Armenian Journal of Physics, 2025, vol. 18, issue 4, pp. 64-71

Modal Analysis of a Cylindrical Resonator Partially Filled with Dielectric

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(Received: December 10, 2025; Revised: January 12, 2026; Accepted: January 18, 2026)

Abstract. The interaction of resonant modes with a partially filled, radially distributed dielectric in a cylindrical resonator is investigated. In non-uniform filling configurations, the sensitivity of resonance-based dielectric evaluation depends strongly on the spatial overlap between the resonant electric field and the dielectric volume. The commonly used TM_{010} mode concentrates electric energy near the cavity axis, leading to weak interaction with off-axis dielectric samples. An alternative modal structure based on the TM_{110} mode is therefore examined in the X-band (8–12 GHz), where its off-axis electric-field localization provides substantially improved dielectric participation. Eigenmode analysis demonstrates that TM_{110} exhibits a 13-68% higher electric energy filling factor, and produces 1.4-4 times larger frequency shifts than TM_{010}. Driven-port simulations further confirm that TM_{110} is more effectively excited, with a $S_{11} \approx -23$ dB compared to $S_{11} \approx -18$ dB for TM_{010}.

Keywords: Cylindrical resonator, partial dielectric loading, eigenmodes, TM_{110} mode

DOI: 10.54503/18291171-2025.18.4-64

1. Introduction

Cylindrical microwave resonators are widely used in dielectric evaluation and sensing because of their well-defined modal field distributions, high quality factors, and compatibility with resonance-based measurement techniques. In perturbation methods, dielectric loading modifies the stored electromagnetic energy, producing measurable shifts in resonant frequency and quality factor, which can be related to the complex permittivity of the inserted material [1,2]. The accuracy and sensitivity of such measurements depend strongly on the spatial overlap between the resonant electric field and the dielectric volume. Therefore, most established cavity-based approaches assume that the dielectric sample is placed in a region of strong electric-field intensity, such as the cavity axis for TM_{01p} modes or near the cavity wall for TE_{m11} modes [3,4]. However, many practical dielectric inserts are only partially filled in the finite radial direction, while the cavity center remains air-filled. In these configurations, the commonly used TM_{010} mode may exhibit poor field–dielectric overlap, leading to reduced energy participation and lower frequency sensitivity. Previous studies have shown that alternative resonant modes can improve sensitivity when conventional modes are poorly matched to the sample geometry [5,6]. An alternative, TM_{110} mode, exhibits off-axis electric-field localization that can improve interaction with radially distributed dielectric, motivating a comparison of TM_{010} and TM_{110} modes [7]. In this work, full-wave eigenmode and driven-port simulations were performed to compare the TM_{010} and TM_{110} modes in a partially filled cylindrical resonator in the X-band (8–12 GHz), where many dielectric materials exhibit stable, well-characterized properties, making their characterization in this frequency range practically relevant for microwave sensing applications.

2. Materials and Methods

Conventional cylindrical cavity-based permittivity extraction and sensing methods implicitly assume that the dielectric sample occupies regions of strong electric-field intensity, typically aligned with the axial field maximum of TM_{01p} modes. For annular dielectrics, this assumption is violated, as the field maximum lies within the hollow cavity core rather than the dielectric material. As a result, the effective electric energy stored within the dielectric is reduced, leading to diminished sensitivity and increased ambiguity in permittivity estimation.
To characterize the annular geometry, the *relative inner radius α* is defined as.

$$\alpha = \frac{R_{in}}{R_{out}}, 0 < \alpha < 1. \tag{1}$$

where R_{in} and R_{out} are inner and outer radii of annular dielectric, respectively.
To quantify the interaction between the cavity modes and the dielectric, the total electric energy stored within the dielectric is computed as.

