b \right\rangle$. Let the atom, as in experiments ([1-3]), is on the ground state before the interaction and has some momentum P_0 along the z-axis. Then

$$A(0,z) = \frac{1}{\sqrt{2\pi\hbar}}\exp(\frac{i}{\hbar}P_0 z) \equiv \chi(P_0);\; B(0,z) = 0\,. \tag{4}$$

The system (3) has well-known periodic solutions with the Rabi frequency $\nu = U/\hbar$. At the end of the first interaction stage ($t = \tau = 2L/c$) we have

$$A(\tau,z) = \chi(P_0)\cos\nu\tau;\, B(\tau,z) = i\chi(P_0 + \hbar k)\sin\nu\tau \tag{5}$$

which are the initial values for the second stage of interaction(with the standing wave). During the second time interval ($\tau \le t \le \tau_p$, where τ_p is the light pulse duration) the equations have the form (3) too, where the replacement $U\exp(\pm ikz) \to 2U\cos kz$ must be done. At the end of interaction with the standing wave $\left(t = \tau_p\right)$ the solutions are

$$\begin{aligned} A(\tau_p,z) &= \left[f(\tau_p,\tau:z)\cos\nu\tau - \exp(ikz)g(\tau_p,\tau:z)\sin\nu\tau\right]\chi(P_0) \\ B(\tau_p,z) &= i\left[f(\tau_p,\tau:z)\sin\nu\tau + \exp(-ikz)g(\tau_p,\tau:z)\cos\nu\tau\right]\chi(P_0 + \hbar k) \end{aligned} \tag{6}$$

where the following notations are used

$$f\left(\tau_p,\tau;z\right) = \cos\left(2\nu\left(\tau_p - \tau\right)\cos kz\right),\; g\left(\tau_p,\tau;z\right) = \sin\left(2\nu\left(\tau_p - \tau\right)\cos kz\right).$$

When the atom interacts with the "tail" travelling wave ($\tau_p \le t \le\; \le \tau_p + \tau$), which has the form $E(t,z) = E\exp(ikz - \omega t) + c.c$, the amplitudes can be written

$$A(t,z) = A(\tau_p,z)\cos\nu(t-\tau_p) + i\exp(ikz)B(\tau_p,z)\sin\nu(t-\tau_p) \tag{9a}$$

$$B(t,z) = B(\tau_p,z)\cos\nu(t-\tau_p) + i\exp(-ikz)A(\tau_p,z)\sin\nu(t-\tau_p)\,. \tag{9b}$$

At the moment $t = \tau_p + \tau$ the light pulse is passing, leaving the atom in the free state. After the above-mentioned three stages of interaction, atom flies and falls on detector, spontaneously emitting from the excited states. But it is clear that really the contribution is small from these incoherent one-photon processes and it can be left behind.

In distinction from the initial moment $t = 0$, atom hasn't a definite value of momentum along the z-axis. But (including ([1-3]) too) the atoms in experiments, as a rule, are registered under the definite angles in comparison with initial direction, i.e. in the states with the definite values of P.Therefore, it is worth to expand the atomic total amplitudes (6) in a series of amplitudes with the definite values of momentum. For this we use the known formula:

$$e^{\pm ix\cos\theta} = \sum_{n=-\infty}^{n=\infty} (\pm i)^n J_n(x) e^{in\theta}, \tag{10}$$

where $J_n(x)$ is the Bessel function. Then the square of modulus of the coefficient at the each exponential expression presents the atomic probability to have the momentum $P = P_0 + n\hbar k$ on the ground (for A's series) or excited (for B's series) energy values. For them we obtain the following expressions ($\tau_p \le t \le \tau_p + \tau$):

$$W_{ground}^{(n)} = \frac{1+(-1)^n}{2}[J_n(u)\cos\nu\tau\cos\nu(t-\tau_p) + J_{n-1}(u)\sin\nu(t-\tau_p+\tau) + \\ + J_{n-2}(u)\sin\nu\tau\sin\nu(t-\tau_p)]^2, \tag{11a}$$

$$W_{excited}^{(n)} = \frac{1-(-1)^n}{2}[J_{n+1}(u)\cos\nu\sin\tau\nu(t-\tau_p) - J_n(u)\sin\nu(t-\tau_p+\tau) - \\ - J_{n-1}(u)\cos\nu\tau\cos\nu(t-\tau_p)]^2, \tag{11b}$$

where $u = 2\nu\tau_s$.

