ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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Solution of Special Mixed Dynamic Problems of Anisotropic Plates

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Keywords: anisotropy, dynamics, 3D mixed boundary problems, resonance, asymptotic solution

The spatial mixed dynamic problem for anisotropic plates is solved. It is assumed, that the plate has a plane of elastic symmetry, the facial surface is imparted normal displacements that change harmonically over time, and the shear stresses there are equal to zero. The lower facial surface of the plate is rigidly fixed. For this class of problems, the hypotheses of the classical and well-known refined theories of plates (Reissner E., Ambartsumyan S., Timoshenko's type aren't applicable. The asymptotic solution to the problem is obtained. It is shown that longitudinal oscillations in the vertical direction are dominant, which also generate tangential oscillations, the amplitudes of which, however, are an order of magnitude smaller than the longitudinal ones. The conditions for the occurrence of resonance were established and the values of resonant frequencies were determined. If the displacements subjected to the facial surface depend polynomially on the tangential coordinates, the solution becomes mathematically exact. The illustrative example is given.

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К решению пространственных смешанных динамических задач анизотропных пластин.

Ключевые слова: анизотропия, динамика, 3D смешанные краевые задачи, резонанс, асимптотическое решение

Решена пространственная смешанная динамическая задача для анизотропных пластин. Считается, что пластина имеет плоскость упругой симметрии, лицевой поверхности сообщены нормальные перемещения гармонически изменяющиеся во времени, а касательные напряжения там равны нулю. Нижняя лицевая поверхность пластины жёстко закреплена. Для этого класса задач гипотезы классической и известных уточненных теорий пластин (Рейснер Е., Амбарцумян С., типа Тимошенко) неприменимы. Получено асимптотическое решение задачи. Показано, что доминирующими являются продольные колебания в вертикальном направлении, которые порождают также тангенциальные колебания, амплитуда которых, однако, на порядок меньше продольных. Установлены условия возникновения резонанса и определены значения резонансных частот. Если сообщаемые лицевой поверхности перемещения полиномиально зависят от тангенциальных координат, решение становится математически точным. Приведён иллюстрационный пример.

Աղալովյան Լ.Ա., Աղալովյան Մ.Լ., Զաքարյան Տ.Վ., Թովմասյան Ա.Բ.

Անիզոտրոպ սալերի տարածական խառը դինամիկական խնդիրների լուծման մասին

Հիմնաբառեր՝ անիզոտրոպություն, դինամիկա, 3D խառը եզրային խնդիրներ, ռեզոնանս, ասիմպտոտիկ լուծում

Անիզոտրոպ սալերի համար լուծված է տարածական խառը դինամիկական խնդիրը: Ենթադրվում է, որ սալն ունի առաձգականության սիմետրիայի հարթություն, դիմային մակերևույթին հաղորդված է նորմալ տեղափոխություն՝ ըստ ժամանակի հարմոնիկ փոփոխվող, շոշափող լարումներն այնտեղ զրո են: Սալի ստորին մակերևույթը կոշտ ամրակցված է: Այս դասի խնդիրների լուծման համար սալերի դասական և ճշգրտված հայտնի տեսությունները (Ռեյսներ, Համբարձումյան, Տիմոշենկոյի տիպի) կիրառելի չեն: Ստացված է խնդրի ասիմպտոտիկ լուծումը: Ցույց է տրված, որ գլխավոր են ուղղաձիգի ուղղությամբ երկայնական տատանումները, առաջանում են նաև սահքային տատանումներ, սակայն նրանց ամպլիտուդը կարգով փոքր է: Արտածված են ռեզոնանսի առաջացման պայմանները, որոշված են ռեզոնանսային հաճախությունները: Բերված է իլյուստրացիոն օրինակ՝ երբ ասիմպտոտիկ լուծումը դառնում է մաթեմատիկորեն ճշգրիտ:

Introduction

Most of crystals and modern composite materials are anisotropic, part of them have a plane of elastic symmetry. For example, such are, crystals of the monoclinic system [1,2]. There are relatively few works on determining the stress-strain states of plates and shells, having the plane of elastic symmetry, even according to the classical theory of plates and shells. The classical theory of plates and shells, as well as the well-known refined theories of Reissner [3,4], Ambartsumyan [5], Timoshenko's type [6], consider only one class of problems: it is assumed that on the facial surfaces of plates and shells the conditions of the first boundary value problem of elasticity theory are given, i.e. the values of the corresponding components of the stress tensor. These theories aren't applicable when on the facial surfaces are specified the conditions of the second (displacements) or mixed boundary value problems. For example, the conditions of the second boundary value problem can't be satisfied, since in classical theory it is assumed that the normal displacement (W) doesn't depend on the transverse coordinate (z). By classical and refined theories a variety of static and dynamic problems for isotropic, orthotropic and layered plates and shells have been solved. In recent decades, many important results in the field of theory of plates and shells have been obtained on the base of the asymptotic theory of plates and shells [7-13]. The asymptotic method turned out to be particularly effective for solving the second and mixed, both static and dynamic, boundary value problems of plates and shells. The fundamentally new asymptotics, compared to the classical one, was established, asymptotics, which made it possible to effectively solve these problems [14-20]. In this paper forced vibrations of plates, having the plane of elastic symmetry, when to the facial surface were subjected normal displacements that changed harmonically over time, and the lower facial surface was rigidly fixed were studied.

1.The statement of the problem: basic equations and relationships.

