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On Ambartsumian Equation and its Applications

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V. Ambartsumian's Principle of Invariance (PI) ([1-7]) honoured of wide application in linear problems of Mathematical Physics and Mathematics of Non-Linear functional equations- Ambartsumian Eq. (AE) and its generalizations. This paper is a breaf review of some results in this direction, obtained by representatives of Byurakan school of Mathematical Physics.

OPERATOR FORM OF AE IN RADIATIVE TRANSFER ([7]).

Let in space is given the cartesian system of cordinates (x,y,z) and $\mathbf{n}$ is the ort of axies OZ. Let $\Omega = \{\omega\}$ be an unite sphere of directions with measure $d\omega$ (body angle). Lets $\omega\omega'$ be the scalar product of $\omega, \omega' \in \Omega$. Let $\Omega^+ \subset \Omega$ is the half-space of positive directions of ω for which $\omega \cdot \mathbf{n} > 0$ and $\prod(a,b)$ is the plane layer, bounded by planes z=a and z=b, where $-\infty \le a \le b \le +\infty$.

Consider the linear stationary Radiative Transfer (RT) problem in the homogeneous medium $\prod_r = \prod(0,r), r \le +\infty$. We 'll assume that the intensity of radiation J into a medium depends on z, direction ω and on set w of other characteristics of radiation field (energy or frequency, polarization degree and etc.): $J = J(z,\omega,w)$. Let W={w} be a phase space with measure dw.

We denote $J^{\pm}(z,\omega,w) = J(z,\pm\omega,w)$, $\omega \in \Omega^+$. Let the vector – functions $J^{\pm}(z)$ are follows:

$$J^{\pm}(z)(\omega,w) = J \pm (z,\omega,w).$$

The Integral – Differential RT equation for wide class of Linear RT problems in $\prod_r$, in vector – operator notations, permits the following form:

$$\pm\frac{dJ^{\pm}(z)}{dz} = -A(z)J^{\pm}(z) + L^{+}(z)J^{\pm}(z) + L^{-}(z)J^{\pm}(z) \tag{1}$$

Here A(z) is the operator of multiplication on function $a(z,w)(\omega\mathbf{n})^{-1}$, where $a(z,w)>0$ is the volume coefficients of absorption, $L^{\pm}$ are the integral operators, describing the redistribution of radiation by direction, energy and etc. at elementary act of scattering:

$$L^{\pm}f(\omega,w)=\int_{\Omega xw}L^{\pm}(\omega,w,\omega',w')f(\omega',w')d\omega'dw'$$

The condition of absence of radiation fission is the following:

$$\iint\left[L^{+}(\omega,w,\omega',w')+L^{-}(\omega,w,\omega',w')\right]'dwd\omega=\lambda a(w)(\omega n)^{-1},\ \ \lambda\le 1 \qquad (2)$$

The functions $L^{\pm}$ possesse the condition of symmetry:

$$(\omega\mathbf{n})L^{\pm}(\omega,w,\omega',w')=(\omega'\mathbf{n})L^{\pm}(\omega',w'\omega,w) \qquad (3)$$

The following boundary conditions are joined to Eq.(1)

$$J^{+}(0)=J_0,\ \ J^{-}(r)=J_0 \qquad (4)$$

At $r=+\infty$ (the case of homogeneous half-space) the boundary conditions for (1) are

$$J^{+}(0)=J_0,\ \ J^{-}(z)=o\left(e^{Az}\right),\ z\to+\infty \qquad (5)$$

In particualar case of coherent isotropic scattering the operators A and $L^{\pm}$ are (where $\eta=\omega n$)

$$(Af)(\eta)=\frac{1}{\eta}f(\eta),\qquad (L^{\pm}f)(\eta)=\frac{\lambda}{2\eta}\int_0^1 f(\zeta)d\zeta,\ \ 0\le\lambda\le 1 \qquad (6)$$

PI (in operator form) for the problem (1),(5) means that exists integral operator $\rho=\text{const}$, such that

$$J^{-}(z)=\rho J^{+}(z),\ \ z\ge 0. \qquad (7)$$

From (1) and (7) AE for ρ (in general operator form) is obtained

$$A\rho+\rho A=L^{-}+\rho L^{+}+L^{+}\rho+\rho L^{-}\rho \qquad (8)$$

If ρ is known, then J^{+} is determined by Cauchy problem

$$\frac{dJ^{+}(z)}{dz}=-GJ^{+},\ \ J^{+}(0)=J_0,\qquad G=A-L^{+}-L^{-}\rho \qquad (9)$$

J^{-} is determined by (7).

