Mathematical Modelling and Implementation of an Online Time-Varying Adaptive Noise Reduction System

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Abstract. Noise remains a fundamental challenge in modern communication and signal processing systems, significantly affecting signal quality, reliability, and information accuracy. In dynamic and non-stationary environments, traditional fixed-coefficient filters often fail to provide satisfactory performance, creating a growing demand for adaptive real-time noise reduction techniques. This thesis investigates online noise reduction using time-varying filters, which dynamically adjust their parameters to track changing noise characteristics during signal acquisition or transmission. The study explores the theoretical foundations of time-varying filtering, with particular emphasis on adaptive filtering algorithms such as Least Mean Squares (LMS) and Recursive Least Squares (RLS). It analyzes key implementation challenges, including system stability, convergence speed, computational efficiency, and latency constraints, especially in embedded and real-time systems. Mathematical modeling of discrete-time signals is presented to describe the dynamic behavior of time-varying filters and their role in minimizing noise influence while preserving signal integrity.

1. Introduction

In modern communication and signal processing systems, noise remains one of the fundamental challenges affecting the quality, reliability, and intelligibility of transmitted and recorded signals. Whether in telecommunications, audio engineering, biomedical signal processing, or real-time data acquisition systems, unwanted noise components degrade system performance and reduce the accuracy of information extraction. As digital systems continue to evolve and operate in increasingly dynamic and unpredictable environments, the demand for efficient real-time noise reduction techniques has significantly increased.

Noise can originate from various sources, including thermal fluctuations in electronic components, electromagnetic interference, quantization errors, environmental disturbances, and transmission channel imperfections. In practical systems, noise is often non-stationary, meaning its statistical properties change over time. Traditional filtering approaches, such as fixed-coefficient linear filters, are typically designed under the assumption of stationary noise and may not perform effectively in time-varying conditions. Therefore, adaptive and time-varying filtering techniques have become essential in modern signal processing applications.

Online noise reduction refers to the process of suppressing unwanted noise components in a signal while the signal is being acquired or transmitted, without significant delay. Unlike offline processing, where the entire signal is available for analysis, online processing requires algorithms that operate sequentially and in real time. This imposes strict constraints on computational complexity, memory usage, and algorithm stability. Real-time systems such as voice communication platforms, hearing aids,

radar systems, and biomedical monitoring devices rely heavily on efficient online noise reduction techniques to ensure continuous and reliable operation.

Time-varying filters represent a powerful approach to addressing the limitations of fixed filtering methods. A time-varying filter is characterized by filter coefficients that change over time according to the signal characteristics or environmental conditions. By continuously adjusting its parameters, such a filter can track variations in noise statistics and maintain optimal performance under dynamic conditions. This adaptability makes time-varying filters particularly suitable for non-stationary signal environments. One of the theoretical foundations of time-varying filtering lies in adaptive filtering theory, including algorithms such as the Least Mean Squares (LMS) and Recursive Least Squares (RLS) methods. These algorithms iteratively update filter coefficients to minimize an error criterion, typically the mean squared error between the desired signal and the filter output. While classical adaptive filters are widely used, modern implementations often integrate additional techniques such as spectral estimation, statistical modeling, and machine learning-based adaptation mechanisms to improve robustness and convergence speed.

2. System Setup and validation

The implementation of online noise reduction using time-varying filters involves several critical considerations. First, system stability must be ensured despite continuous parameter updates. Second, convergence speed and tracking ability must be balanced to avoid excessive sensitivity to rapid fluctuations. Third, computational efficiency must be optimized, especially in embedded systems and portable devices with limited processing power. Furthermore, latency constraints require that filtering operations introduce minimal delay to preserve real-time responsiveness.

In audio signal processing, time-varying filters are widely used for speech enhancement, where background noise levels may change due to environmental conditions such as traffic, wind, or crowd noise. In biomedical applications, such as electrocardiogram (ECG) and electroencephalogram (EEG) monitoring, time-varying filtering helps remove interference caused by muscle activity or power-line noise while preserving critical physiological information. In wireless communication systems, adaptive filtering techniques compensate for channel fading and interference, improving signal-to-noise ratio (SNR) and overall communication quality.

Let the observed signal be a mixture of the useful signal and additive noise:

$$x(n) = s(n) + v(n) \tag{1}$$

where:
- $x(n)$ is the measured input (noisy) signal,
- $s(n)$ is the desired clean component,
- $v(n)$ is additive noise.