It is required to find in the area $D=\{(x,y,z):0\le x\le a,\ 0\le y\le b,-h\le z\le h,$ $2h<<l,\ l=\min(a,b)\}$, which is occupied by a plate (Fig. 1) solutions to the equations of motion of the three-dimensional problem of elasticity theory:

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}+\frac{\partial\sigma_{xz}}{\partial z}=\rho\frac{\partial^2 u}{\partial t^2},\quad \frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{yz}}{\partial z}=\rho\frac{\partial^2 v}{\partial t^2},\tag{1}$$

$$\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}=\rho\frac{\partial^2 w}{\partial t^2},$$

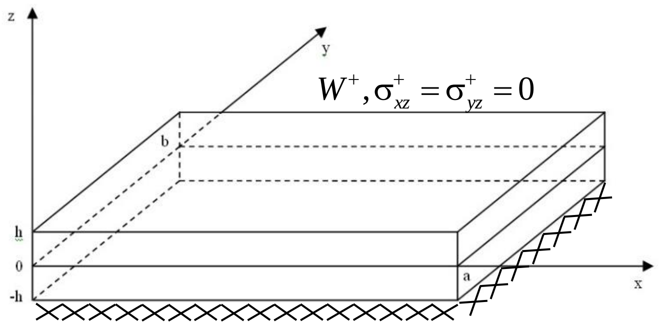

Fig.1. The structure of plate

and the relations of an anisotropic body, which have a plane of elastic symmetry [1,2]:

$$\begin{aligned}
&\frac{\partial u}{\partial x} = a_{11}\sigma_{xx} + a_{12}\sigma_{yy} + a_{13}\sigma_{zz} + a_{16}\sigma_{xy} \\
&\frac{\partial v}{\partial y} = a_{12}\sigma_{xx} + a_{22}\sigma_{yy} + a_{23}\sigma_{zz} + a_{26}\sigma_{xy} \\
&\frac{\partial w}{\partial z} = a_{13}\sigma_{xx} + a_{23}\sigma_{yy} + a_{33}\sigma_{zz} + a_{36}\sigma_{xy} \\
&\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = a_{44}\sigma_{yz} + a_{45}\sigma_{xz} \\
&\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = a_{45}\sigma_{yz} + a_{55}\sigma_{xz} \\
&\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = a_{16}\sigma_{xx} + a_{26}\sigma_{yy} + a_{36}\sigma_{zz} + a_{66}\sigma_{xy}
\end{aligned} \qquad (2)$$

at the following boundary conditions at $z = h$

$$W(x, y, h, t) = -W^{+}(\xi, \eta)\exp(i\Omega t), \qquad \xi = \frac{x}{l}, \;\; \eta = \frac{y}{l},$$

$$\sigma_{xz}(x, y, h, t) = 0, \qquad \sigma_{yz}(x, y, h, t) = 0, \qquad (3)$$

and at $z = -h$

$$u(x, y, -h, t) = 0, \; v(x, y, -h, t) = 0, \;\; w(x, y, -h, t) = 0 \qquad (4)$$

where Ω – the frequency of forced action.
The conditions on the lateral surfaces of the plate we will not specify for now; by them is caused the appearance of boundary layer [13].

2.The asymptotic solution of the problem.

The solution of the formulated problem will be sought in the form

$$\sigma_{\alpha\beta}(x,y,z,t)=\sigma_{ij}(\xi,\eta,\zeta)\exp(i\Omega t),\quad \alpha,\beta=x,y,z,\quad i,j=1,2,3,$$
$$u(x,y,z,t)=u_x(\xi,\eta,\zeta)\exp(i\Omega t),\quad (u,v,w;u_x,u_y,u_z) \tag{5}$$

By substituting (5) in equations (1) and elasticity relations (2) and in the newly obtained system, moving to dimensionless coordinates and displacements

$$\xi=\frac{x}{l},\ \eta=\frac{y}{l},\ \zeta=\frac{z}{h},\ U=\frac{u_x}{l},\ V=\frac{u_y}{l},\ W=\frac{u_z}{l}, \tag{6}$$

as a result we will obtain the system singularly perturbed by the small parameter $\varepsilon=\frac{h}{l}$:

$$\begin{aligned}
&\frac{\partial\sigma_{11}}{\partial\xi}+\frac{\partial\sigma_{12}}{\partial\eta}+\varepsilon^{-1}\frac{\partial\sigma_{13}}{\partial\zeta}+\varepsilon^{-2}\Omega_*^2U=0,\\
&\frac{\partial\sigma_{12}}{\partial\xi}+\frac{\partial\sigma_{22}}{\partial\eta}+\varepsilon^{-1}\frac{\partial\sigma_{23}}{\partial\zeta}+\varepsilon^{-2}\Omega_*^2V=0,\\
&\frac{\partial\sigma_{13}}{\partial\xi}+\frac{\partial\sigma_{23}}{\partial\eta}+\varepsilon^{-1}\frac{\partial\sigma_{33}}{\partial\zeta}+\varepsilon^{-2}\Omega_*^2W=0,\\
&\frac{\partial U}{\partial\xi}=a_{11}\sigma_{11}+a_{12}\sigma_{22}+a_{13}\sigma_{33}+a_{16}\sigma_{12},\\
&\frac{\partial V}{\partial\eta}=a_{12}\sigma_{11}+a_{22}\sigma_{22}+a_{23}\sigma_{33}+a_{26}\sigma_{12},\\
&\varepsilon^{-1}\frac{\partial W}{\partial\zeta}=a_{13}\sigma_{13}+a_{23}\sigma_{22}+a_{33}\sigma_{33}+a_{36}\sigma_{12},\\
&\frac{\partial W}{\partial\eta}+\varepsilon^{-1}\frac{\partial V}{\partial\zeta}=a_{44}\sigma_{23}+a_{45}\sigma_{13},\\
&\frac{\partial W}{\partial\xi}+\varepsilon^{-1}\frac{\partial U}{\partial\zeta}=a_{45}\sigma_{23}+a_{55}\sigma_{13},\\
&\frac{\partial V}{\partial\xi}+\frac{\partial U}{\partial\eta}=a_{16}\sigma_{11}+a_{26}\sigma_{22}+a_{36}\sigma_{33}+a_{66}\sigma_{12},\\
&\Omega_*^2=\rho h^2\Omega^2,
\end{aligned} \tag{7}$$