Solution of problem (9) has the form

$$J^{+}(z)=X(z)J_0 \qquad (10)$$

where $X(z)=e^{-Gz}$ is the semigroup of operators determining by Cauchy problem

$$\frac{dX}{dz}=-GX,\ \ X(0)=I \qquad (11)$$

I- is the identity operator.
From (8) one can pass to the functional form of AE in regard to kernel ρ of integral operator ρ. For problem (6), (8) we obtain

$$(\frac{1}{\eta}+\frac{1}{\zeta})\rho(\eta,\zeta)=\frac{\lambda}{2\eta}[1+\int_0^1\rho(\eta',\zeta)d\eta'][1+\eta\int_0^1\rho(\eta,\eta')\frac{d\eta'}{\eta'}] \qquad (12)$$

Denote

$$\varphi(\zeta)=1+\int_0^1\rho(\eta',\zeta)d\eta' \qquad (13)$$

From (12) one can obtain the Ambartsumian φ - Equation

$$\varphi(\eta)=1+\frac{\lambda}{2}\eta\varphi(\eta)\int_0^1\frac{\varphi(\zeta)d\zeta}{\eta+\zeta} \qquad (14)$$

The Eq. (12) and (14) were obtained in ([1,2]) by means of application of PI to the problem of diffuse reflection from homogeneous half space. The functional form of AE for case of coherent anisotropic scattering was obtained in ([2,3]).

The physical solution of AE

The solution of AE (14) is nonunique. In ([8]) it was shown, that it is possible to construct "physical solution" (PS) of (14). Let consider the following iterations for (8):

$$A\rho_{m+1}+\rho_{m+1}A=L^{-}+\rho_m L^{+}+L^{+}\rho_m+\rho_m L^{-}\rho_m, \qquad \rho_0=0, \quad m=0,1,2,..... \qquad (15)$$

Since A is an operator of multiplication on positive function, then from (15) are subsequently determined the kernels ρ_m of integral operators ρ_m. The sequence ρ_m strongly converges in corresponding functional spaces, to the integral operator ρ. This limit ρ is called the Basic solution (BS) of AE (8). Its kernel ρ possesses the properties

i. ρ is the minimal positive solution of AE .

ii. ρ satisfies to inequality

$$\int_{\Omega x W}\rho(\omega,w,\omega',w')d\omega dw\le\lambda\le1 \qquad (16)$$

which is in accordance with physical meaning of ρ.

iii. ρ satisfies to the principle of mutuality

$$(\omega n)\rho(\omega,w,\omega',w')=(\omega' n)\rho(\omega',w',\omega,w) \qquad (17)$$

iv. ρ - is the strong limit at $r\to\infty$ of reflection operator $R(r)$ from homogeneous layer Π_r.

v. ρ - is the limit $(\text{at } t \to +\infty)$ of the operator of reflection for corresponding nonstationary problem

vi. By means of ρ the solution of problem (1), (4), or (1), (5) is constructed.

ρ is the unique solution of AE (8), possessing all above enumerated properties and therefore is the physical solution of (8).

Miln's problem of anisotropic scattering

One of classical problems of RT is the conservative Miln's problem in anisotropic coherent scattering.

This problem is the following: to construct a positive solution of conservative RT equation in homogeneous half-space at absence initial sources of energy in finite part of medium.

The Integral-Differential RT Equation has a form.

$$\frac{\partial J(\tau,\omega)}{\partial\tau} + J(\tau,\omega) = \int_{\Omega} g(\omega,\omega')J(\tau,\omega')d\omega' \tag{18}$$

with boundary conditions

$$J^{+}(0,\omega) = 0;\; J^{-}(\tau,\omega) = o\left(\exp\left\{\tau(\omega n)^{-1}\right\}\right) \text{ at } \tau \to +\infty \tag{19}$$

Here g - is the indicatrix of scattering:

$$g \geq 0;\quad \int_{-1}^{1} g(\mu)d\mu = 1$$

In work ([9]) the theory of Eq. (18) and Miln's problem is developed, assuming that the function g is integrable with quadrat $\left(g \in L_2(-1;1)\right)$. This assumption allows to apply the methods of Hilbert spaces. In ([10,11]) for the first time Miln's problem in natarally assumption ($g \in L_1(-1,1)$ was studied and solved.