$$U = \frac{1}{2}\iiint_{Vd} \varepsilon_r |E|^2 dV \tag{2}$$

where ε_r is relative permittivity of the medium and E is the electric field magnitude.
In eigenmode analysis, the absolute magnitude of the electric field is arbitrary. However, the relative energy distribution between modes provides a meaningful measure of their effectiveness in interacting with the dielectric material, provided that the fields are normalized such that the total stored electromagnetic energy equals 1 J.
To characterize the coupling between the annular dielectric and the electric field of a given mode, the *electric energy filling factor η* is used as the figure of merit. The filling factor quantifies the fraction of resonant electric energy stored inside the dielectric volume relative to the total cavity energy, directly indicating how efficiently a resonant mode concentrates electric energy within the dielectric region and is therefore suitable for mode comparison [8]:

$$\eta = \frac{\iiint_{Vd} \varepsilon_r |E|^2 dV}{\iiint_{Vt} \varepsilon_r |E|^2 dV} \tag{3}$$

where V_d and V_t are volumes of the dielectric and total cavity, respectively.

The fractional frequency shift, serving as a sensitivity measure for permittivity changes, can be approximated using perturbation theory. [8,9]

$$\frac{\Delta f}{f_0} \approx -\frac{1}{2}\frac{\iiint_{Vd} \Delta\varepsilon_r |E|^2 dV}{\iiint_{Vt} \varepsilon_r |E|^2 dV} \tag{4}$$

Using the filling factor definition above (and assuming background permittivity in the denominator is approximately constant), the sensitivity per unit permittivity is then

$$\delta = \frac{\Delta f}{\Delta\varepsilon} \approx \frac{-f_0}{2}\eta \tag{5}$$

This indicates that frequency sensitivity scales directly with the filling factor: higher η leads to larger frequency shifts, and enables a direct comparison of modes in terms of their effectiveness in detecting permittivity variations.

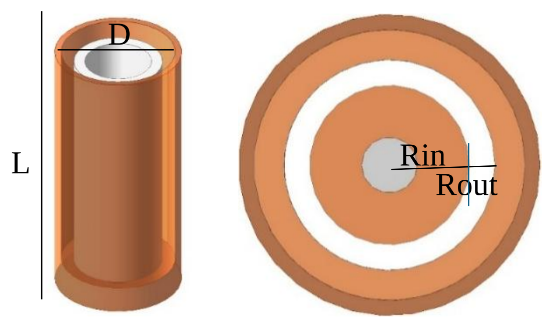

Fig. 1. Model of the cylindrical resonator with annular dielectric.

To evaluate the impact of cavity mode characteristics on dielectric loading and frequency response, a cylindrical copper cavity loaded with an alumina (Al_2O_3) dielectric rod with ε_r = 9.5 was modeled (Fig.1). The cavity has an inner diameter D = 20 mm and length L = 43 mm, while the dielectric rod has inner and outer radii of R_{in} = 12 mm, R_{out} = 16 mm, and length of 40 mm. Eigenmode and driven-port simulations were performed in the COMSOL Multiphysics environment to quantify dielectric loading and compare the electromagnetic responses of TM_{010} and TM_{110} modes.

3. Results and Discussion

Eigenmode simulations were first performed without dielectric loading to identify the natural resonant frequencies and intrinsic field distributions of the cavity modes of interest, as summarized in Table 1.

Table 1. Eigenmode resonant frequencies of the empty cavity

Mode	Frequency [GHz]
TM_{010}	11.6
TM_{110}	18.4

Fig.2 shows the electric field distributions normalized such that the total stored energy equals 1 J, for the TM_{010} and TM_{110} eigenmodes, when placing the dielectric rod inside the cavity. The field pattern of the TM_{010} (Fig. 2a) is concentrated along the cavity axis, which coincides with the hollow air-core, consequently, only a small fraction of the mode's electric energy overlaps the dielectric volume. By contrast, the TM_{110} mode (Fig. 2b) exhibits off-axis electric-field maxima with strong radial and azimuthal components that fall within the annular region, indicating substantially larger field–dielectric overlap.