The solution of the singularly perturbed system (7) is the sum of the solutions of the external problem $(\mathrm{I}^{\mathrm{out}})$ and the boundary layer $(\mathrm{I_b})$ [13]: $\mathrm{I}=\mathrm{I}^{\mathrm{out}}+\mathrm{I_b}$. The solutions of the external problem $(\mathrm{I}^{\mathrm{out}})$ and the boundary layer $(\mathrm{I_b})$ can be constructed separately. The boundary layer solution $(\mathrm{I_b})$ localized near the lateral surface and all quantities decrease exponentially from the lateral surface into the inside of the plate [13]. To this also corresponds the phenomenon of the edge effect [21]. That's why, we don't consider it in this paper. The solution for the boundary layer and its conjugation with the solution of the external problem $\left(I^{out}\right)$ can be implemented in the way described in [13].

The solution to the external problem we will seek out in the form of

$$\sigma_{ij}^{out} = \varepsilon^{-1+s}\sigma_{ij}^{(s)}(\xi,\eta,\zeta),\ i,j=1,2,3,\ s=\overline{0,N}$$
$$\left(U^{out},V^{out},W^{out}\right)=\varepsilon^{s}\left(U^{(s)},V^{(s)},W^{(s)}\right), \tag{8}$$

where notation $s=\overline{0,N}$ means summation by repeating (umbral) index s from 0 to number of approximations N. From (8) it follows that the stresses must have the same intensity.
By substituting (8) into (7) and in each equation equating coefficients at the same powers ε, we will obtain the following consistent system for determining unknown functions $\sigma_{ij}^{(s)}, U^{(s)}, V^{(s)}, W^{(s)}$:

$$\frac{\partial\sigma_{11}^{(s-1)}}{\partial\xi}+\frac{\partial\sigma_{12}^{(s-1)}}{\partial\eta}+\frac{\partial\sigma_{13}^{(s)}}{\partial\zeta}+\Omega_*^2U^{(s)}=0,$$
$$\frac{\partial\sigma_{12}^{(s-1)}}{\partial\xi}+\frac{\partial\sigma_{22}^{(s-1)}}{\partial\eta}+\frac{\partial\sigma_{23}^{(s)}}{\partial\zeta}+\Omega_*^2V^{(s)}=0$$
$$\frac{\partial\sigma_{13}^{(s-1)}}{\partial\xi}+\frac{\partial\sigma_{23}^{(s-1)}}{\partial\eta}+\frac{\partial\sigma_{33}^{(s)}}{\partial\zeta}+\Omega_*^2W^{(s)}=0,$$

$$\frac{\partial U^{(s-1)}}{\partial\xi}=a_{11}\sigma_{11}^{(s)}+a_{12}\sigma_{22}^{(s)}+a_{13}\sigma_{33}^{(s)}+a_{16}\sigma_{12}^{(s)},$$
$$\frac{\partial V^{(s-1)}}{\partial\eta}=a_{12}\sigma_{11}^{(s)}+a_{22}\sigma_{22}^{(s)}+a_{23}\sigma_{33}^{(s)}+a_{26}\sigma_{12}^{(s)},$$
$$\frac{\partial W^{(s)}}{\partial\zeta}=a_{13}\sigma_{11}^{(s)}+a_{23}\sigma_{22}^{(s)}+a_{33}\sigma_{33}^{(s)}+a_{36}\sigma_{12}^{(s)}, \tag{9}$$
$$\frac{\partial W^{(s-1)}}{\partial\eta}+\frac{\partial V^{(s)}}{\partial\zeta}=a_{44}\sigma_{23}^{(s)}+a_{45}\sigma_{13}^{(s)},$$
$$\frac{\partial W^{(s-1)}}{\partial\xi}+\frac{\partial U^{(s)}}{\partial\zeta}=a_{45}\sigma_{23}^{(s)}+a_{55}\sigma_{13}^{(s)},$$
$$\frac{\partial V^{(s-1)}}{\partial\xi}+\frac{\partial U^{(s-1)}}{\partial\eta}=a_{16}\sigma_{11}^{(s)}+a_{26}\sigma_{22}^{(s)}+a_{36}\sigma_{33}^{(s)}+a_{66}\sigma_{12}^{(s)},$$

From the elasticity relations of system (9) all stresses can be expressed through displacements by formulas

$$\sigma_{13}^{(s)} = \frac{1}{\Delta_2}\left(a_{44}\frac{\partial U^{(s)}}{\partial \zeta} - a_{45}\frac{\partial V^{(s)}}{\partial \zeta} + \sigma_{13*}^{(s)} \right),$$

$$\sigma_{23}^{(s)} = \frac{1}{\Delta_2}\left(-a_{45}\frac{\partial U^{(s)}}{\partial \zeta} + a_{55}\frac{\partial V^{(s)}}{\partial \zeta} + \sigma_{23*}^{(s)} \right),$$

$$\sigma_{11}^{(s)} = \frac{1}{\Delta_1}\left(A_{31}\frac{\partial W^{(s)}}{\partial \zeta} + \sigma_{11*}^{(s)} \right), \qquad \sigma_{22}^{(s)} = \frac{1}{\Delta_1}\left(A_{32}\frac{\partial W^{(s)}}{\partial \zeta} + \sigma_{22*}^{(s)} \right) \tag{10}$$

$$\sigma_{33}^{(s)} = \frac{1}{\Delta_1}\left(A_{33}\frac{\partial W^{(s)}}{\partial \zeta} + \sigma_{33*}^{(s)} \right), \qquad \sigma_{12}^{(s)} = \frac{1}{\Delta_1}\left(A_{34}\frac{\partial W^{(s)}}{\partial \zeta} + \sigma_{12*}^{(s)} \right),$$