The solution of problem it became possible in virtue of application of AE and a number of mathematical constructions.It was shown, that together with properties (16), (17), for reflection function ρ the equality takes place

$$\int_{\Omega} \rho(\omega,\omega')d\omega = 1$$

It is essential for construction of solution of Miln's problem.

The formal procedure of construction of solution of Miln's problem it was early partially known, and consist from the following stages.

i. The function ρ is constructed.

ii. The function f is determined by formulae

$$f(\omega) = C\left[\omega n + \int_{\Omega} \rho(\omega,\omega')(\omega' n)d\omega'\right];\; C = \text{const} \tag{20}$$

iii. J^{+} - is determined from Cauchy problem.

$$\frac{dJ^{+}}{d\tau} = L^{-}f + \left[-A + L^{+} + L^{-}\rho\right]J^{+};\; J^{+}(0) = 0 \tag{21}$$

iv. J^- - is determined by formulae

$$J^- = f + \rho J^+$$

Solvability, asymptotic behaviour and other properties of solution are established in [10].

On method of Spheric Harmonics

One of general methods of solution anisotropic scattering is the method of spheric harmonic, ([3,5,6]), which is based on replacement of indicatrix g by finite linear combinatoon of Legendre polinomials P_k

$$g(\mu) \approx g_m(\mu) = \sum_{k=0}^{n} C_k P_k(\mu)$$

Then Eq.(18) (and more general equations of anisotropic scattering) on separate equaitions in regard to asimuthal harmonic are reduced.

In the capacity of g_m it is usualy taken the partial sum of expansion of g in series of Fourier by Legendre polinomials. It provides best root square proximity g_m to g for fixed m. It turn out to be, hewever, that proximity between solutions of initial and reduced equations is provided by proximity g_m to g in metric L_1, i.e. in sense

$$\delta_n = \int_{-1}^{1} \left| g(\mu) - g_m(\mu) \right| d\mu \quad \text{(for fixed n)}$$

It is also desirable provision of inequality $0 \le g_m \le g$.

Such approximation problem it is still possible to solve by numerical methods.

The Layer of finite thickness

It's well known that RT problems in layer of finite thickness are essentially complicated comparing with RT in homogeneous half-space. In ([4,5]) basing on PI was suggested method of singular Eq. for layer of finite thickness (φ and ψ functions of Ambartsumian or X,Y functions of Chandreseckhar). However this method was not obtain wide applications.

Riccati equation method, reducing the boundary-value Transfer problem to Cauchy problems doesn't take into account specification of homogeneity of medium.

In ([7]) the new variant of PI was developed. By this way for the first time the solution method for RT problem in homogeneous layer Π_r, $r < +\infty$ by means of Ambartsumian function ρ for Π_∞ was found.

Let $R = R(r)$ and $T = T(r)$ are operators of reflection and transmission for Π_r. Let $U = \rho \mathbf{X}(r)$, where the semigroup $\mathbf{X}(r)$ is determined according to (11). The following relations take place.:

$$\rho = R(r) + TU(r), \qquad \mathbf{X}(r) = \mathbf{T}(r) + R(r)U(r) \tag{22}$$

The linear system (22) in regard to R and T is easy solved. Then the estimations $\|\rho\| \le \lambda$ and $\|X(r)\| \le 1$ following from (16), the important role are played.

The mathematical substantiation and development of this method are contained in ([11,12,13]).

The Inverse problems

The operator approach and other analytical constructions, describing on previous sections, were allowed to solve a number of Inverse problems on determination of local act of interaction of radiation with medium by known reflection and transmission properties of all layer ([15,14,13]). Such problems are of interest in Atmosphere optics, Neutron physics, Messbauer spectroscopy, and etc. Below we'll consider only two Inverse problems.

Inverse problem 1. (Division of layer into half).

Let's the operators R(r) and T(r) for homogeneous layer Π_r are given. It is required to construct $R\left(\frac{r}{2}\right)$ and $T\left(\frac{r}{2}\right)$. From (22) we have

$$U(r) = [R(r) + T(r)U(r)][T(r) + R(r)]U(r) \tag{23}$$

The Eq. (23) is an nonlinear Eq. in regard to W(r). It's physical solution is the limit of simple iterations W_n, $W_0 = 0$. It follows from $\|R + T\| \leq 1$ (no Fission). We have $\|U\| \leq 1$.