These formulas present, in fact, the momentum distributions for the ground and excited atoms consequently. Each of them is asymmetric relative to $n \to -n$ transformation in general. To see the principal result of this paper, the CAMPA phenomenon, it is enough to compare these expressions for two moments of time: $t = \tau_p$, when the influence of standing wave has just finished, and $t = \tau_p + \tau$ when the interaction runs fully. Expressions strongly differ from each other in general. It can be confirmed also by the illustration on Fig.2(a,b).

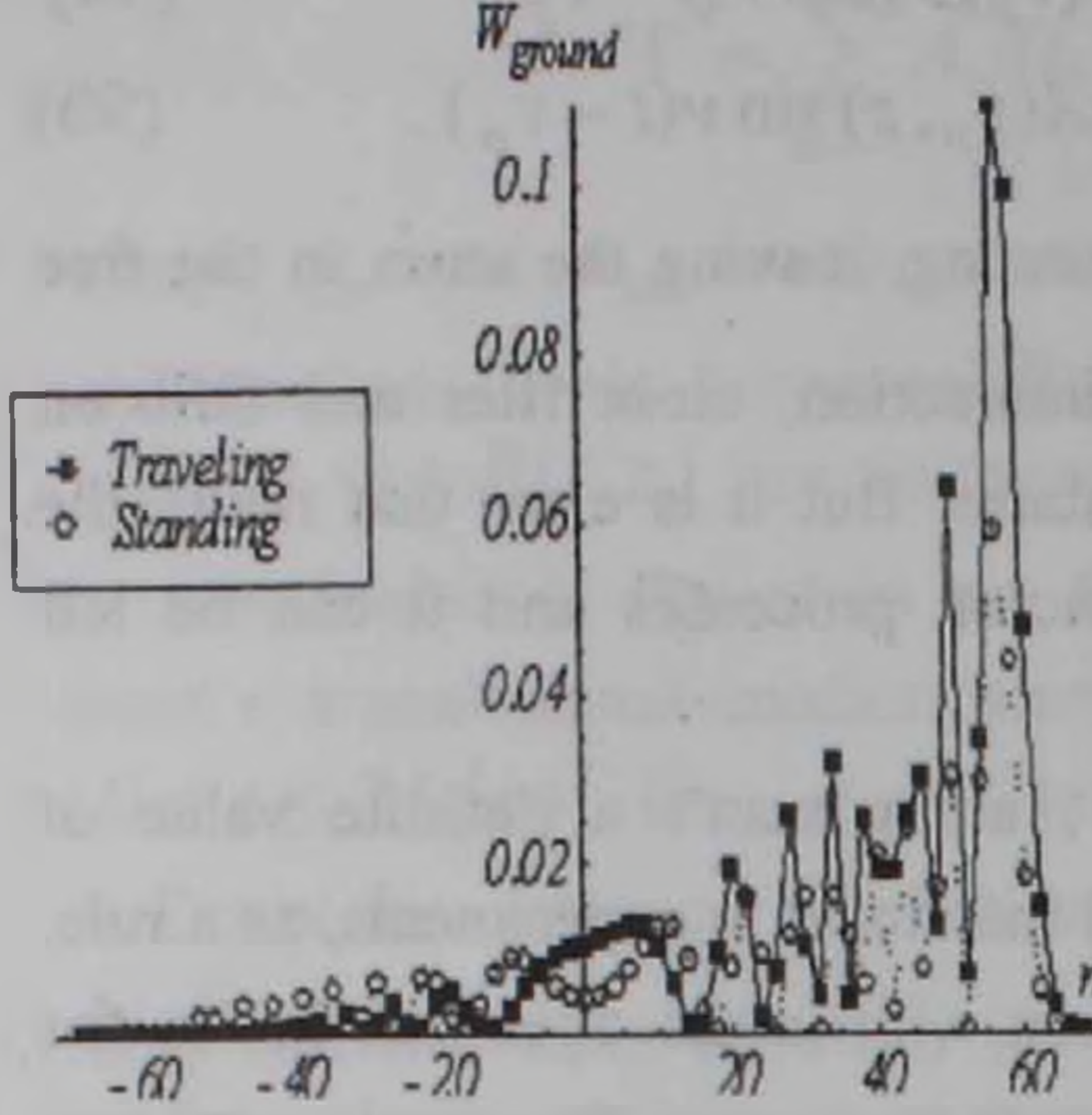

Fig.2a. The $W^{(n)}_{ground}$ probability versus the number n of reemitted photons at the moment $t = \tau_p$, when the influence of standing wave is just finished, and at the moment $t = \tau_p + \tau$ when the interaction runs fully. The parameters are $|A(\tau)|^2 = \frac{1}{2}, |B(\tau)|^2 = \frac{1}{2}$, $\nu\tau = 20, \nu\tau_p = 50$.