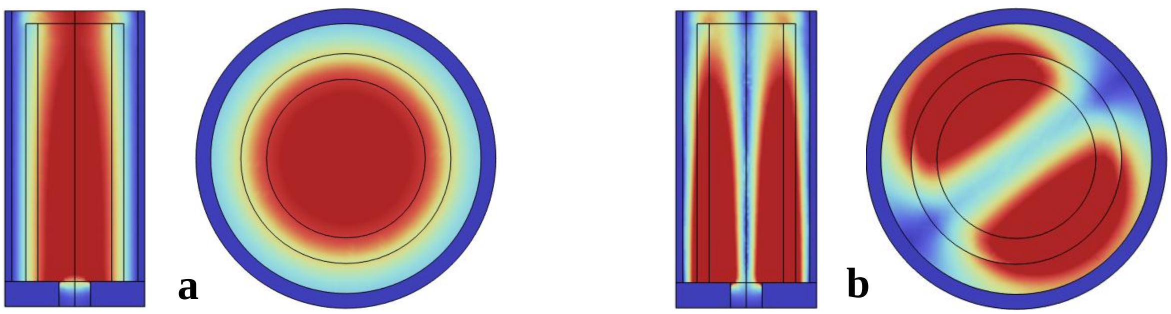

Fig. 2. Normalized E-field distributions in longitudinal and transverse planes for a) TM_{010} b) TM_{110}.

Electric energy filling factor

To quantify dielectric participation, the electric energy filling factor η was computed from the eigenmode fields while sweeping the relative inner radius. Fig.3 presents η as a function of α for both modes.

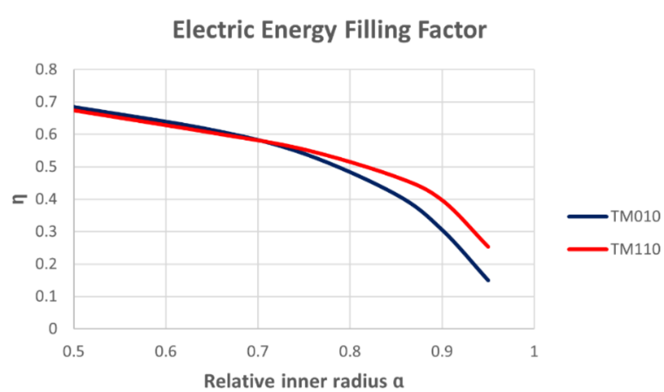

Fig. 3. Electric energy filling factor as a function of relative inner radius.

This shows that, for smaller values of α, corresponding to thicker dielectric region extending closer to the cavity axis, both modes exhibit comparable filling factors, indicating substantial overlap between the stored electric energy and the dielectric volume. However, as α increases, the dielectric becomes increasingly annular and moves away from the cavity axis; consequently, the filling factor of TM_{010} decreases rapidly beyond α=0.75 since its electric field is predominantly concentrated within the hollow central region. In contrast, TM_{110} mode maintains a higher filling factor due to its off-axis electric-field maxima. For α=0.85, TM_{110} mode exhibits 13% higher electric energy filling factor, increasing to about 68% for α=0.95.

Sensitivity of resonant modes

The suitability of each resonant mode for interaction with annular dielectrics was further evaluated by analyzing the sensitivity of the resonant frequency to variations in the relative permittivity. Fig.4 presents the eigenfrequency shift as a function of dielectric permittivity for both modes, with the relative inner radius of α= 0.75 and α= 0.95.

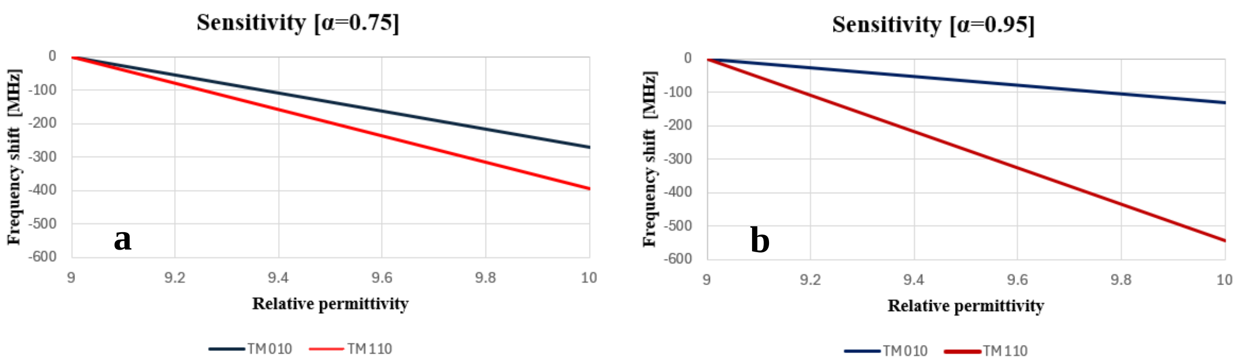

Fig. 4. Eigenmode frequency shifts for TM_{010} and TM_{110} modes. a) α= 0.75 b) α= 0.95.

