Communications of BAO, Vol. 72, Issue 1, 2025, pp. 106-112

Residence time of spectral lines photons in the optically active media

A. Nikoghossian*

NAS RA V.Ambartsumian Byurakan Astrophysical Observatory, Byurakan 0213, Aragatsotn Province, Armenia

Abstract

As part of our recent series of works on the evolution of spectral lines formed in optically active media under the influence of non-stationary energy sources, the paper examines temporal changes in certain statistical averages characterising the radiation field in the medium under study. Line radiation transfer in 1D atmosphere of finite optical thickness is considered for monochromatic scattering. Of the statistical averages, the greatest attention is paid to the average residence time of various photons during diffusion in the medium. At the same time, the average number of scatterings occurring in the medium and the average number of photons undergoing thermalisation are also determined. The specified values, together with the profiles of the lines formed, are given for both photons reflected from the medium and photons passing through it. First, the problem of steady-state radiation is considered, after which the evolutionary picture of all the above-mentioned values under the action of non-stationary sources of illumination of the medium is described.

1. Introduction

In the series of papers (Nikoghossian, 2022, 2023, 2024), we studied the evolution of spectral lines formed in a finite medium in the presence of time-dependent energy sources. The effects of various physical factors such as frequency redistribution, medium inhomogeneity, existence of hydrodynamical motions were treated. The chronography of the observed spectral line profiles was created for a medium of some predetermined optical thickness and optical properties. However, in practice, for fully understanding the nature of the observed temporal changes, the information of averages on some statistical characteristics of the radiation diffusion process also is of some importance. Two of them, namely, the average number of scattering events underwent by reflected and transmitted photons, as well as the time of their residence in the medium are found.

In the paper these averages are studied for a finite medium where the transfer of radiation is accompanied by absorption and scattering both in the spectral line and continuum. In addition to the steady-state problem, we consider the line formation time-dependent problem due to temporal variations of external sources of energy. Here, we will limit ourselves to considering the case where the functional form of the change in energy of external sources over time has the form of the Dirac $\delta(t)-$ function.

The outline of the paper is as follows: we will begin in the opening Sect.2 by introducing the key functions constructing their Neumann expansions and setting the problem under consideration. The following (Sect.3) is a detailed description of the multiple scattering process in the steady-state regime.

2. Statement of the problem

We consider, for expository reasons, the transfer of radiation in a 1D absorbing and scattering atmosphere of finite optical thickness τ_0 calculated in the center of a given spectral line. One of its

*nikoghoss@yahoo.com

doi: https://doi.org/10.52526/25792776-25.72.1-106

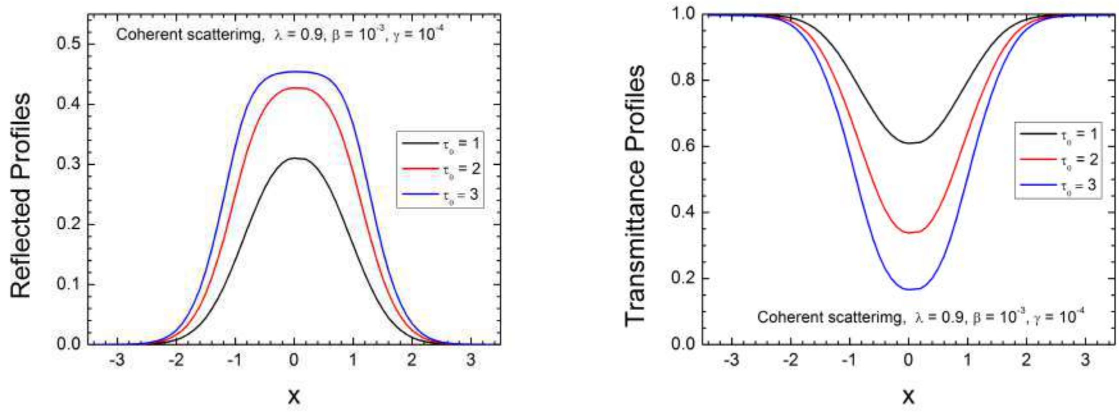

Figure 1. Profiles of the lines formed as a result of reflection from the medium (left panel) and those resulting from transmission through medium (right panel).

boundaries, e.g., $\tau = 0$, is illuminated by the continuous radiation flux $I_0(t)$ which undergoes multiple scattering with the proper coefficient λ. We assume that during diffusion in a medium, photons are scattered and absorbed also in a continuous spectrum. The interactions in the latter case are taken into account by introducing the parameters β and γ, which are defined as the ratio of the absorption and scattering coefficients in the continuum to the absorption coefficient at the centre of the spectral line, respectively. Scattering in the continuous spectrum will be considered independent of frequency, as is the case, for example, in such an important and frequently encountered in astrophysical applications case as Thomson scattering on free electrons. One of the advantages of the described formulation of the problem is that the influence of scattering in a continuous spectrum is taken into account relatively simply (see, for instance, Nikoghossian, 1986, 2022). First, we limit ourselves to considering the steady-state radiation regime in a medium, assuming that it is illuminated by radiation of unit intensity in a continuous spectrum. In addition to the reflection and transmission coefficients $\rho(x,\tau_0)$ and $q(x,\tau_0)$, we will also be interested in various statistical characteristics of the process of multiple scattering of radiation, such as the average number of scattered and transmitted photons, the mean time spent by these photons in the medium before they leave it, and the spectrum of photons that are thermalise in the medium during diffusion. Farther, these values are determined in cases where the medium is illuminated by a non-stationary energy source.