Where

$$\sigma_{13*}^{(s)} = a_{44}\frac{\partial W^{(s-1)}}{\partial \xi} - a_{45}\frac{\partial W^{(s-1)}}{\partial \eta}, \qquad \sigma_{23*}^{(s)} = -a_{45}\frac{\partial W^{(s-1)}}{\partial \xi} + a_{55}\frac{\partial W^{(s-1)}}{\partial \eta},$$

$$\sigma_{11*}^{(s)} = A_{11}\frac{\partial U^{(s-1)}}{\partial \xi} + A_{21}\frac{\partial V^{(s-1)}}{\partial \eta} + A_{41}\left(\frac{\partial U^{(s-1)}}{\partial \eta} + \frac{\partial V^{(s-1)}}{\partial \xi} \right),$$

$$\sigma_{22*}^{(s)} = A_{12}\frac{\partial U^{(s-1)}}{\partial \xi} + A_{22}\frac{\partial V^{(s-1)}}{\partial \eta} + A_{42}\left(\frac{\partial U^{(s-1)}}{\partial \eta} + \frac{\partial V^{(s-1)}}{\partial \xi} \right),$$

$$\sigma_{33*}^{(s)} = A_{13}\frac{\partial U^{(s-1)}}{\partial \xi} + A_{23}\frac{\partial V^{(s-1)}}{\partial \eta} + A_{43}\left(\frac{\partial U^{(s-1)}}{\partial \eta} + \frac{\partial V^{(s-1)}}{\partial \xi} \right), \tag{11}$$

$$\sigma_{12*}^{(s)} = A_{14}\frac{\partial U^{(s-1)}}{\partial \xi} + A_{24}\frac{\partial V^{(s-1)}}{\partial \eta} + A_{44}\left(\frac{\partial U^{(s-1)}}{\partial \eta} + \frac{\partial V^{(s-1)}}{\partial \xi} \right),$$

$$\Delta_1 = \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{16} \\ a_{12} & a_{22} & a_{23} & a_{26} \\ a_{13} & a_{23} & a_{33} & a_{36} \\ a_{16} & a_{26} & a_{36} & a_{66} \end{vmatrix}, \ \Delta_2 = a_{44}a_{55} - a_{45}^2,$$

By substituting $\sigma_{13}^{(s)}, \sigma_{23}^{(s)}$ into two first equations of system (9), for determining $U^{(s)}, V^{(s)}$ we will obtain the system

$$a_{44}\frac{\partial^2 U^{(s)}}{\partial \zeta^2} - a_{45}\frac{\partial^2 V^{(s)}}{\partial \zeta^2} + \Delta_2 \Omega_*^2 U^{(s)} = R_u^{(s)}(\xi, \eta, \zeta)$$

$$-a_{45}\frac{\partial^2 U^{(s)}}{\partial \zeta^2} + a_{55}\frac{\partial^2 V^{(s)}}{\partial \zeta^2} + \Delta_2 \Omega_*^2 V^{(s)} = R_v^{(s)}(\xi, \eta, \zeta) \tag{12}$$

$$R_u^{(s)} = -\Delta_2\left(\frac{\partial\sigma_{11}^{(s-1)}}{\partial\xi} + \frac{\partial\sigma_{12}^{(s-1)}}{\partial\eta}\right) - \frac{\partial\sigma_{13*}^{(s)}}{\partial\zeta},$$

$$R_v^{(s)} = -\Delta_2\left(\frac{\partial\sigma_{12}^{(s-1)}}{\partial\xi} + \frac{\partial\sigma_{22}^{(s-1)}}{\partial\eta}\right) - \frac{\partial\sigma_{23*}^{(s)}}{\partial\zeta},$$

By substituting the value of $\sigma_{33}^{(s)}$ into the third equation of system (9), for determination of $W^{(s)}$ we will obtain the equation

$$\frac{\partial^2 W^{(s)}}{\partial\zeta^2} + \frac{\Delta_1}{A_{33}}\Omega_*^2 W^{(s)} = R_w^{(s)}(\xi,\eta,\zeta \tag{13}$$

$$R_w^{(s)} = -\frac{1}{A_{33}}\left[\frac{\partial\sigma_{33*}^{(s)}}{\partial\zeta} + \Delta_1\left(\frac{\partial\sigma_{13}^{(s-1)}}{\partial\xi} + \frac{\partial\sigma_{23}^{(s-1)}}{\partial\eta}\right)\right],$$

From system (12) $V^{(s)}$ can be expressed through $U^{(s)}$ by formula

$$V^{(s)} = -\frac{1}{a_{45}\Omega_*^2}\frac{\partial^2 U^{(s)}}{\partial\zeta^2} - \frac{a_{55}}{a_{45}}U^{(s)} + R_v^{*(s)} \tag{14}$$

$$R_v^{*(s)} = \frac{1}{\Delta_2\Omega_*^2}\left(\frac{a_{55}}{a_{45}}R_u^{(s)} + R_v^{(s)}\right)$$

and for determining $U^{(s)}$ taking into acount (14) we will obtain the equation

$$\frac{\partial^4 U^{(s)}}{\partial\zeta^4} + \left(a_{44} + a_{55}\right)\Omega_*^2\frac{\partial^2 U^{(s)}}{\partial\zeta^2} + \Delta_2\Omega_*^4 U^{(s)} = \bar{R}_u^{(s)} \tag{15}$$

$$\bar{R}_u^{(s)} = \frac{1}{\Delta_2}\left(a_{55}\frac{\partial^2 R_u^{(s)}}{\partial\zeta^2} + \Delta_2\Omega_*^2 R_u^{(s)} + a_{45}\frac{\partial^2 R_v^{(s)}}{\partial\zeta^2}\right)$$