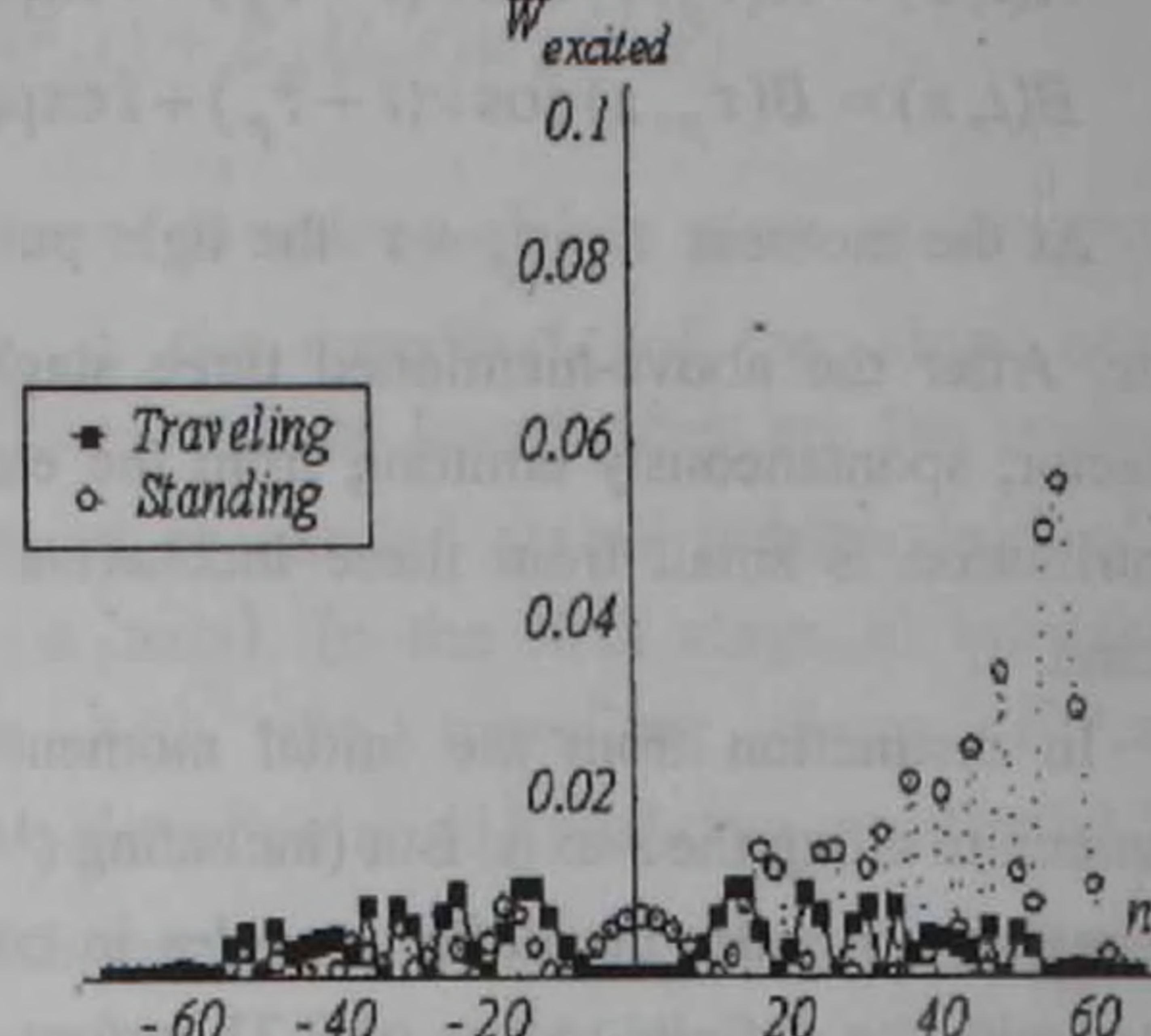

Fig. 2b. The $W^{(n)}_{excited}$ probability versus the number n of reemitted photons at the moment $t = \tau_p$, when the influence of standing wave is just finished, and at the moment $t = \tau_p + \tau$. The parameters are the same as on Fig.2.

The average momentum obtained by an atom for any interaction moment in the, can be presented as a sum of two terms: $\langle P\rangle = = \langle P\rangle^{in} + \langle \Delta P\rangle$. For the final interaction range ($\tau_p \le t \le \tau_p + \tau$) the quantities $\langle P\rangle^{in}$ and $\langle \Delta P\rangle$ are average momentum obtained during the interaction with standing wave and obtained in the ''tail'' travelling wave before the consideration of moment t, consequently. Additional average momentum $\langle P\rangle^{in}$ and $\langle \Delta P\rangle^{in}$, like the corresponding probabilities (11a) and (11b), obtained by means of CAMPA, oscillate on Rabi frequencies On Fig.3a we show these oscillations for the case $|A(\tau)|^2 = = \frac{1}{2}, B(\tau) = \frac{1}{2}$. The preceding time ranges are presented too. The oscillation amplitudes approximately equal to the ''entrance'' value. So, one photon coherent process can vary atomic momentum on the ground and excited states just equal to the variation, induced in the standing wave! Of course, the total additional momentum varies only in the limits of one photon momentum $\hbar k$. Fig.3b illustrates similar oscillations for the case $|A(\tau)|^2 = 1, B(\tau) = 0$ (symmetric scattering in the standing wave).