The resulting frequency shift values are summarized in Table 2.

Table 2. Frequency shift values for TM_{010} and TM_{110} modes

Mode	$\delta_{(\alpha=0.75)}$ [MHz/per unit ε]	$\delta_{(\alpha=0.95)}$ [MHz/per unit ε]
TM_{010}	-272	-130
TM_{110}	**-395**	**-540**

The TM_{010} mode exhibits a relatively weak frequency dependence on permittivity, consistent with its limited electric-field overlap with the annular dielectric region. At α= 0.75, TM_{010} mode produces *-272 MHz* shift per unit change of permittivity, which decreases to *-130 MHz* at α= 0.95. In contrast, the TM_{110} mode produces a larger shift, yielding *-395 MHz* at α= 0.75 and increasing to *-540 MHz* at α= 0.95. This confirms that TM_{110} provides stronger dielectric interaction and is better suited for partially filled annular geometries.

Modal excitation

Driven-port simulations were performed to verify the selective excitation of specific resonant modes in the cylindrical cavity and to assess the frequency response to annular dielectric loading. Two coaxial probe configurations were considered (Fig.5): a) center-aligned probe, intended to predominantly excite the TM_{010} mode, and b) an offset probe, to excite the TM_{110} mode. The alumina sample (ε_r = 9.5, loss tangent = $8 \cdot 10^{-4}$ and α = 0.75) was included in the model to evaluate its effect on resonant frequency, coupling behavior, and field distribution.

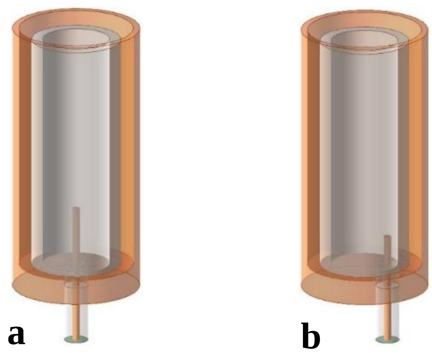

Fig. 5. Simulation model of the cylindrical cavity with annular dielectric and coaxial probe configurations: a) Center-aligned probe b) Offset probe.

The center-aligned probe efficiently couples to modes characterized by strong axial electric fields along the cavity axis, whereas the offset probe preferentially excites modes with electric-field maxima located away from the axis. The introduction of the annular dielectric significantly alters the cavity loading and modifies the external coupling conditions. Therefore, the probe insertion depth and lateral offset were systematically optimized to approach near-critical coupling for each excitation configuration. Multiple probe lengths and placements were evaluated, and the configuration yielding optimal coupling was selected for further analysis.