The goal of online noise reduction is to estimate a denoised signal $\hat{s}(n)$ sequentially, with minimal delay, while the noise statistics may vary over time.

In adaptive noise reduction systems, a desired/reference signal is often defined as $d(n)$. Depending on the architecture, $d(n)$ may represent the primary noisy measurement or an externally provided target.

The mathematical modeling of time-varying filters typically involves discrete-time signal representations. Consider an input signal composed of a desired signal component and an additive noise component. The goal of the filtering process is to estimate the clean signal by minimizing the influence of noise. Unlike fixed linear time-invariant (LTI) systems, time-varying filters are described by difference equations whose coefficients depend explicitly on time. This time dependency enables the system to respond dynamically to changes in signal characteristics.

A finite impulse response (FIR) filter of length M produces an output:

$$y(n) = \sum_{k=0}^{M-1} w_k(n) \, x(n-k) \tag{2}$$

where $w_k(n)$ is the time-varying coefficient (tap weight) at time index n.

Vector form:

$$y(n) = w^H(n)x(n) \tag{3}$$

with

$$x(n) = [x(n) \quad x(n-1) \quad … \quad x(n-M+1)]^T, w(n) = [w_0(n) \quad w_1(n) \quad … \quad w_{M-1}(n)]^T$$

and $(\cdot)^H$ is the Hermitian transpose (transpose + complex conjugate), used in complex/IQ processing.

Because $w(n)$ changes with time, the system is Linear Time-Varying (LTV).

A typical online implementation follows these steps at each sample n:
1. Form input vector $x(n)$ from current and past samples.
2. Compute filter output $y(n) = w^H(n)x(n)$.
3. Compute error $e(n) = d(n) - y(n)$.
4. Update coefficients $w(n+1) = w(n) + \mu x(n)e^*(n)$.
5. Output denoised signal $\hat{s}(n) = e(n)$ (or other defined output).

This loop structure (fig.1.) matches typical graphical implementations (e.g., LabVIEW For-Loop with shift registers storing $w(n)$).
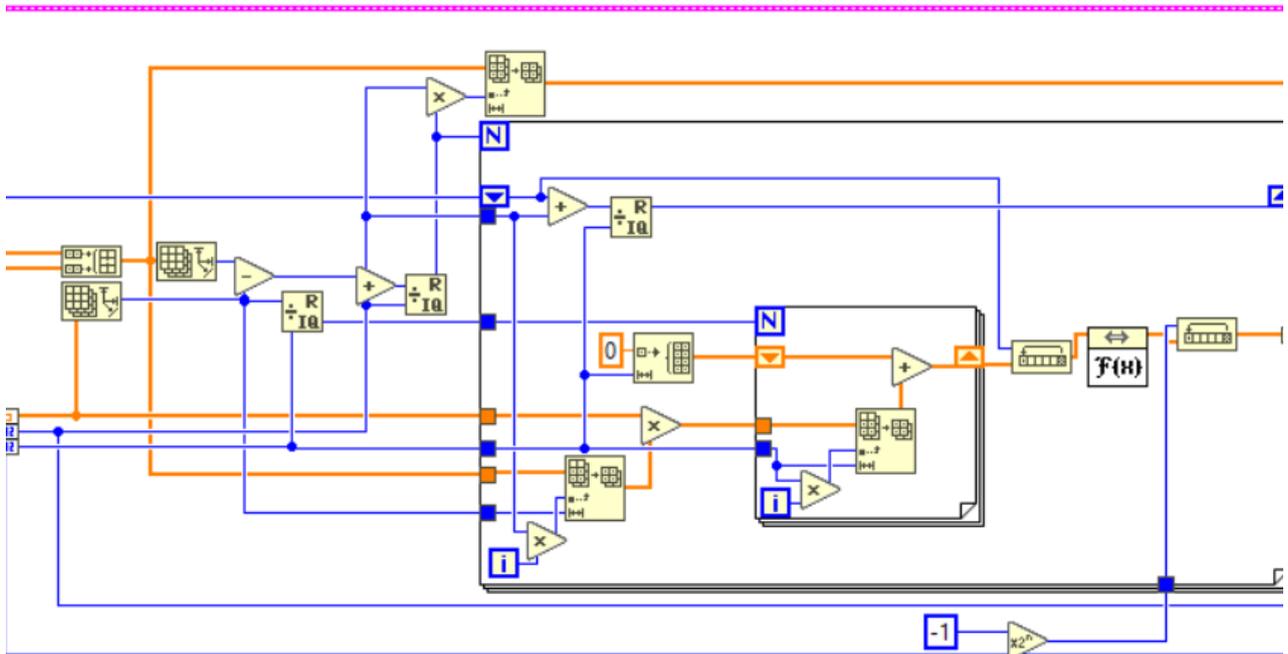

Fig. 1. Mathematical implementation in LabVIEW.