Solution of problem 1 consists of following stages.

i. The physical solution of (8) is constructed.
ii. By means of formulae (22) the operators ρ and $X(r)$ are determined
iii. The positive quadric root from $X(r)$ is extracted $X\left(\frac{r}{2}\right) = [X(r)]^{1/2}$. Then $X\left(\frac{r}{2}\right) = \rho X\left(\frac{r}{2}\right)$ is determined.
iv. The linear system in ragerd to $R\left(\frac{r}{2}\right)$ and $T\left(\frac{r}{2}\right)$ is solved. This system is obtained from (22) by means of replacement r by $\frac{r}{2}$.

Inverse problem 2. By means of known operators $R(r)$ and $T(r)$ how to determine the operators A and $L^{\pm}$.

The solution of this problem is based on relations

$$A - L^{+} = \lim_{\tau \to 0} \frac{1}{\tau}[I - T(\tau)], \; L^{-} = \lim_{\tau \to 0} \frac{1}{\tau} R(\tau). \tag{24}$$

By using the approach, described above, one can construct the operators $X(2^{-k} r)$ $k \geq 1$, $R(2^{-k} r)$ and $T(2^{-k} r)$. If we take k enough greater then from (24) may be approximately determined $A - L^{+}$ and L^{-}.

The operators A and $L^{\pm}$ may be immideately determined in the following two cases.

i. The symmetric case $L^{+} = L^{-} = L$.

ii. The case, where the operator A is known. (for example $a(\omega, w) = 1$).

The case i. includes in particularly the problems of isotropic scattering and anisotropic coherent scattering where the indicatrix of scattering is a even function.

In general case the full solution of problem is based on the relation (2).

Wiener - Hopf Eguation

PI and Ambartsumian Eq. have important applications in theory and effective solution of various classes of Integral equations. They are in organic connection with volterrian factorization of Integral operators. More complete results in this direction are obtained in the case of Wiener-Hop Eq. (WHE (16-22)) and some more general convolution Eq.(22-24). Let consider WHE

$$f(x) = g(x) + \int_0^\infty K(x-t) f(t) dt \tag{25}$$

or in operator form

$$(\mathrm{I} - \mathbf{K})\mathrm{f} = \mathrm{g}, \quad (\mathbf{K}f)(x) = \int_0^\infty K(x-t) f(t) dt$$

We assume that the kernel K is an even function and represents in the form

$$K(x) = \int_a^b e^{-|x|s} G(s) ds \equiv \int_a^b e^{-|x|s} d\sigma(s), 0 \le a < b \le +\infty \tag{26}$$

where

$$\mathrm{G(s)} \ge 0; \ \sigma(\mathrm{s}) = \int_0^{\mathrm{s}} \mathrm{G(p)dp}; \ \lambda = 2\int_{\mathrm{a}}^{\mathrm{b}} \frac{\mathrm{G(s)ds}}{\mathrm{s}} \le 1$$

The considered Equation has an important applications in RT, Kinetic theory of Gases and etc.

Consider the factorization

$$\mathrm{I} - \mathbf{K} = (\mathrm{I} - \mathbf{V}_-)(\mathrm{I} - \mathbf{V}_+) \tag{27}$$

where $\mathrm{V}_\pm$ are volterrian operators to be found

$$(\mathrm{V}_+\mathrm{f})(\mathrm{x}) = \int_0^{\mathrm{x}} \mathrm{V(x-t)f(t)dt} \qquad (\mathrm{V}_-\mathrm{f})(\mathrm{x}) = \int_{\mathrm{x}}^\infty \mathrm{V(t-x)f(t)dt} \tag{28}$$

Let φ -is a solution of AE

$$\varphi(\mathrm{s}) = 1 + \varphi(\mathrm{s}) \int_{\mathrm{a}}^{\mathrm{b}} \frac{\varphi(\mathrm{p})\mathrm{G(p)dp}}{\mathrm{s+p}} \tag{29}$$

satisfying the condition

$$\mu \equiv \int_a^b \frac{1}{s} |\varphi(s)| G(s) ds \le +\infty$$

Then the factorization (27) holds, where

$$V(x)=\int_a^b e^{-xs}\varphi(s)G(s)ds.$$

The factorization interpretation of AE is very simple and fruitfull.
AE (29) possesses Basic solution (BS) φ, which is the limit of simple iterations. It possesses the following properties.