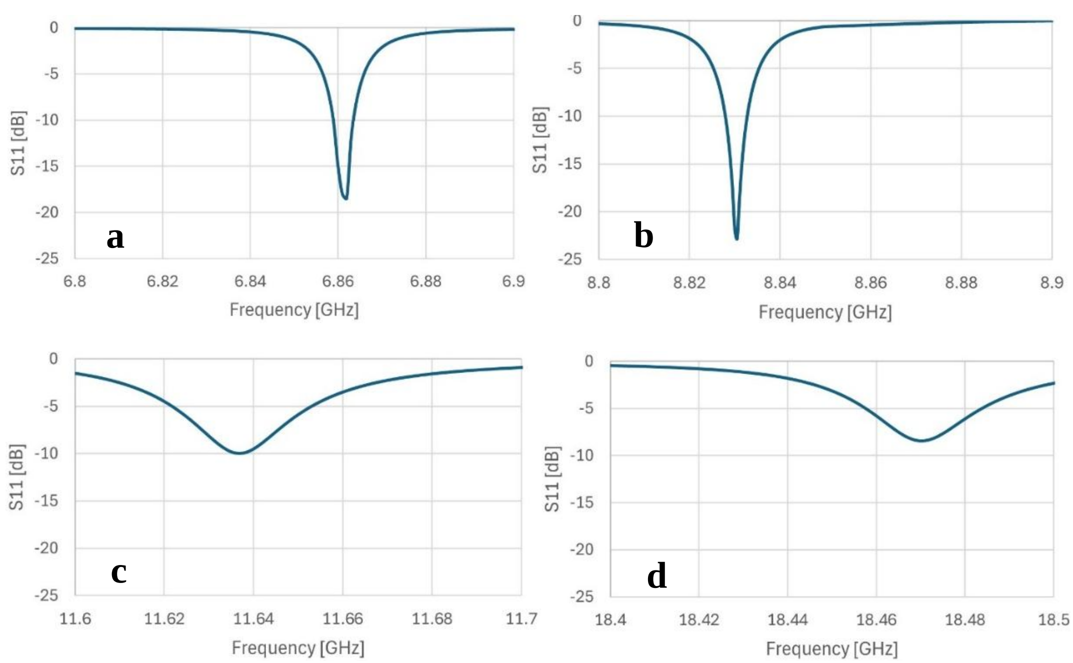

Fig. 6. Simulated S11 reflection coefficients of the cavity resonator: a) TM_{010} (loaded) b) TM_{110} (loaded), c) TM_{010} (empty) d) TM_{110} (empty).

Fig.6 presents the simulated S_{11} reflection coefficients obtained from driven-port simulations for both TM_{010} and TM_{110} modes, under empty-cavity and dielectric-loaded configurations. In the dielectric-filled case (Fig.6 a, b), both modes exhibit clear resonance dips, confirming that modes are stably excited. Notably, the TM_{110} mode achieves stronger coupling, with a resonance minimum of approximately *-23 dB*, compared to approximately *-18 dB* for TM_{010} mode. In the empty-cavity case (Fig. 6 c, d), the resonance dips become significantly weaker, with S_{11} minima remaining around *-10 dB* for TM_{010} and about *-9 dB* for TM_{110}, reflecting reduced coupling in the absence of dielectric loading. Overall, the driven-port results confirm that the TM_{110} is not only more effective in terms of dielectric participation (as shown by eigenmode analysis), but is also more efficiently excited under realistic coupling conditions. The corresponding resonant frequencies are summarized in Table 3.

Table 3. Resonant frequencies of TM_{010} and TM_{110} modes

Configuration	**F (TM_{010}) [GHz]**	**F (TM_{110}) [GHz]**
Empty cavity	11.64	18.47
Loaded cavity	6.86	8.83

The significant frequency reduction under dielectric loading results from the high relative permittivity ($\varepsilon_r = 9.5$) of the alumina. Because the resonant frequency scales approximately as [9]

$$f \propto \sqrt{\frac{1}{\varepsilon_{eff}}} \tag{6}$$

the increased effective permittivity of the cavity leads to the observed downshift, particularly for the TM_{110} mode due to its stronger field–dielectric overlap.