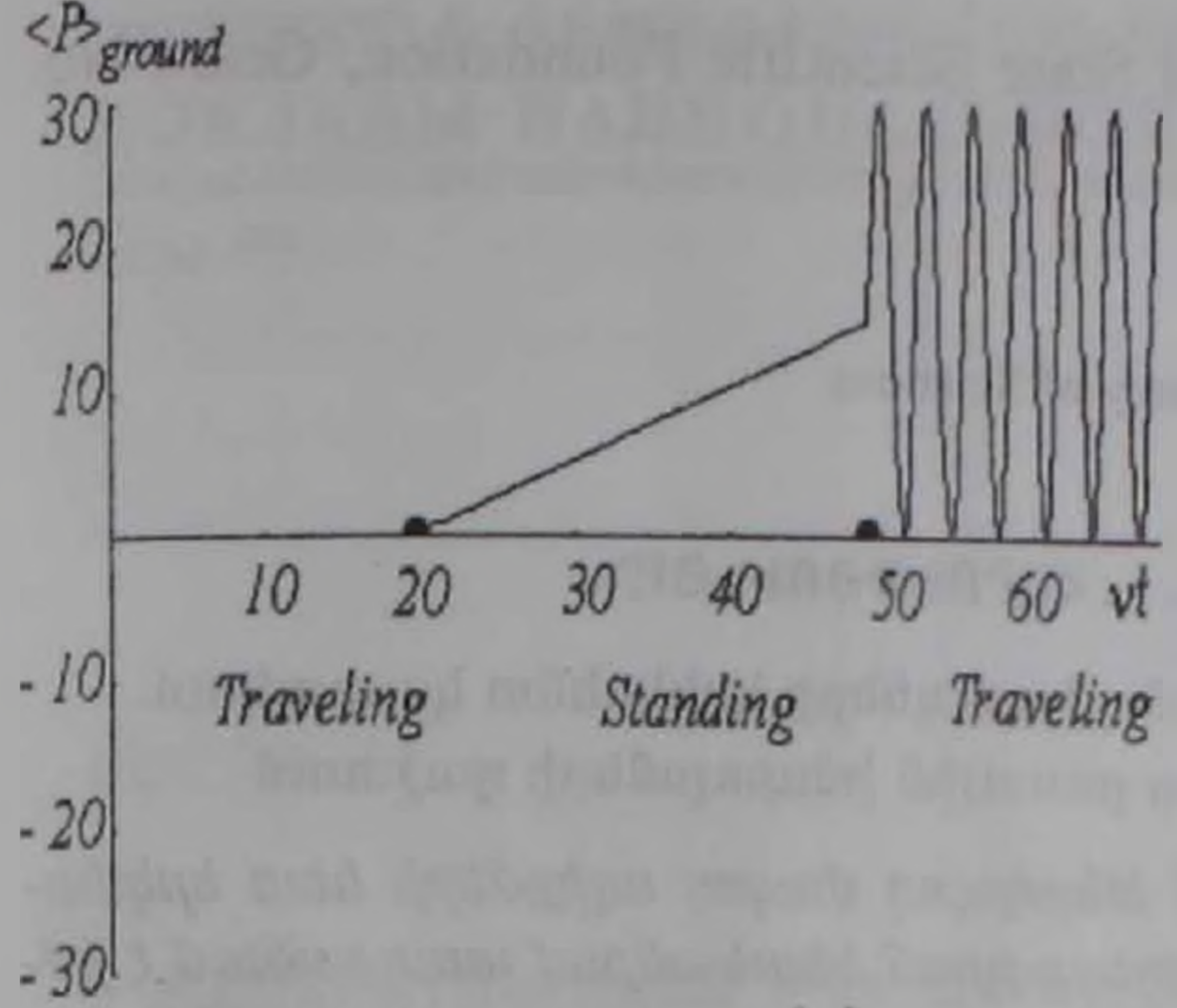

Fig.3a. The time evolution of $\langle P\rangle_g$ for the case of maximum asymmetry of momentum distribution after the interaction with the standing wave $|A(\tau)|^2=\frac{1}{2},|B(\tau)|^2=\frac{1}{2}$).
The parameters are $\nu\tau=20,\ \nu\tau_p=50$.