$$\varphi(s)\downarrow s;\ \varphi\in C[0,+\infty);\ \varphi(+\infty)=1;\ \varphi(0)=(1-\lambda)^{-1/2}.$$

In conservative case $\lambda=1$, we have $\varphi(0)=+\infty$. Let α_k be moments of function φ

$$\alpha_m = m!\int_a^b \frac{\varphi(s)G(s)ds}{s^{m+1}}ds\ (\le +\infty)$$

Then $\alpha_0=1;\ \alpha_1=\sqrt{\int_a^b \frac{G(s)ds}{s^3}}\ \le +\infty$

It $\alpha_1 < +\infty$, then

$$\varphi(s)=\frac{1}{\alpha_1 s}+\frac{\alpha_2}{2\alpha_1^2}+\varphi_1(s),$$

where $\varphi_1(0^+)=0$.

The factorization (27) reduced WHE (25) to the solution of two volterrian-type Eq.

$$F(x)=g(x)+\int_x^\infty V(t-x)F(t)dt,\ f(x)=F(x)+\int_0^x V(x-t)f(t)dt$$

The resolvent function Φ is determined by Renewal Eq.

$$\Phi(x)=V(x)+\int_0^x V(x-t)\Phi(t)dt$$

and allows the representation ([25])

$$\Phi(x)=\int_\alpha^\beta e^{-xp}d\rho(p);\quad \rho(p)\uparrow p$$

The method of approximate solution of Eq. (25), (26) based on constructions, described above, will be discussed bellow.

The AE for non-symantic kernel Kand system of WHE were studied in ([16-21]).

Using Ambartsumyan function in works ([26-28]) the effective solution method on convolution type integral equation of finite interval with kernel (26) is suggested.

Convolution-type equation with direct and inverse shifts.

Consider the following Equation

$$f(x)=g(x)+\int_0^\infty K(x-t)f(t)dt+\int_0^\infty K_o(x+t)f(t)dt \tag{30}$$

where k is given by (26) and

$$K_o(x)=\int_{a_0}^{b_0} e^{-xs}G_0(s)ds,\; g(x)=\int_{a_1}^{b_1} e^{-xs}G_1(s)ds.$$

Such equation arises in RT and in Kinetic Theory of Gases, (KTG), when we take into account reflection of rediation (or gas) from boundary medium. In virtue of factorization method of ([23]) and factorization approach of AE, it became possible to receive exact analytical results on Eq. (30).

Rewrite (30) in operator form $(\mathbf{I}-\mathbf{K}-\mathbf{K}_o)f=g$.

Let φ -be the Basic Solution of (29). Then the following factorization holds.

$$\mathbf{I}-\mathbf{K}-\mathbf{K}_o=(\mathbf{I}-\mathbf{V}_-)(\mathbf{I}-\mathbf{T})(\mathbf{I}-\mathbf{V}_+) \tag{31}$$

Here $\mathbf{V}_\pm$ are given by (28) and **T** is the following operator

$$(\mathbf{T}f)(x)=\int_{a_0}^{b_0} T(x+t)f(t)dt,\; T(x)=\int_{a_0}^{b_0} e^{-xs}\varphi^2(s)G_0(s)ds$$

As application of factorization (31) let's consider an important problem on construction of possitive solution S of conservative $(\lambda=1)$ homogeneous equation (30). In particulary, when $K_o=\varepsilon K$, at $0\le\varepsilon\le1$, the factorization (31) allows to reduce this problem to the solution of three coupled equations:

$$(\mathbf{I}-\mathbf{V}_-)F=0,\; (\mathbf{I}-\mathbf{T})H=F,\; (\mathbf{I}-\mathbf{V}_+)S=H \tag{32}$$

The first equation possesses non-trivial solution $F(x)=C=\text{const}$. The second Eq. (where $F(x)=C$) possesses unique bounded solution $H(x)=ch(x)$. The solution S of last Eq. (32) possesses asymptotic $S(x)\sim ax$ at $x\to+\infty$.
The following formulae is of interest in KTG

$$S(x)-ax\sim\frac{a\alpha_2}{\alpha_1}+\frac{1}{\alpha_1}\int_0^\infty H(x)dx$$

On an approximate solution of the problems (25) and (30).

