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Application of Ambartsumian Principle of Invariance to Problems of Radiation Transfer in Solids

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The theory of radiative transfer ([1-3]) developed for multiple scattering of light in gaseous media and the scattering of neutrons may be applied for the solution of a member of new problems of transfer in solids. The present work is a brief review of works carried out in IAPP, NAS, RA. The problem of γ-radiation in crystals containing Mössbauer isotopes is an important example of the aforesaid. Of great interest in the gamma resonance spectroscopy are angular and frequency distribution of the of γ-radiation scattered in the crystal that gives information about the properties and structure of the medium.

The Mössbauer γ-quantum noncoherent scattering has been theoretically analysed in many papers ([4-7]). But they take into account only a single quantum scattering which is true either for a thin scatterer ($\tau_0 << 1$, τ_0 is the scattering layer effective thickness) or for scattering with a small probability of re-emission $\lambda = 1/(1+\alpha)$ (α is the inner conversion coefficient). But in the experimental particle we often have to deal with thick samples $\tau_0 > 1$ and Mössbauer nuclei with a small conversion coefficient in which the single scattering approximation is a priori wrong.

The first attempt to take the multiple scattering into account in a one-dimensional medium was made in ([8]) where the theory of the optical radiation transfer was applied to solve the problem of the γ-radiation transfer in isotropic media containing Mössbauer nuclei.

The intensity of radiation at multiple isotropic scattering with complete redistribution in frequencies on crystals of finite thickness is determined. It is shown that the width of resonance line in the energy spectrum of γ-radiation increases as the thickness of scatterer.

For homogeneous semi-infinite media the problem of radiation transfer in the simplest models of a scattering act, i.e., the monochromatic scattering with complete redistribution in frequencies (in the isotropic case) admits exact analytical solutions ([1,2,9,10]).

In Refs ([11-12]) approximate analytical solutions have been obtained for general problems of radiation transfer in a layer of finite thickness in case of complete redistribution in frequencies inside the spectral line (both in the one-dimensional approximation and in the three-dimensional case). The accuracy of these solutions increases with increasing layer thicknesses. The calculations were carried out for the case of isotropic scattering. Using the method of ([14-16]) that permits to solve the transfer problem in the slab of finite thickness with the help of the Ambartsumian function for half-space. The Ambartsumian functions φ and ψ were calculated for cases of Doppler and Lorentz profiles of the coefficient. In the three-dimensional case these functions have the following form([12]):

$$\varphi(\tau_0,z)=\varphi(z)-\frac{2}{\lambda}\cdot\frac{\widetilde{F}(\tau_0,z)}{\varphi(z)}\beta(\tau_0,z)\left(1-\frac{\lambda}{2}\varphi_0\right)-\frac{\lambda}{2}\varphi(z)\widetilde{F}(\tau_0,z)\psi_0,$$
$$\psi(\tau_0,z)=\varphi(z)\widetilde{F}(\tau_0,z)\left(1-\frac{\lambda}{2}\varphi_0\right)+\frac{2}{\lambda}\cdot\frac{\beta(\tau_0,z)}{\varphi(z)}\left(1-\frac{\lambda}{2}\psi_0\widetilde{F}(\tau_0,z)\right), \quad (1)$$

On the basis of results ([11-13]), in Refs ([17,18]) the problems of γ-radiation transfer were studied in cases of both isotropic and anisotropic scattering with complete redistribution in frequencies. The problems of diffuse reflectectia and transmission of Mössbauer radiation were solved.

In ([17]) deals with the problem of the γ-quantum transfer in the three-dimensional case of plane-parallel layers taking account of the multiple scattering at various elementary processes of scattering.

In the media containing Mössbauer nuclei there are two main competing channels scattering, the nuclear resonance scattering and the electron scattering ([6]) (Rayleigh and Compton scattering). However, since the cross-sections of Rayleigh and Compton scattering are much less than that of the resonance scattering, their contributions are not taken into account.

In the case of the nuclear resonance scattering we deal with nonconservative scattering, i.e., $\lambda<1$.

