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Line Formation in an Atmosphere with Arbitrary Velocity Gradient

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1. Introduction

In studying the spectra of various cosmic objects we find them very often being affected essentially by the macroscopic motions within a medium where they are formed. Gaseous nebulae, shells of the novae or supernovae, a wide range of the non-stationary phenomena in the solar atmosphere, stellar winds represent typical examples of such objects. The presented list is obviously far from being exhaustive. It should be emphasized that the theory of the spectral line formation with allowance for the velocity field in an atmosphere is much more involved compared to that for the static media. This primarily concerns the strong resonance lines formed in the optically thick atmosphere when the effects of the multiple scattering become dominating. This kind of problem is encountered, for instance, in treating the extended atmospheres or the dense winds of the LBV (Luminous Blue Variable) stars (see e.g.([1]) and list of references therein).

The main objective of this report is to demonstrate that Ambartsumian's invariance principle in conjunction with the previous results of the present authors ([2,3]) may be used successfully in tackling the problem of the spectral line formation in a scattering atmosphere characterized by the internal motions. For expository reasons, we shall limit the discussion to considering the model problem of non-coherent scattering in an one-dimensional semi-infinite atmosphere. No constraints will be imposed on the velocity gradient in the atmosphere. Note that a kind of such problem but for the velocity increasing linearly with depth was treated by Sobolev ([4]), who obtained approximate solutions of the problem under various assumptions concerning the velocity gradient.

The invariance technique employed below in deriving the basic equations is largely similar to that by Sobolev ([5]) elaborated for the non-homogeneous atmosphere. For simplicity, we shall adopt the probabilistic language and deal with quantities possessing transparent physical significance.

2. Description of the scattering process.

Let us start with specifying the elementary event of scattering. Throughout the paper the scattering process is assumed to be isotropic and following by the partial redistribution over frequencies. For the redistribution function (averaged over directions) we introduce the commonly used notation $r(x',x)$, where x and x' are dimensionless frequencies (measured from the center of the line ν_0 in the units of Doppler widths $\Delta\nu_D = (u/c)\nu_0$, where u is the thermal velocity) of the incident and scattered quanta, respectively. The function $r(x',x)$ obeys the condition

$$\int_{-\infty}^{\infty} r(x',x)dx' = \alpha(x), \tag{1}$$

where $\alpha(x)$ is the line absorption profile normalized as follows

$$\int_{-\infty}^{\infty} \alpha(x)dx = \sqrt{\pi}. \tag{2}$$

Note also that in some specific but important cases of the frequency redistribution the function $r(x',x)$ admits bilinear expansion over the set of certain functions $\{\alpha_\kappa(x)\}$

$$r(x',x) = \sum_{k=0}^{\infty} A_k \alpha_k(x')\alpha_k(x), \tag{3}$$

where A_k are some constants. Referring the reader for the details of this point to the papers ([6,7]), we note that for pure Doppler redistribution in frequencies when

$$r(x',x) = \int_{\max(|x|,|x'|)}^{\infty} e^{-t^2} dt \tag{4}$$

and $\alpha(x) = e^{-x^2}$, we have $A_k = 1/(2k+1)$, and

$$\alpha_k(x) = \left(\pi^{1/4} 2^k \sqrt{(2k)!}\right)^{-1} e^{-x^2} H_{2k}(x), \tag{5}$$

where $H_k(x)$ is the Hermit polynomial of the kth order.

During multiple scattering in an atmosphere, the photon may be thermalized by absorbing in the continuous spectrum. This process is characterized by the ratio β of the absorption coefficient in the continuum to that in the center of the spectral line.

Before proceeding directly to our problem we introduce two important characteristics of the diffusion process in the source-free homogeneous and stationary atmosphere. One of them is the reflectance ρ of the semi-infinite atmosphere, also referred to as 'reflection function' or 'reflection coefficient'. The probabilistic meaning assigned to function ρ in case of non-coherent scattering (occurring in a one-dimensional medium) is given as follows: $\rho(x',x)dx$ is the probability that a photon of frequency x' incident over the semi-infinite atmosphere will be reflected in the frequency domain $(x, x+dx)$. Being applied to the reflectance of atmosphere, Ambartsumian's invariance principle leads to the functional equation

$$\begin{aligned}(2/\lambda)\left[v(x') + v(x)\right]\rho(x',x) = r(x',x) + \int_{-\infty}^{\infty} r(x',x'')\rho(x'',x)dx'' + \\ + \int_{-\infty}^{\infty} \rho(x',x'')r(x'',x)dx'' + \int_{-\infty}^{\infty} \rho(x',x'')dx'' \int_{-\infty}^{\infty} r(x'',x''')\rho(x''',x)dx''',\end{aligned} \tag{6}$$

where $v(x) = \alpha(x) + \beta$, and λ is the single-scattering albedo. Eq. (6) in the specific case of $\beta = 0$ was written for the first time by Sobolev ([8]) (see also ([9,10])).