The described approach and some other numerical method are suggested in works ([29-31]). The principal methods of approximate analytical and numerical solution of Eq. (25) and (30) are based on the modification of method of discrete ordinates (MDO) of Chandrasekhar ([5]). The application of MDO is equivalent to the replacement of kernel K in (26) by finite linear combination of exponentials.

$$K(x) \approx T(x) = \sum c_m e^{-|x|s_m} = \int_a^b e^{-|x|s} d\tilde{\sigma}(s)$$

where $a < s_1, < s_2 < \cdots < s_n < b \leq +\infty$, and $\tilde{\sigma}$ is the step by step constant function. The proximity of solutions of Eq. (25) and reducing Equation (with kernel T) are estimated by means of quantity (see ([29]))

$$\delta = \|K - T\|_{L_1(-\infty,+\infty)}$$

In conservative case it is essentially to provide carring out of condition $T \leq K$.

The problem of optimal selection of nodes $\{s_k\}$ and weight factors $\{c_m\}$, which are minimize the quantity δ for fixed n, is suggested in ([29]).

In virtue of MDO, AE is reduced to the finite nonlinear algebraic systems. It may be solved by iteration.

$$\varphi_i = 1 + \varphi_i \sum_{k=1}^{n} \frac{\varphi_k c_k}{s_i + s_k}; i = 1,2,\cdots n, \varphi_i = \varphi(s_i)$$

The function Φ has a form

$$\Phi(x) \approx \tilde{\Phi}(x) = \sum q_m e^{-p_m x}$$

where p_m - are determined from the characteristic equation $\sum_{k=1}^{n} \frac{c_k \varphi_k}{s_k - p} = 1$. They are arrange according to $0 < p_1 < s_1 < p_2 < s_2 < \cdots < p_n < s_n$. The numbers $q_m > 0$ are determined from linear algebraic system with Cauchy matrix:

$$\sum_{k=1}^{n} \frac{q_k}{s_j - p_k} = 1, \quad j = 1,2,\cdots n$$

Let's consider the equation (25), in case, when $g(x) = \int_{a_0}^{b_0} e^{-xs} G_o(s) \approx \sum_{m=1}^{n} b_m e^{-xs_m}$.

Then the approximate solution of initial equation (25) is expressed by numbers (φ_k), (p_k), (q_k), (b_m)

$$f(x) = \sum_{m=1}^{n} b_m \varphi_m e^{-xs_m} + \sum_{k=1}^{n} \sum_{m=1}^{n} b_m q_k \varphi_m \frac{e^{-xp_k} e^{-xs_m}}{s_m - p_k}.$$

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Համբարձումյանի հավասարումը և նրա կիրառությունները

Աշխատանքը հանդիսանում է Վ.Համբարձումյանի հավասարման (ՀՀ), նրա ընդհանրացումների և կիրառությունների ուղղությամբ Մաթեմատիկական Ֆիզիկայի Բյուրականյան դպրոցի ներկայացուցիչների կատարած որոշ աշխատանքների ակնարկ: Բերված են ՀՀ-ի օպերատորային և սկալյար տեսքերը, նրա ֆիզիկական լուծման կառուցման անալիտիկ և թվային մեթոդներ: Շարադրված են համասեռ կիսատարածությունում և վերջավոր հաստության շերտում ճառագայթման տեղափոխման և Գազերի կինետիկ տեսության որոշ ուղիղ և համադարձ խնդիրների լուծման մեթոդներ, որոնք հիմնված են Համբարձումյանի հավասարման կիրառման վրա:

Н.Б. Енгибарян, А.Х. Хачатрян, М.Г. Мурадян

Уравнение Амбарцумяна и его применения

Работа является обзором некоторых работ представителей Бюраканской школы Математической физики по уравнвнию В. Амбарцумяна (УА), его обобщениям и приложениям. Приведены операторная и скалярная форма УА, аналитические и численные методы построения Физичекого решения УА. Изложены методы решения ряда прямых и обратных задач Теории переноса излучения и Кинетической теории газов в однородном полупространстве и в плоском слое конечной толщины, с применением УА.