Another important feature of the scattering process is the probability $g(x_1, x_2)$ that after absorbing a quantum with the frequency x_1 the nucleus will emit a quantum with the frequency x_2, where $x=(\omega-\omega_0)/(\Gamma_a/2)$ is the deviation of the γ-quantum frequency ω from the resonance frequency ω_0 in units of the half-width of the nuclear excited state $\Gamma_a/2$. In the first part of ([17]) paper deal with the model already proposed in ([4-5]) which assumes that $x_1=x_2$ (monochromatic scattering), i.e., $g(x_1, x_2)$ is a δ-function $g(x_1,x_2)=\delta(x_1-x_2)$. The second part is devoted to the model of complete frequency redistribution $(g(x_1,x_2)=(1/\pi)\alpha(x_2))$, corresponding to the case when the nucleus forgets the absorbed quantum frequency (a nucleus "without memory").

The resonance scattering angular distribution, which is non-spherical in the general case due to the crystal structure and its physical properties, is another characteristic quantity of the elementary process of scattering. Below consider the case of an isotropic medium corresponding to the case of a spherical distribution $x(\gamma)=1$.

In addition to these assumptions, using the so-called probability method ([9]) for calculating the intensity $I_\eta^+(\upsilon)$ of the transmitted resonant γ-quantum beam and for the intensity $I_\eta^-(\upsilon)$ of the reflected one, we have

$$I_\eta^+(\upsilon)=\frac{1}{\eta}\int_0^{\tau_0}d\tau\int_{-\infty}^{\infty}p(\tau_0-\tau,\eta,x_1,x_2,\upsilon)dx_1dx_2,$$
$$I_\eta^-(\upsilon)=\frac{1}{\eta}\int_0^{\tau_0}d\tau\int_{-\infty}^{\infty}p(\tau,\eta,x_1,x_2)L(\tau,x_1,\upsilon)dx_1dx_2, \quad (2)$$

where $p(\tau,\eta,x_1,x_2)dx_2$ is the probability that the quantum with frequency x_1 absorbed in the effective thickness τ leaves the medium with the frequency in the range (x_2, x_2+dx_2), $L(\tau,x_1,\upsilon)dx_1$ is the energy absorbed in the medium per second, υ is the source velocity, and $\eta=\cos\vartheta$.

It should be noted that in our treatment the radiation sources may be found both inside and outside the scattering medium.

We discuss the case when the γ-radiation incident on the absorber at an angle ϑ_0 has the intensity $I(x_1, \upsilon)=I_0\alpha_0(x_1-\upsilon)$; then the energy absorbed at the depth τ from the surface may be expressed by

$$L(\tau, x_1, \upsilon) = I_0\alpha_0(x_1 - \upsilon)\alpha(x_1)e^{-\alpha(x_1)\tau/\eta_0}, \tag{3}$$

where $\alpha_0(x)$ is the radiation line-shape and $\alpha(x)$ the absorption lineshape (both lines are assumed to be Lorentzians $\alpha(x)=\alpha_0(x)=1/(x^2+1)$), $\eta_0=\cos\vartheta_0$, and πI_0 is the resonant quantum complete intensity.

For the probability p it is easy to find a certain integral relation. Actually, the emission probability of a quantum absorbed at the depth τ is $(\lambda/4\pi)g(x_1, x_2)$, while the probability that a quantum leaves the medium with the frequency x_2 at the angle ϑ after multiple scattering is $p(\tau,\eta,x_1,x_2)$. But the quantum may also leave the medium without scattering, the probability being $(1/4\pi)e^{-\alpha(x_2)\tau/\eta}$, or it may first be absorbed at the point τ', then re-emitted and leave the medium, the probability being the same p. Summing over all those points τ' we obtain

$$p(\tau,\eta,x_1,x_2) = \frac{\lambda}{4\pi}g(x_1,x_2)e^{-\alpha(x_2)\tau/\eta} + \frac{\lambda}{2}\int_{-\infty}^{\infty} g(x_1,x)\alpha(x)dx \times$$

$$\times \int_0^{\tau_0} p(\tau',\eta,x_1,x_2)d\tau' \int_0^1 e^{-\alpha(x)|\tau-\tau'|/\eta'}\frac{d\eta'}{\eta'}. \tag{4}$$

In ([17]) the solution of this integral equation is discussed for two (monochromatic, complete frequency redistribution) cases of the function $g(x_1, x_2)$.