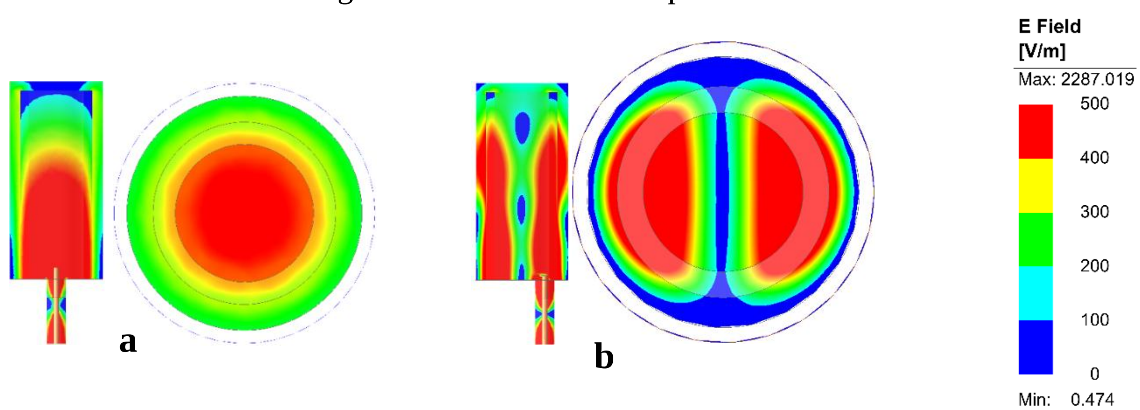

Fig. 7. Driven-port E-field distributions, in longitudinal and transverse planes
a) TM_{010} mode b) TM_{110} mode.

As shown in Fig. 7a TM_{010} electric field is concentrated along the cavity axis, which coincides with the hollow core of the annular dielectric; consequently, only a small fraction of the stored electric energy overlaps the dielectric volume. In contrast, the TM_{110} mode (Fig. 7b) exhibits off-axis electric-field maxima with strong radial and azimuthal components that fall within the annular region, indicating substantially higher dielectric interaction. Overall driven-port simulations demonstrate that appropriate probe placement enables selective excitation of the desired modes while accurately capturing dielectric-induced frequency shifts under practical excitation conditions. The results therefore establish a direct link between eigenmode field distributions and experimentally accessible S-parameter measurements.

4. Conclusions

This work presents a full-wave modal analysis of a cylindrical resonator with partial dielectric filling, focusing on the TM_{010} and TM_{110} modes under radially distributed dielectric loading. Eigenmode analysis showed that improved field overlap translates directly into larger eigenfrequency shifts per unit permittivity change. Driven-port simulations further demonstrated that each mode can be selectively excited through appropriate probe placement, with the TM_{110} mode showing stronger coupling and better port matching. These findings provide clear guidance for resonant mode selection and excitation strategies in cylindrical resonators with non-uniform dielectric filling and support future work on experimental validation and dielectric characterization.

References

[1] M. N. M. Kehn, L. Shafai, F. Safari, and S. Noghanian, "Permittivity measurement of disk and annular dielectric samples using coaxial transmission-line fixtures," Can. J. Electr. Comput. Eng., vol. 34, no. 1, pp. 31–41, 2009.

[2] "Method and apparatus for measuring dielectric properties using ring sample placement in a resonant cavity," Feb. 18, 2014. U.S. Patent 8,653,819 B2,

[3] P. Sharma, L. Lao, and G. Falcone, "A microwave cavity resonator sensor for water-in-oil measurements," Sens. Actuators B: Chem., vol. 262, pp. 200–210, 2018.

[4] R. R. Mett, J. W. Sidabras, I. S. Golovina, and J. S. Hyde, "Dielectric microwave resonators in TE011 cavities for electron paramagnetic resonance spectroscopy," Rev. Sci. Instrum., vol. 79, no. 9, Art. no. 094702, 2008.

[5] M. D. Janezic, U. Arz, S. Begley, and P. Bartley, "Improved permittivity measurement of dielectric substrates by use of the TE111 mode of a split-cylinder cavity," in Proc. 73rd ARFTG Microwave Measurement Conf., Boston, MA, USA, 2009.

[6] J. Zou, C.-J. Li, C. Zheng, et al., "A novel strategy for detecting permittivity and loss tangent of low-loss materials based on a cylindrical resonant cavity (TE111)," Sensors, vol. 23, no. 12, Art. no. 5469, 2023, doi: 10.3390/s23125469.

[7] W. Verhoeven, J. F. M. van Rens, A. H. Kemper, et al., "Design and characterization of dielectric-filled TM110 microwave cavities for ultrafast electron microscopy," Rev. Sci. Instrum., vol. 90, no. 8, Art. no. 083703, 2019.

[8] R. F. Harrington, Time-Harmonic Electromagnetic Fields. New York, NY, USA: McGraw-Hill, 1961.

[9] D. M. Pozar, Microwave Engineering, 4th ed. Hoboken, NJ, USA: Wiley, 2011.