3. Experimental results

This figure illustrates the original time-domain signal contaminated by additive noise prior to any processing. The horizontal axis represents the discrete sample index, while the vertical axis corresponds to the signal amplitude.

The waveform exhibits highly irregular oscillatory behavior, characterized by rapid amplitude fluctuations and random spikes. Although the underlying chirp structure is present, it is not clearly distinguishable due to the significant influence of broadband noise. The noise introduces unpredictable

variations across the entire signal duration, masking the smooth frequency evolution of the chirp component.

The amplitude distribution appears widely spread, with peaks reaching both positive and negative extremes. These fluctuations are primarily caused by stochastic noise components superimposed on the deterministic signal. As a result, the time-domain representation alone does not allow for easy identification of the signal's instantaneous frequency behavior.
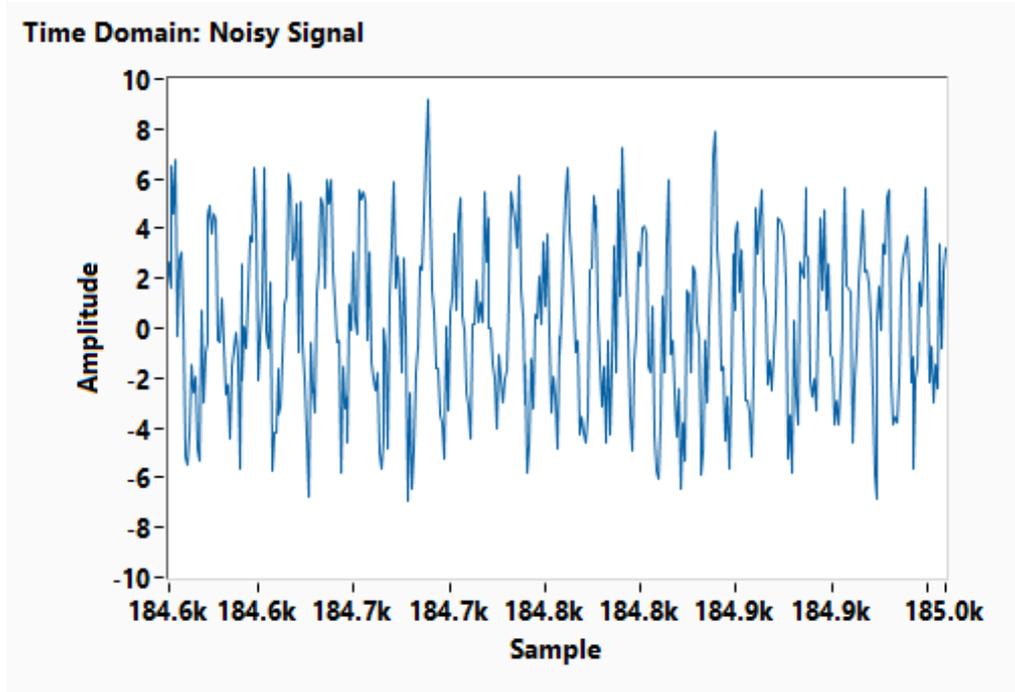

Fig. 2. Time Domain: Noisy Signal

This figure presents the time–frequency representation of the noisy signal obtained using the Gabor transform. The horizontal axis represents the discrete time samples, while the vertical axis corresponds to frequency (Hz). The color intensity indicates the magnitude of the Gabor coefficients, with brighter regions representing higher energy concentration.

Two dominant time-varying frequency components are clearly visible in the image. These components correspond to the chirp signal embedded in noise. The upward-sloping bright ridge represents a frequency component that increases over time, while the downward-sloping ridge represents a frequency component decreasing over time. Such behavior is characteristic of chirp signals, where the instantaneous frequency changes continuously.

The background of the image exhibits a granular blue texture distributed across the entire time–frequency plane. This diffuse energy represents broadband noise affecting all frequency bands. Unlike the structured chirp components, the noise does not form coherent ridges and instead appears randomly distributed.