In the general case, however, the scattering distribution is nonspherical and, consequently, the problem must be solved taking into account the anisotropy of scattering. In ([18]) is devoted to anisotropic incoherent scattering of Mössbauer radiation taking account of its multiple interaction with the nuclei. It should be noted that in ([5,6]) only single scattering has been considered, which obviously leads to wrong results.

In the ([18]) paper is also devoted to the study of the frequency-modulated γ-radiation scattering in a plane parallel layer.

The angular distribution of the nuclear resonance scattering of γ-quanta (the scattering indicatrix) described by the general theory of angular correlations ([19,20]) is only of the main features of the elementary scattering process. In case of quadrupole interaction between the nuclei and crystalline fields in a polycrystalline medium the angular distribution is written as

$$\chi(\theta) = \sum_{i=0}^{n} A_i C_i P_i(\cos\theta) \tag{5}$$

where A_i are constants, C_i the coefficients of the angular distribution attenuation, and $P_i(\cos\theta)$ Legendre polynomials.

For the spin transition 1/2→3/2→1/2 (L = 1, 2) the angular distribution has a relatively simple form,

$$\chi(\theta) = 1 + \frac{C_2}{4} P_2(\cos\theta) \tag{6}$$

where the coefficient C_2 changes from 0.2 (for strong quadrupole or magnetic interaction) up to 1 (for weak quadrupole or magnetic interaction).

The anisotropy of the factor f_a ([22]) is also taken into account in ([21]) and it is shown that in this case the value C_2 is found in the same range.

Thus, the anisotropic scattering of Mössbauer radiation in finite layers will be considered in the indicatrix approximation (5).

In calculations the Rayleigh channel of γ-quantum scattering is neglected. The terminology and definitions are taken from ([8]).

Let us introduce $p(\tau, x_1, x_2, \eta', \eta, \varphi)d\Omega dx_2$ denoting the probability that a quantum with the frequency x_1 (in units Γ/2), incident at an angle arccos η' to the external normal of the medium surface $\tau = 0$ and absorbed at the optical depth τ, will emerge from the medium through the surface $\tau = 0$ at the angle arccos η to the normal, at the azimuth φ within the solid angle $d\theta$, and frequency in the range x_2, $x_2 + dx_2$. Taking into account multiple scattering we have the equation for the quantum emergence probability from the medium (see e.g. ([23]))

$$p(\tau, x_1, x_2, \eta_0, \eta, \varphi) = \frac{\lambda}{4\pi} g(x_1, x_2)\chi(\gamma)e^{-\alpha(x)\tau/\eta} +$$

$$+\frac{\lambda}{4\pi}\int_{-\infty}^{\infty} dx \int_0^{2\pi} d\varphi' \left[\int_0^1 \chi(\gamma')\frac{d\eta'}{\eta'}\int_0^{\tau} e^{-\alpha(x)(\tau-\tau')/\eta'} g(x_1, x)\alpha(x)p(\tau', x, x_2, \eta', \eta, \varphi-\varphi')d\tau' -\right.$$

$$\left.-\int_{-1}^{0} \chi(\gamma')\frac{d\eta'}{\eta'}\int_{\tau}^{\tau_0} e^{-\alpha(x)(\tau-\tau')/\eta'} g(x_1, x)\alpha(x)p(\tau', x, x_2, \eta', \eta, \varphi-\varphi')d\tau'\right]$$

where γ and γ' are the scattering angles, $g(x_1, x_2)$ is the probability of the frequency redistribution (see ([24]), λ the probability for the quantum to survive in the process of interaction with the nucleus, and $\alpha(x)$ the line contour ($\alpha(x) = 1/x^2 + 1$).