REFERENCES

[1] V.A.Ambartsumian Dokl. Acad. Nauk SSSR 1943. Vol.38 No 8, p.257. [2] V.A.Ambartsumian Scientific works, Vol. 1, Erevan 1960. [3] V.A.Ambartsumian J. Experim. Theor. Fiziki 1943. Vol. 224. No 9-10,p.224. [4] V.A.Ambartsumian A Life in Astrophysics, Selected Papers , Allerton Press Inc. [5] S.Chandrasechar Radiative Transfer Oxford 1950. [6] V.V.Sobolev Radiative Transfer Gostechizd. Moscow 1956. [7] N.B.Yengibarian, M.A.Mnatsakanian Dokl. Acad. Nauk SSSR 1974. Vol.217 No 3, pp. 533-53(Russian). [8] N.B.Yengibarian, E.A.Melkonian Principle of Invariance and its Applications Yerevan 1989, pp.326-333(Russian). [9] M.V.Maslennikov Trudi Steklov Math. Inst. Acad.Nauk SSSR 1961 Vol.61 (Russian). [10] N.B.Yengibarian, M.M.Makarian Math. Annal. And Applications, Yerevan 1980, pp. 72-80 (Russian). [11] M.M.Makarian Dokl. Acad. Nauk SSSR. 1986. Vol. 291 No 1. pp. 72-75 (Russian). [12] M.G.Mouradian J. Quant. Spectrosc. Radiat. Transfer 1991 Vol. 46 No 5 , pp. 477-481. [13] M.G Mouradian J. Vichisl.Math. Math.Phizik. 1993 Vol. 33 No 2 , pp.271-282 (Russian). [14] N.B.Yengibarian, M.G.Mouradian Dokl. Akad. Nauk Arm. SSR 1988. Vol. 86.No 3,pp. 122-126(Russian). [15] N.B.Yengibarian, M.G.Mouradian, R.S.Vardanian Trudi XXII Chtenii po Kosmonavtike ,Moscow 1989, pp. 14-20 (Russian). [16] N.B.Yengibarian Dokl. Akad. Nauk Arm.SSR 1972 . Vol.54 No 3, pp. 129-133 (Russian). [17] N.B.Yengibarian, A.A.Arutiunian Math. USSR Sbornik, 1975. Vol. 26 No 1, pp. 31-54. [18] N.B.Yengibarian Izvestiya Acad. Nauk Arm. SSR, Mathem. 1980. Vol. 15 No 3, pp.233-244 (Russian). [19] L.G.Arabajian, N.B.Yengibarian Math. Annal. Itogi Nauki i Techniki . 1984. Vol.22 , pp. 175-244 (Russian). [20] N.B.Yengibarian, L.G.Arabajian Math. USSR Sbornik 1985. Vol. 52. No 1, pp. 181-208. [21] L.G.Arabajian, N.B.Yengibarian Izvestiya Acad. Nauk Arm.SSR, Mathem. 1990. Vol . 25. No 2, pp.135-155. [22] N.B.Yengibarian, A.Kh. Khachatrian J. Vichisl. Mathem. Math.Fiziki 1998. Vol. 38 No 3, pp. 408-424 (Russian). [23] N.B.Yengibarian,L.G.Arabajian Diff. Uravnenia 1990. Vol. 26. No 8, pp. 1441- 1453 (Russian). [24] N.B.Yengibarian, A.Kh. Khachatrian J. Quant. Spectrosc. Radiat. Transfer 1991. Vol. 46. No. 6, pp. 565-575 (Russian). [25] N.B.Yengibarian, A.A.Poghssian Math. Zametki 1990. Vol. 47. No 6, pp. 23-30 (Russian). [26] N. B. Yengibarian, M. A. Mnatsakanian Math. Zametki 1976. Vol. 19. No 6, pp. 927-932 (Russian). [27] R. S. Vardanian, N. B. Yengibarian J. Vichisl. Mathem. Math. Phys. 1989. Vol. 29 No 9, pp. 1291-1300 (Russian). [28] A. Kh. Khacatrian, K. V. Papoyan, Co Journ. Contemprorary Math. Annal. (Armania) 1997. Vol.1. [29] N. B. Yengiabarian, E. A. Melkonian, Dokl. Akadem. Nauk SSSR 1987. Vol. 292 No 2, pp. 322-326. [30] A. Kh. Khachatrian, A. A. Akopian, E. A. Melkonian, J. Quant. Spectrosc. Radiat. Transfer 1991. Vol. 45. No 6, pp. 367-376. [31] A. Kh. Khachatrian Astr. And Space Science 1994. Vol. 215, pp. 17-35.