This visualization demonstrates the advantage of time–frequency analysis: the useful signal components are localized and structured, whereas the noise energy is spread across the plane. Such separation in the time–frequency domain provides the theoretical basis for applying thresholding techniques to suppress noise while preserving the significant signal components.
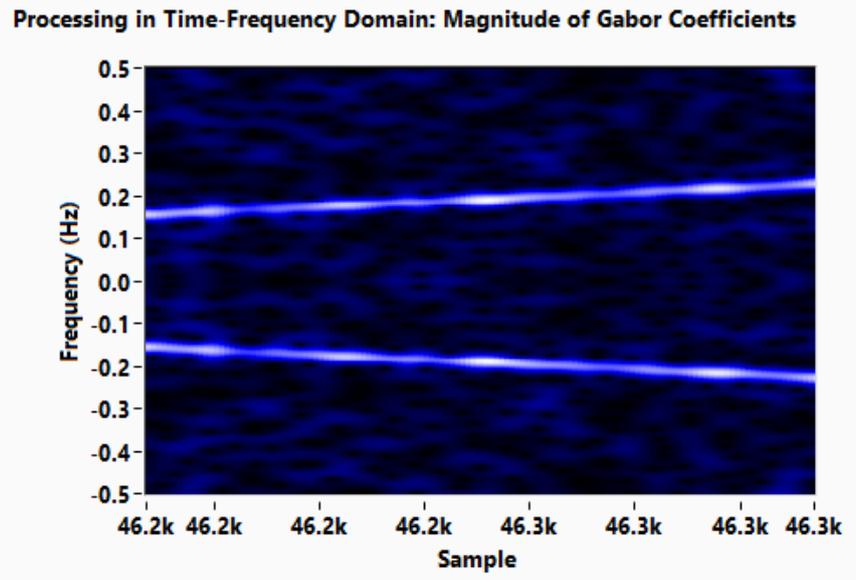
Fig. 3. Processing in Time–Frequency Domain: Magnitude of Gabor Coefficients

This figure illustrates the reconstructed time-domain signal after applying thresholding in the Gabor time–frequency domain and performing inverse Gabor expansion.

The horizontal axis represents discrete time samples, while the vertical axis shows signal amplitude. Compared to the original noisy signal (not shown here), the waveform appears significantly cleaner and more regular. The oscillatory structure of the chirp signal is clearly visible, and the amplitude variations follow a smooth envelope.

The reduction of random high-frequency fluctuations indicates successful suppression of broadband noise. The remaining oscillations correspond to the meaningful signal components preserved during the thresholding process. Importantly, the waveform does not exhibit noticeable distortion, which confirms that the threshold level was selected appropriately to balance noise suppression and signal preservation.

This result demonstrates the effectiveness of time–frequency domain processing: by attenuating low-energy Gabor coefficients associated with noise and reconstructing the signal using only significant components, the method achieves substantial improvement in signal clarity while maintaining temporal structure.
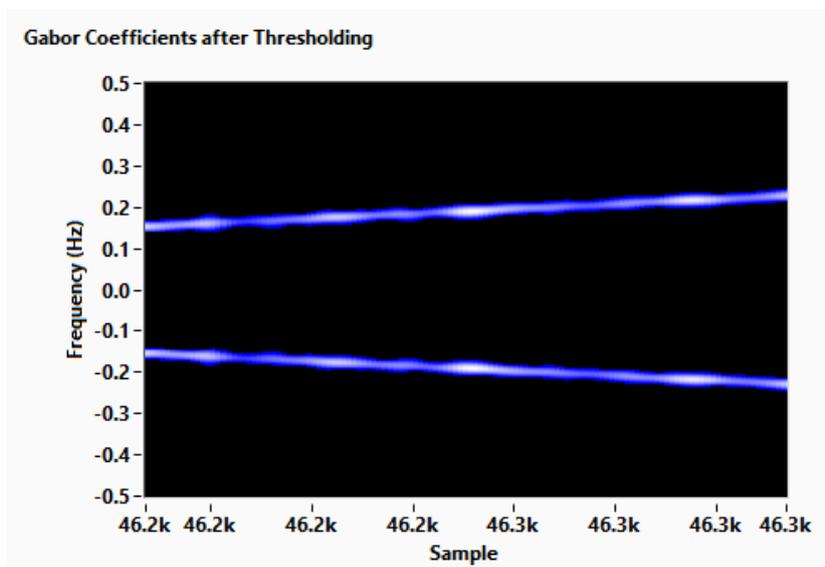
Fig. 4. Time Domain: Denoised Signal

This figure shows the magnitude of the Gabor coefficients after applying thresholding in the time–frequency domain.