Further simplification of this equation is connected with the substitution of physical models for the scattering indicatrix $\chi(\gamma)$ and the probability of the frequency redistribution $g(x_1, x_2)$. The scattering indicatrix may be always expanded by Legendre polynomials (see ([8])), therefore, the property of these polynomials may be applied to further calculations.

For the frequency redistribution probability there may evidently exist two models: the model of monochromatic scattering and the model of complete frequency redistribution.

The fact that the redistribution function $g(x_1, x_2)$ does not depend on the absorbed quantum frequency simplifies the equation for the quantum emergence probability which now will not depend on the absorbed quantum frequency x_1.

In the Mössbauer spectroscopy the scattering given the intensities of the radiation emitted from the medium through the surfaces $\tau = 0$ and $\tau = \tau_0$ depending on the source velocity v (in the units Γ/2).

$$I_\eta^-(\nu) = \frac{1}{2\pi\eta} \int_0^{2\pi} d\varphi \int_{-\infty}^{\infty} dx_2 \int_0^{\tau_0} p(\tau, x_2, \eta_0, \eta, \varphi) L(\tau, \nu) d\tau,$$

$$I_\eta^+(\nu) = \frac{1}{2\pi\eta} \int_0^{2\pi} d\varphi \int_{-\infty}^{\infty} dx_2 \int_0^{\tau_0} p(\tau_0 - \tau, x_2, \eta_0, \eta, \varphi) L(\tau, \nu) d\tau, \tag{8}$$

where $L(\tau, \nu)$ is the integral intensity of the γ-quanta absorbed at the optical depth τ,

$$L(\tau, \nu) = \int_{-\infty}^{\infty} \alpha_0(x_1 - \nu)\alpha(x_1) e^{-\alpha(x_1)\tau/\eta_0} dx_1, \tag{9}$$

where $\alpha_0(x)$ is the shape of the radiation line.

Therefore, to calculate the intensities $I^-(\nu)$ and $I^+(\nu)$ it is necessary to fined the probability $p(\tau, x_2, \eta_0, \eta, \varphi)$. On the other hand, this problem is of independent interest, particularly in the theory of the optical radiation transfer. In ([18]) taking account of the specific nature of the Mössbauer measurements and further considerations are similar to those in ([25,26])

In the ([18]) paper is also devoted to the study of the frequency-modulated γ-radiation scattering in a plane parallel layer.

. In this case also have to deal with another type of Mössbauer spectroscopy, namely with the modulation Mössbauer spectroscopy for scattering, which may suggest new possibilities for the investigation of physical and chemical properties of condensed media.

All the results given above are also valid for this case. The only difference is the incident radiation intensity. If the Mössbauer radiation in the source is frequency modulated by acoustic oscillations, then the lineshape of the incident radiation has the form ([26,27]).

$$\alpha_0(x - \nu) = \sum_{n=-\infty}^{\infty} \frac{I_n^2(\alpha)}{(x - \nu - n\Omega)^2 + 1}, \tag{10}$$

where $I_n(\alpha)$ is the Bessel function, $\alpha = A/\lambda$ the modulation index, A and Ω are amplitude and frequency of ultrasonic oscillations (in units $\Gamma/2$), and λ is the given γ-quantum wavelength.

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Համբարձումյանի ինվարիանտության սկզբունքի կիրառումը պինդ մարմնում ճառագայթման տեղափոխման խնդիրներում

Աշխատանքը հանդիսանում է Համբարձումյանի ինվարիանտության սկզբունքի կիրառմամբ, պինդ մարմնում ճառագայթման տեղափոխման խնդիրների ուղղությամբ ՀՀ ԳԱԱ ՖԿԳԻ-ում կատարված աշխատանքների համառոտ ակնարկ։

Академик НАН Армении **А. Р. МКРТЧЯН, Х. В. КОТАНДЖЯН**

Применение принципа инвариантности Амбарцумяна к задачам переноса излучения в твердых телах

Работа представляет собой краткий обзор исследований, выполняемых в ИППФ НАН РА по проблеме переноса излучения в твердом теле с применением принципа инвариантности Амбарцумяна.

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