The solution to the equation (15) is $U^{(s)} = U_0^{(s)} + U_\tau^{(s)}(\xi,\eta,\zeta)$, where $U_\tau^{(s)}$ – is particular solution:

$$U_0^{(s)} = D_1^{(s)}\cos b_1\Omega_*\zeta + D_2^{(s)}\sin b_1\Omega_*\zeta + D_3^{(s)}\cos b_2\Omega_*\zeta + D_4^{(s)}\sin b_2\Omega_* \tag{16}$$

$$b_{1,2} = \sqrt{\frac{(a_{44} + a_{55}) \mp \sqrt{(a_{44} - a_{55})^2 + 4a_{45}^2}}{2}}$$

According to (14), (16) we have

$$V^{(s)} = D_1^{(s)}d_1\cos b_1\Omega_*\zeta + D_2^{(s)}d_1\sin b_1\Omega_*\zeta +$$
$$+D_3^{(s)}d_2\cos b_2\Omega_*\zeta + D_4^{(s)}d_2\sin b_2\Omega_*\zeta + V_\tau^{(s)} \tag{17}$$

$$d_1 = \frac{1}{a_{45}}\left(b_1^2 - a_{55}\right), \qquad d_2 = \frac{1}{a_{45}}\left(b_2^2 - a_{55}\right)$$

$$\mathrm{V}_\tau^{(s)} = -\frac{1}{a_{45}\Omega_*^2}\frac{\partial^2 U_\tau^{(s)}}{\partial\zeta^2} - \frac{a_{55}}{a_{45}}U_\tau^{(s)} + \frac{1}{\Delta_2\Omega_*^2}\left(\frac{a_{55}}{a_{45}}R_u^{(s)} + R_v^{(s)}\right),$$

According to (10), (16), (17)

$$\sigma_{13}^{(s)} = \frac{1}{\Delta_2}(D_1^{(s)}d_{11}\sin b_1\Omega_*\zeta - D_2^{(s)}d_{11}\cos b_1\Omega_*\zeta +$$

$$+D_3^{(s)}d_{12}\sin b_2\Omega_*\zeta - D_4^{(s)}d_{12}\cos b_2\Omega_*\zeta + \sigma_{13\tau}^{(s)}) \qquad (18)$$

$$\sigma_{23}^{(s)} = \frac{1}{\Delta_2}\Big(D_1^{(s)}d_{21}\sin b_1\Omega_*\zeta - D_2^{(s)}d_{21}\cos b_1\Omega_*\zeta +$$

$$+D_3^{(s)}d_{22}\sin b_2\Omega_*\zeta - D_4^{(s)}d_{22}\cos b_2\Omega_*\zeta + \sigma_{23\tau}^{(s)}\Big)$$

$$d_{11} = \Omega_* b_1\left(b_1^2 - a_{55} - a_{44}\right), \qquad d_{12} = \Omega_* b_2\left(b_2^2 - a_{55} - a_{44}\right)$$

$$d_{21} = \Omega_* b_1\left(a_{45} - a_{55}d_1\right), \qquad d_{22} = \Omega_* b_2\left(a_{45} - a_{55}d_2\right)$$

$$\sigma_{13\tau}^{(s)} = a_{44}\frac{\partial U_\tau^{(s)}}{\partial\zeta} - a_{45}\frac{\partial V_\tau^{(s)}}{\partial\zeta} + \sigma_{13*}^{(s)},$$

$$\sigma_{23\tau}^{(s)} = -a_{45}\frac{\partial U_\tau^{(s)}}{\partial\zeta} + a_{55}\frac{\partial V_\tau^{(s)}}{\partial\zeta} + \sigma_{23*}^{(s)},$$

The solution to the equation (13) is

$$\mathrm{W}^{(s)} = \mathrm{W}_0^{(s)} + \mathrm{W}_\tau^{(s)}(\xi,\eta,\zeta)$$

at $\frac{\Delta_1}{A_{33}} > 0,$

$$\mathrm{W}_0^{(s)} = \mathrm{D}_5^{(s)}\cos\sqrt{\frac{\Delta_1}{A_{33}}}\Omega_*\zeta + \mathrm{D}_6^{(s)}\sin\sqrt{\frac{\Delta_1}{A_{33}}}\Omega_*\zeta, \qquad (19)$$

$$\sigma_{33}^{(s)} = \sqrt{\frac{A_{33}}{\Delta_1}}\Omega_*\left(-\mathrm{D}_5^{(s)}\sin\sqrt{\frac{\Delta_1}{A_{33}}}\Omega_*\zeta + \mathrm{D}_6^{(s)}\cos\sqrt{\frac{\Delta_1}{A_{33}}}\Omega_*\zeta\right) + \sigma_{33\tau}^{(s)},$$

at $\frac{\Delta_1}{A_{33}} < 0,$

$$\mathrm{W}_0^{(s)} = \mathrm{D}_5^{(s)}\mathrm{ch}\sqrt{\left|\frac{\Delta_1}{A_{33}}\right|}\Omega_*\zeta + \mathrm{D}_6^{(s)}\mathrm{sh}\sqrt{\left|\frac{\Delta_1}{A_{33}}\right|}\,\Omega_*\zeta,$$

$$\sigma_{33}^{(s)}=\sqrt{\frac{A_{33}}{\Delta_1}}\Omega_*\left(D_5^{(s)}\mathrm{sh}\sqrt{\left|\frac{\Delta_1}{A_{33}}\right|}\Omega_*\zeta+D_6^{(s)}\mathrm{ch}\sqrt{\left|\frac{\Delta_1}{A_{33}}\right|}\Omega_*\zeta\right)+\sigma_{33\tau}^{(s)}, \qquad (20)$$

$$\sigma_{33\tau}^{(s)}=\frac{1}{\Delta_1}\left(A_{33}\frac{\partial W_\tau^{(s)}}{\partial\zeta}+\sigma_{33*}^{(s)}\right),$$