The second quantity needed in further discussion is the function P defined as to have the following probabilistic meaning: $P(\tau,x',x)dx$ is the probability that a photon of frequency x', *moving* initially at depth τ in either direction, will escape it as a result of multiple scattering as a photon with frequency from the interval $(x, x+dx)$. The principle of invariance, which requires that the addition (or removal) of an infinitesimal layer to (from) the surface of the semi-infinite atmosphere must not change the mentioned probability, yields ([2])

$$\frac{dP(\tau,x',x)}{d\tau} = -\nu(x)P(\tau,x',x) + \int_{-\infty}^{\infty} P(\tau,x',x'')\alpha(x'')p(0,x'',x)dx'', \qquad (7)$$

where $p(\tau,x',x)$ is the photon escape probability designed for the photon of frequency x' *absorbed* at the optical depth τ. It is obvious that $P(0,x',x) = \rho(x',x)$. Note also that the function $p(0,x',x)$ is also simply expressed in terms of the reflectance ρ and the redistribution function

$$\alpha(x')p(0,x',x) = r(x',x) + \int_{-\infty}^{\infty} r(x',x'')\rho(x'',x)dx''. \qquad (8)$$

In fact, the knowledge of the function P allows to find the outgoing intensity for any given distribution of energy sources in the static atmosphere.

3. Formulation of the problem and basic equations

Now let us consider a semi-infinite atmosphere, the various parts of which are moving relative to each other. Suppose $V(\tau)$ be the velocity field in the atmosphere. This function is assumed to be given and we do not impose any restrictions on its behaviour.

In discussing the scattering process in such an atmosphere one must distinguish from each other the frequency x of a photon in the laboratory or observer's frame and that (denoted hereafter by $x_1(\tau)$ or $x_2(\tau)$ depending on the direction of propagation) in the comoving frame at a given depth τ. These frequencies are related by means of Doppler formula

$$x_1(\tau) = x + \delta(\tau)\,; \quad x_2(\tau) = x - \delta(\tau), \qquad (9)$$

where $\delta(\tau) = V(\tau)/u$.

Our purpose in this section is to evaluate the line intensity radiated by the atmosphere described, in the presence of the internal energy sources, the power of which we denote by $\varepsilon(\tau,x)$. Before turning immediately to this problem we shall discuss first an auxiliary problem of determining the reflectance of the non-stationary atmosphere.

In contrast to the stationary case, the problem of finding the reflectance must be embedded into family of problems that concern atmospheres, the top parts of which, referenced from some optical depth t, are removed. We shall call such atmospheres truncated, and t is the truncation parameter. Thus, we are interested in the task of evaluating the reflectance $\rho(t,x',x)$ for the family of atmospheres dependent on the parameter t. On setting $t = 0$ in equations derived below, we arrive obviously at the requisite results pertaining our particular problem.

The standard invariance procedure, being applied to the reflectance $\rho(t,x',x)$ of a truncated atmosphere, yields

$$(2/\lambda)[v(x_1') + v(x_2)]\rho(t,x',x) - \frac{\partial \rho(t,x',x)}{\partial t} = r(x_1',x_2) +$$
$$+ \int_{-\infty}^{\infty} r(x_1',x_1'')\rho(t,x'',x)dx'' + \int_{-\infty}^{\infty} \rho(t,x',x'')r(x_2'',x_2)dx'' + \qquad (10)$$
$$+ \int_{-\infty}^{\infty} \rho(t,x',x'')dx'' \int_{-\infty}^{\infty} r(x_2'',x_1''')\rho(t,x''',x)dx''',$$

where $x_{1,2}(t) = x \pm \delta(t)$, $x_1'(t) = x' \pm \delta(t)$, and so on. For simplicity of exposition, the dependence of x and x', on t in Eq.(10) is not marked explicitly.