In comparison with the original Gabor representation (Figure 1), the background noise has been significantly reduced. The majority of low-energy coefficients have been suppressed, resulting in a predominantly dark background. Only the high-energy ridges corresponding to the chirp components remain visible.

The preservation of the two main frequency trajectories indicates that the thresholding process successfully retained the informative parts of the signal while eliminating components associated with noise. The time-varying nature of the signal is still clearly observable, demonstrating that the essential structure of the chirp was not distorted.

This figure visually confirms the effectiveness of the denoising strategy. By applying a magnitude threshold in the time–frequency domain, the algorithm performs selective filtering that removes distributed noise energy while preserving coherent signal features. This selective energy retention forms the basis for accurate reconstruction of the denoised time-domain signal shown in Figure 3.
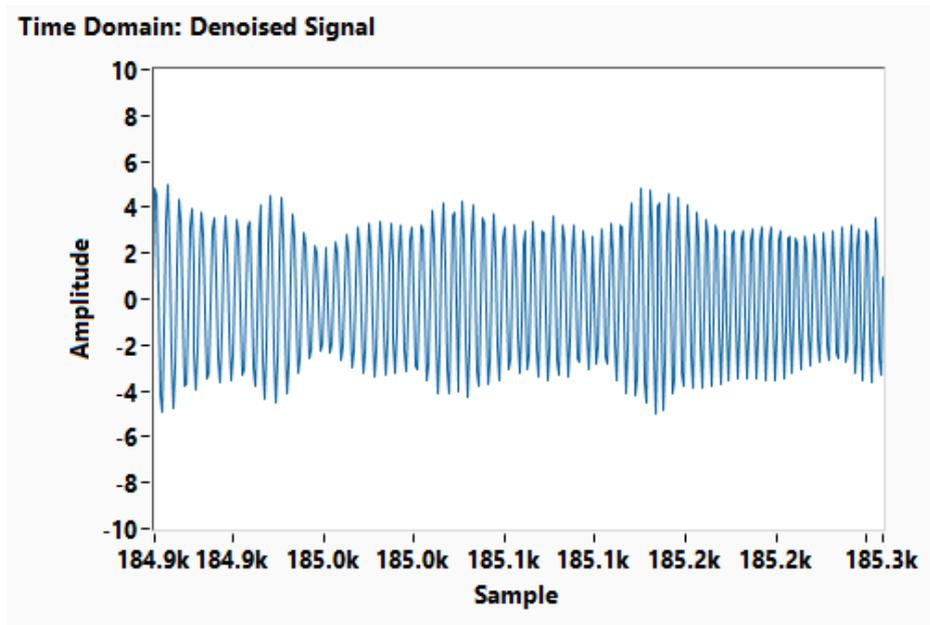

Fig. 5. Time Domain: Denoised Signal

Conclusions

This work presented an online noise reduction method based on a time-varying time–frequency approach using the Gabor transform, magnitude thresholding, and inverse Gabor expansion for reconstruction. The main idea is to convert the noisy time-domain signal into the time–frequency domain, where useful components (such as chirp trajectories) appear as structured, high-energy ridges, while noise is spread more diffusely across the plane. By applying a properly selected threshold to the Gabor coefficients, low-energy components associated with broadband noise are suppressed, and the remaining significant coefficients preserve the essential signal content.

The obtained results confirm that thresholding in the Gabor domain significantly reduces background noise while maintaining the key time-varying frequency structure of the signal. After reconstruction, the denoised waveform exhibits improved regularity and clarity compared to the original noisy signal, with noticeably fewer random fluctuations. The comparison between the original time–frequency representation and the post-thresholding representation visually demonstrates the effectiveness of the method: the noisy background is strongly attenuated while the informative

components remain dominant.

Overall, the proposed workflow is suitable for real-time (online) processing, because it follows a sequential pipeline—transform, coefficient processing, and reconstruction—compatible with streaming implementations. Future improvements can include adaptive or data-driven threshold selection, more advanced shrinkage rules (soft/garrote thresholding), and quantitative evaluation using metrics such as SNR improvement, MSE, and convergence/tracking behavior under different non-stationary noise conditions. were observed. These findings confirm that the proposed hybrid framework achieves a favorable balance between robustness and computational efficiency.

The system was validated across multiple experimental trials to ensure repeatability. Results demonstrated that while waterfall features alone are sensitive to noise, and bispectral features alone are computationally demanding, their combination under the CS-DSB framework provides a robust and efficient solution for RF receiver identification. The integration of LabVIEW for real-time signal acquisition and processing further confirms the practical applicability of the proposed method.

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