The general solution of the external problem contains 6 yet unknown functions $D_1^{(s)}(\xi,\eta)-D_6^{(s)}(\xi,\eta)$, which are uniquely determined using boundary conditions (3), (4).

3. The satisfaction of the boundary conditions

Using solutions (16), (17), (18), (19), (20) we will satisfy the boundary conditions (3), (4). Satisfaction of boundary conditions (3) with respect to σ_{xz},σ_{yz} and condition (4) relatively to u,v leads to the solution of the algebraic system

$$\begin{aligned}
&D_1^{(s)}d_{11}c_1-D_2^{(s)}d_{11}c_2+D_3^{(s)}d_{12}c_3-D_4^{(s)}d_{12}c_4=-\sigma_{13*}^{(s)}(\zeta=1)\\
&D_1^{(s)}d_{21}c_1-D_2^{(s)}d_{21}c_2+D_3^{(s)}d_{22}c_3-D_4^{(s)}d_{22}c_4=-\sigma_{23*}^{(s)}(\zeta=1)\\
&D_1^{(s)}c_2-D_2^{(s)}c_1+D_3^{(s)}c_4-D_4^{(s)}c_3=-U_\tau^{(s)}(\zeta=-1)\\
&D_1^{(s)}d_1c_2-D_2^{(s)}d_1c_1+D_3^{(s)}d_2c_4-D_4^{(s)}d_2c_3=-V_\tau^{(s)}(\zeta=-1)
\end{aligned} \qquad (21)$$

where

$$c_1=\sin b_1\Omega_*,\ \ c_2=\cos b_1\Omega_*,\ \ c_3=\sin b_2\Omega_*,\ \ \ c_4=\cos b_2\Omega_*$$

by Cramer's formula

$$D_j^{(s)}=\frac{\Delta_j^{(s)}}{\Delta},\ j=1,2,3,4 \qquad (22)$$

$$\Delta=\begin{vmatrix} d_{11}c_1 & d_{11}c_2 & d_{12}c_3 & d_{12}c_4\\ d_{21}c_1 & d_{21}c_2 & d_{22}c_3 & d_{22}c_4\\ c_2 & c_1 & c_4 & c_3\\ d_1c_2 & d_1c_1 & d_2c_4 & d_2c_3 \end{vmatrix}=(d_2-d_1)(d_{11}d_{22}-d_{12}d_{21})\cos 2b_1\Omega_*\cos 2b_2\Omega_*$$

$\Delta_j^{(s)}$ is obtained from Δ by replacing j-th column with column from free terms $j=1,2,3,4$.

According to (22) we have

$$D_1^{(s)}=\frac{\sin b_1\Omega_*}{\delta_1}\left(d_{12}\sigma_{23*}^{(s)}(\zeta=1)-d_{22}\sigma_{13*}^{(s)}(\zeta=1)\right)+$$
$$+\frac{\cos b_1\Omega_*}{\delta_2}\left(d_2U_\tau^{(s)}(\zeta=-1)-V_\tau^{(s)}(\zeta=-1)\right),$$

$$D_2^{(s)}=\frac{\cos b_1\Omega_*}{\delta_1}\left(d_{12}\sigma_{23*}^{(s)}(\zeta=1)-d_{22}\sigma_{13*}^{(s)}(\zeta=1)\right)+$$
$$+\frac{\sin b_1\Omega_*}{\delta_2}\left(d_2\mathrm{U}_\tau^{(s)}(\zeta=-1)-\mathrm{V}_\tau^{(s)}(\zeta=-1)\right),$$
$$D_3^{(s)}=\frac{\sin b_2\Omega_*}{\delta_3}\left(d_{21}\sigma_{13*}^{(s)}(\zeta=1)-d_{11}\sigma_{23*}^{(s)}(\zeta=1)\right)+$$
$$+\frac{\cos b_2\Omega_*}{\delta_4}\left(-d_1\mathrm{U}_\tau^{(s)}(\zeta=-1)+\mathrm{V}_\tau^{(s)}(\zeta=-1)\right), \qquad (23)$$
$$D_4^{(s)}=\frac{\cos b_2\Omega_*}{\delta_3}\left(d_{21}\sigma_{13*}^{(s)}(\zeta=1)-d_{11}\sigma_{23*}^{(s)}(\zeta=1)\right)+$$
$$+\frac{\sin b_2\Omega_*}{\delta_4}\left(-d_1\mathrm{U}_\tau^{(s)}(\zeta=-1)+\mathrm{V}_\tau^{(s)}(\zeta=-1)\right),$$
$$\delta_1=(d_{11}d_{22}-d_{12}d_{21})\cos 2b_1\Omega_*,\quad \delta_2=(d_1-d_2)\cos 2b_1\Omega_*$$
$$\delta_3=(d_{11}d_{22}-d_{12}d_{21})\cos 2b_2\Omega_*,\quad \delta_4=(d_1-d_2)\cos 2b_2\Omega_*$$