If the redistribution function $r(x',x)$ admits the bilinear expansion of the type given by Eq.(3), the formal solution of Eq.(10) may be written as

$$\rho(t,x',x) = (\lambda/2)\sum_{k=0}^{\infty} A_k \int_t^{\infty} \varphi_k(\tau,x_1')\overline{\varphi_k}(\tau,x_2)\exp\left(-\int_t^{\tau}\{v[x_1'(t')] + v[x_2(t')]\}dt'\right)d\tau, \qquad (11)$$

where

$$\varphi_k[t,x_1'(t)] = \alpha_k[x_1'(t)] + \int_{-\infty}^{\infty} \rho(t,x',x'')\alpha_k[x_2''(t)]$$
$$\overline{\varphi}_k[t,x_2(t)] = \alpha_k[x_2(t)] + \int_{-\infty}^{\infty} \rho(t,x'',x)\alpha_k[x_1'(t)]dx''. \qquad (12)$$

Similarly, by using the invariance technique one can find the following equation

$$\frac{\partial P(t,\tau,x',x)}{\partial \tau} - \frac{\partial P(t,\tau,x',x)}{\partial t} = -v(x_2)P(t,\tau,x',x) + \qquad (13)$$
$$+ \int_{-\infty}^{\infty} P(t,\tau,x',x'')\alpha(x_2'')p(t,0,x'',x)dx''$$

where the functions $P(t,\tau,x',x)$ and $p(t,\tau,x',x)$ have the probabilistic meaning similar to that of functions $P(\tau,x',x)$ and $p(\tau,x',x)$ introduced in the preceding paragraph, with only difference that these new ones concern the truncated atmosphere.

On the other hand, taking into account the probabilistic meaning of $P(t,\tau,x',x)$ one can immediately write down the expression for the intensity of outgoing radiation

$$I(t,x) = \int_{-\infty}^{\infty} dx' \int_0^{\infty} \{\varepsilon[t+\tau,x_1'(t+\tau)] + \varepsilon[t+\tau,x_2'(t+\tau)]\}P(t,\tau,x',x)d\tau. \qquad (14)$$

Then, by using Eqs.(13) and (14) one may obtain

$$v(x_2)I(t,x) - \frac{\partial I(t,x)}{\partial t} = \int_{-\infty}^{\infty} I(t,x')\alpha(x_2')p(t,0,x',x)dx' + \qquad (15)$$
$$+\varepsilon(t,x_2) + \int_{-\infty}^{\infty} \varepsilon(t,x_1')\rho(t,x',x)dx'$$

It is easily seen that for $\delta(t) = 0$ Eqs. (10) and (13) transform into Eqs.(6) and (7), respectively.

When the frequency redistribution law $r(x',x)$ admits the bilinear expansion (3), the formal solution of the Eq.(15) may be written as follows

$$I(t,x) = (\lambda/2)\sum_{k=0}^{\infty} A_k \int_t^{\infty} \overline{\varphi}_k[\tau,x_2(\tau)]I_k(\tau)\exp\left(-\int_t^{\tau} v[x_2(t')]dt'\right)d\tau +$$

$$+\int_{t}^{\infty}\left\{\varepsilon\left[\tau,x_2(\tau)\right]+\int_{-\infty}^{\infty}\varepsilon\left[\tau,x_1'(\tau)\right]\rho(\tau,x',x)dx'\right\}\exp\left(-\int_{t}^{\tau}v\left[x_2(t')\right]dt'\right)d\tau, \quad (16)$$

where

$$I_k(\tau)=\int_{-\infty}^{\infty}I(\tau,x')\alpha_k\left[x_2'(\tau)\right]dx'. \quad (17)$$

Thus we obtained the analytical solution of the problem. For the practical use of the results it is necessary to elaborate the facilitate methods of numerical calculations which is the subject of a separate study to be performed in the following papers.

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Սպեկտրալ գծերի առաջացումն արագությունների կամայական գրադիենտով մթնոլորտում

Համբարձումյանի ինվարիանտության սկզբունքը կիրառվում է այն դեպքի համար, երբ կիսանվերջ մթնոլորտում գոյություն ունի արագությունների գրադիենտ: Ստացվել են անդրադարձման ֆունկցիայի հատված մթնոլորտից քվանտի դուրս գալու հավասարումները: Նույնանման հավասարում է ստացվել նաև միջավայրից դուրս եկող ճառագայթման ինտենսիվության համար: Ստացված բոլոր հավասարումներն արագությունների գրադիենտի բացակայության դեպքում վերափոխվում են արդեն հայտնի ավելի պարզ հավասարումների:

Г. А. АРУТЮНЯН, А. Г. НИКОГОСЯН

Образование спектральных линий в атмосфере с произвольным градиентом скоростей

Применяется модифицированный принцип инвариантности Амбарцумяна в случае, когда в полубесконечной атмосфере существует градиент скоростей. Получены соответствующие уравнения для функции отражения и вероятности выхода кванта из усеченной атмосферы. Аналогичное уравнение получено также и для интенсивности выходящего из среды излучения. Все полученные уравнения в случае отсутствия градиента скоростей переходят в известные, более простые уравнения.

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