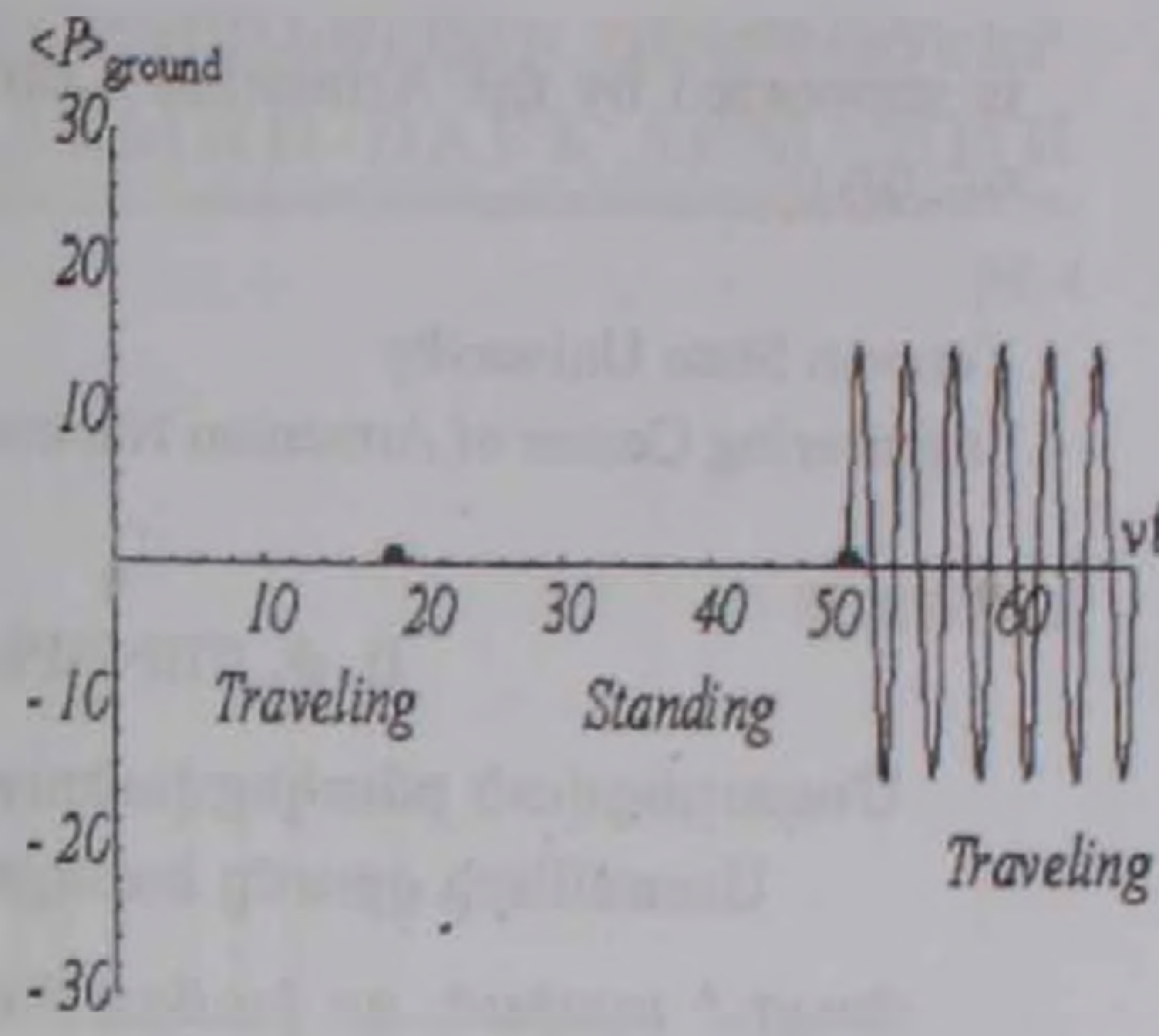

Fig.3b. The time evolution of $\langle P\rangle_g$ for the case of symmetric of momentum distribution after the interaction with the standing wave ($|A(\tau)|^2=\frac{1}{2},|B(\tau)|^2=\frac{1}{2}$). The parameters are $\nu\tau=20,\nu\tau_p=50$.

Let us also note a special future of scattering for final moment of interaction in scheme ([1-3]). It is the relation $\langle P\rangle_e=\langle P\rangle_e^{in}+\langle\Delta P\rangle_e=0$ at the moment $t=\tau_p+\tau$. This relation means that the asymmetry of scattering induced for excited atoms by the existence of first travelling wave is rubbed out by the final travelling wave. It will be expected that the CAMPA phenomenon will appear most sharply, if at the initial moment of interaction with the last travelling wave the atom has

(a) wide and symmetric (for modulus) distribution of momentum for both ground and excited energy levels

(b) equal populations of energy levels.

It is clear that ([1-3]) interaction scheme is not optimal for CAMPA, because the (a) and (b) conditions aren't satisfied simultaneously. Actually, the first condition takes place for $\cos\nu\tau=1$ (or $\sin\nu\tau=0$), but then one of energy levels isn't populated at all, and on the contrary, when the energy levels are populated equally, the momentum distributions are maximally asymmetric and narrow. Nevertheless the CAMPA makes a contribution into the process of the atoms scattering compareable to the contribution of standing wave even in such unfavorable conditions.

We hope that this phenomenon will be taken into account in consistent quantum theory of interaction with the travelling e.m. wave when atomic (molecular) wave function is preliminary modulated by the interaction with the periodic field, in particular for substantial explanation of results ([1-3]).

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Մոդուլացված բնակեցվածության ամպլիտուդների կոհերենտ կուտակում. Ատոմների ցրումը հանդիպակաց լուսային իմպուլսների դաշտում

Ցույց է տրված, որ կանգուն և նրան հաջորդող վազող ալիքների հետ երկմակարդակ ատոմի ռեզոնանսային փոխազդեցության հետևանքով առաջանում է իմպուլսային տարածության մեջ ատոմի կողմից ձեռք բերած իմպուլսների բաշխման ֆունկցիայի կտրուկ խտացման հնարավորություն:

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Когерентная аккумуляция модулированных амплитуд вероятности: Рассеяние атомов в поле встречных световых импульсов

Показано, что вследствие резонансного взаимодействия двухуровневого атома с стоячей волной и последующей бегущей появляется возможность резкой концентрации в импульсном пространстве распределения приобретенных атомом импульсов.

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