At $\cos 2b_1\Omega_*=0$ or $\cos 2b_2\Omega_*=0$ the resonance will occur. The resonant frequencies are

$$\Omega=\frac{1}{hb_1\sqrt{\rho}}\frac{\pi}{4}(2n+1),\ \ \Omega=\frac{1}{hb_2\sqrt{\rho}}\frac{\pi}{4}(2n+1),\ n\in N, \qquad (24)$$

Using (18), (20) and satisfying to conditions (3), (4) relatively we will obtain the system

$$D_5^{(s)}c_5+D_6^{(s)}c_6=-W^{+(s)}-W_\tau^{(s)}(\zeta=1) \qquad (25)$$
$$D_5^{(s)}c_5-D_6^{(s)}c_6=-W_\tau^{(s)}(\zeta=-1)\ \ \sigma_{zz},\ \mathrm{w},$$

from which is follows

$$c_5=\cos\sqrt{\frac{\Delta_1}{A_{33}}}\Omega_*,\quad c_6=\sin\sqrt{\frac{\Delta_1}{A_{33}}}\Omega_*,\quad \text{at } \frac{\Delta_1}{A_{33}}>0 \qquad (26)$$
$$c_5=\mathrm{ch}\sqrt{\left|\frac{\Delta_1}{A_{33}}\right|}\Omega_*,\quad c_6=\mathrm{sh}\sqrt{\left|\frac{\Delta_1}{A_{33}}\right|}\Omega_*,\quad \text{at } \frac{\Delta_1}{A_{33}}<0 \qquad (27)$$
$$D_5^{(s)}=\frac{1}{2c_5}\left(-W^{+(s)}-W_\tau^{(s)}(\zeta=1)-W_\tau^{(s)}(\zeta=-1)\right), \qquad (28)$$
$$D_6^{(s)}=\frac{1}{2c_6}\left(-W^{+(s)}-W_\tau^{(s)}(\zeta=1)+W_\tau^{(s)}(\zeta=-1)\right),$$

The resonant frequencies according to (26) are:

$$\Omega = \frac{1}{h}\sqrt{\frac{A_{33}}{\rho\Delta_1}}\frac{\pi}{2}(2n+1), \qquad \Omega = \frac{1}{h}\sqrt{\frac{A_{33}}{\rho\Delta_1}}\pi n, \qquad n \in N$$

At (27) there is no resonance.

4. On mathematically precise solutions

As it was stated above, if W^+ is an algebraic polynomial, the iteration terminates at the certain approximation, depending on the degree of the polynomial, as a result the solution becomes mathematically exact. Let illustrates this when:

$$W^+ = a_0 + a_1\xi + a_2\eta \tag{30}$$

At $s = 0$, according to formulas (10), (16), (17), (18), (19) we will have:

$$U^{(0)} = 0,\ V^{(0)} = 0,\ \sigma_{13}^{(0)} = 0,\ \sigma_{23}^{(0)} = 0$$

$$W^{(0)} = -W^+\left(\frac{\bar{c}_5}{2c_5} + \frac{\bar{c}_6}{2c_6}\right)$$

$$\bar{c}_5 = \cos\sqrt{\frac{\Delta_1}{A_{33}}}\Omega_*\zeta,\ \ \bar{c}_6 = \sin\sqrt{\frac{\Delta_1}{A_{33}}}\Omega_*\zeta, \tag{31}$$

$$\sigma_{11}^{(0)} = -\frac{A_{31}W^+}{\Delta_1}\phi_1(\zeta), \qquad \sigma_{22}^{(0)} = -\frac{A_{32}W^+}{\Delta_1}\phi_1(\zeta),$$

$$\sigma_{33}^{(0)} = -\frac{A_{33}W^+}{\Delta_1}\phi_1(\zeta), \qquad \sigma_{12}^{(0)} = -\frac{A_{34}W^+}{\Delta_1}\phi_1(\zeta),$$

$$\phi_1(\zeta) = \frac{\Omega_*}{2}\sqrt{\frac{\Delta_1}{A_{33}}}\left(\frac{\bar{c}_5}{c_6} - \frac{\bar{c}_6}{c_5}\right),$$

At $s = 1$ we will have:

$$U^{(1)} = D_1^{(1)}\cos b_1\Omega_*\zeta + D_2^{(1)}\sin b_1\Omega_*\zeta + D_3^{(1)}\cos b_2\Omega_*\zeta + D_4^{(1)}\sin b_2\Omega_*\zeta + U_\tau^{(1)},$$

$$\begin{aligned} V^{(1)} = {} & D_1^{(1)}d_1\cos b_1\Omega_*\zeta + D_2^{(1)}d_1\sin b_1\Omega_*\zeta + \\ & + D_3^{(1)}d_2\cos b_2\Omega_*\zeta + D_4^{(1)}d_2\sin b_2\Omega_*\zeta + V_\tau^{(1)}, \end{aligned} \tag{32}$$

$$\begin{aligned} \sigma_{13}^{(1)} = {} & \frac{1}{\Delta_2}(D_1^{(1)}d_{11}\sin b_1\Omega_*\zeta - D_2^{(1)}d_{11}\cos b_1\Omega_*\zeta + \\ & + D_3^{(1)}d_{12}\sin b_2\Omega_*\zeta - D_4^{(1)}d_{12}\cos b_2\Omega_*\zeta) \end{aligned}$$

$$\begin{aligned} \sigma_{23}^{(1)} = {} & \frac{1}{\Delta_2}\Big(D_1^{(1)}d_{21}\sin b_1\Omega_*\zeta - D_2^{(1)}d_{21}\cos b_1\Omega_*\zeta + \\ & + D_3^{(1)}d_{22}\sin b_2\Omega_*\zeta - D_4^{(1)}d_{22}\cos b_2\Omega_*\zeta\Big) \end{aligned}$$

$$W^{(1)}=0,\ \sigma_{11}^{(1)}=0,\ \sigma_{22}^{(1)}=0,\ \sigma_{33}^{(1)}=0,\quad \sigma_{12}^{(1)}=0,$$

$$U_{\tau}^{(1)}=\frac{k_1}{k_2}\left(\frac{\bar{c}_6}{c_5}-\frac{\bar{c}_5}{c_6}\right),$$

$$V_{\tau}^{(1)}=2\left[\frac{k_1}{k_2a_{45}}\left(\frac{\Delta_1}{A_{33}}-a_{55}\right)+\frac{1}{\Delta_1\Omega_*^2}\left(\frac{k_1a_{55}}{k_2a_{44}}+\frac{\Delta_2\Omega_*^2}{2\Delta_1}\sqrt{\frac{\Delta_1}{A_{33}}}\left(A_{34}a_1+A_{32}a_2\right)-\right.\right.$$

$$\left.\left.-\frac{\Omega_*}{2}\sqrt{\frac{\Delta_1}{A_{33}}}\left(a_{55}a_2-a_{45}a_1\right)\right)\right]ctg\,2\sqrt{\frac{\Delta_1}{A_{33}}}\Omega_*\zeta,$$

$$k_1=\frac{1}{2}\sqrt{\frac{\Delta_1}{A_{33}}}\left(\left(A_{31}a_1+A_{34}a_2\right)\left(\frac{\Delta_2}{\Delta_1}-\frac{a_{55}}{A_{33}}\right)-a_{45}\left(A_{34}a_1+A_{32}a_2\right)\right),$$

$$k_2=\Omega_*\left(\frac{\Delta_1}{A_{33}}\left(\frac{\Delta_1}{A_{33}}-a_{44-}a_{55}\right)+\Delta_2\right),$$

$$D_1^{(1)}=\frac{\sin b_1\Omega_*}{\delta_1}\Big(d_{12}\left(a_{55}a_2-a_{45}a_1\right)-d_{22}\left(a_{44}a_1-a_{45}a_2\right)\Big)+$$

$$+\frac{\cos b_1\Omega_*}{\delta_2}\Big(d_2U_{\tau}^{(1)}\left(\zeta=-1\right)-V_{\tau}^{(1)}\left(\zeta=-1\right)\Big),$$

$$D_2^{(1)}=\frac{\cos b_1\Omega_*}{\delta_1}\Big(d_{12}\left(a_{55}a_2-a_{45}a_1\right)-d_{22}\left(a_{44}a_1-a_{45}a_2\right)\Big)+$$

$$+\frac{\sin b_1\Omega_*}{\delta_2}\Big(d_2U_{\tau}^{(1)}\left(\zeta=-1\right)-V_{\tau}^{(1)}\left(\zeta=-1\right)\Big),$$

$$D_3^{(1)}=\frac{\sin b_2\Omega_*}{\delta_3}\Big(d_{21}\left(a_{44}a_1-a_{45}a_2\right)-d_{11}\left(a_{55}a_2-a_{45}a_1\right)\Big)+$$

$$+\frac{\cos b_2\Omega_*}{\delta_4}\Big(-d_1U_{\tau}^{(1)}\left(\zeta=-1\right)+V_{\tau}^{(1)}\left(\zeta=-1\right)\Big),$$

$$D_4^{(1)}=\frac{\cos b_2\Omega_*}{\delta_3}\Big(d_{21}\left(a_{44}a_1-a_{45}a_2\right)-d_{11}\left(a_{55}a_2-a_{45}a_1\right)\Big)+$$

$$+\frac{\sin b_2\Omega_*}{\delta_4}\Big(-d_1U_{\tau}^{(1)}\left(\zeta=-1\right)+V_{\tau}^{(1)}\left(\zeta=-1\right)\Big),$$

The iteration breaks at the approximation $s=1$, as a result we will have the exact solution.

$$w=lW^{(0)}\exp(i\Omega t),\qquad \sigma_{xx}=\varepsilon^{-1}\sigma_{11}^{(0)}\exp(i\Omega t),\qquad \sigma_{yy}=\varepsilon^{-1}\sigma_{22}^{(0)}\exp(i\Omega t),$$

$$\sigma_{zz}=\varepsilon^{-1}\sigma_{33}^{(0)}\exp(i\Omega t),\ \ \sigma_{xy}=\varepsilon^{-1}\sigma_{12}^{(0)}\exp(i\Omega t), \tag{33}$$

$$u = hU^{(1)}\exp(i\Omega t), \quad v = hV^{(1)}\exp(i\Omega t),$$
$$\sigma_{xz} = \sigma_{13}^{(1)}\exp(i\Omega t), \quad \sigma_{yz} = \sigma_{23}^{(1)}\exp(i\Omega t)$$

Conclusions

The asymptotic solution of mixed dynamic 3D problem for plate having a plane of elastic symmetry is obtained. It is assumed, that the facial surface of the plate is subjected the normal displacements, which changed harmonically over time, and the lower edge of the plate is rigidly fixed. By switching to dimensionless coordinates and displacement in the basic equations and elasticity relations (13 independent constants of elasticity), the system of differential equations singularly perturbed by a small parameter was obtained, which was solved by an asymptotic method. All components of the stress tensor and displacement vector, as well as the values of resonant frequencies were determined. It was shown, that in similar classes of problems, the dominant role played longitudinal oscillations in the vertical direction and the amplitudes of shear oscillations are an order smaller.

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