3. Steady state problem

Provided that the radiation falls on the boundary $\tau = 0$, applying the invariant imbedding technique leads to the following equations for the reflection and transmission coefficients

$$\frac{\partial \rho}{\partial \tau_0} = -2\bar{v}\left(x\right)\rho\left(x,\tau_0\right) + \frac{\tilde{\lambda}\left(x\right)}{2}\left[1+\rho\left(x,\tau_0\right)\right]^2, \tag{1}$$

and

$$\frac{\partial q}{\partial \tau_0} = -\bar{v}\left(x\right)q\left(x,\tau_0\right) + \frac{\tilde{\lambda}\left(x\right)}{2}\left[1+\rho\left(x,\tau_0\right)\right]q\left(x,\tau_0\right) \tag{2}$$

with initial conditions $\rho(x,0) = 0$ and $q(x,0) = 1$. The dimensionless frequency x is measured as displacement from the centre of the line in the Doppler half-widths, $\bar{v}\left(x\right) = \alpha\left(x\right) + \beta + \gamma$, $\tilde{\lambda}\left(x\right) = \left[\lambda\alpha\left(x\right) + \gamma\right]/\bar{v}\left(x\right)$ and $\alpha\left(x\right)$ is profile of the absorption coefficient in the spectral line. The profiles of the lines formed at the boundaries of the medium for $\tau_0 = 1, 2, 3$ are depicted in Fig.1.

The numerical values of the parameters in Fig.1, which are frequently used below, were selected based on their frequency of occurrence in the interpretation of line spectra observed in practice. Since the scattering is assumed to be monochromatic, the spectral composition of photons thermally excited in the medium can be directly derived from the above distributions and is shown in Fig.2.

doi: https://doi.org/10.52526/25792776-25.72.1-106

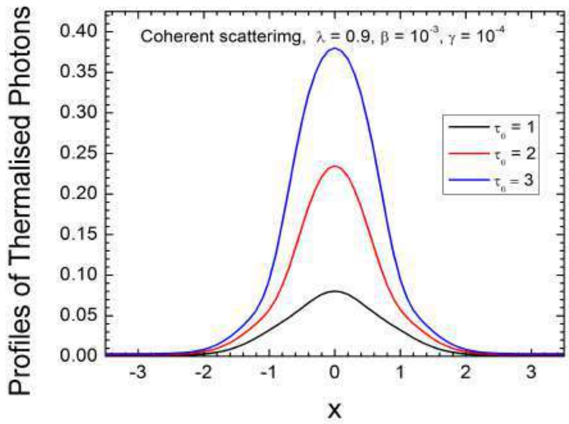

Figure 2. The spectrum of photons underwent thermalisation in the medium.

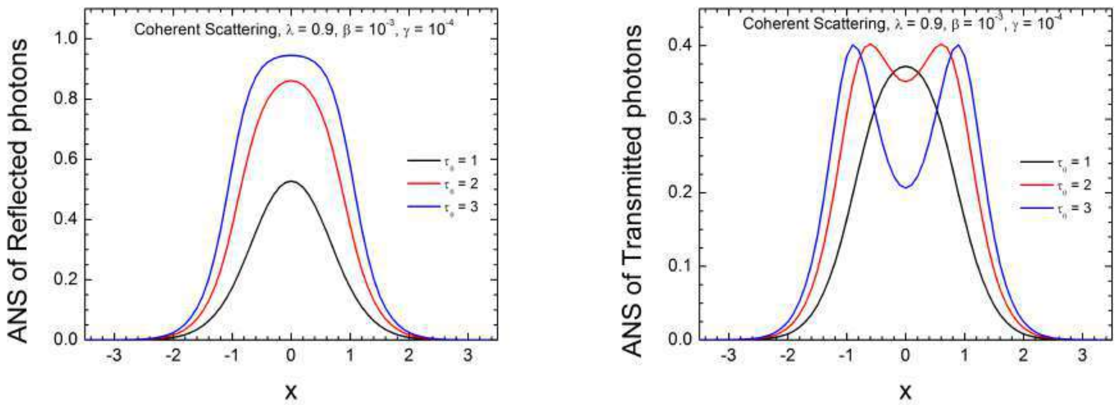

Figure 3. Average numbers of scattering acts correspondingly for photons reflected from the medium (left panel) and transmitted through it (right panel).

As might be expected, thermalisation mainly affects the quanta of central frequencies, which have a higher average number of scattering events. The frequency dependence of this latter quantity in media of different optical thicknesses is shown in Fig.2.

The average number of scatterings of quanta as they wander through the medium plays a significant role in the chronometry of spectral line changes. As is known, for both types of quanta (those reflected from the medium and those passing through it), it can be determined using the formulas of Amrtsumian (1948), but it can be easily determined numerically using the Neumann series expansion of the corresponding functions.

$$\langle N \rangle = \lambda \frac{\partial \ln I}{\partial \lambda} \tag{3}$$

A typical example of the frequency dependence of the average values of interest is shown in Fig. 3. It should be noted that this dependence is largely determined by the frequency redistribution law adopted in the description of multiple scattering in a medium. The greatest differences are found in the wings of the lines.

Scattering in the continuous spectrum also plays a significant role in the wings of the lines. A typical example of this effect is shown in Fig.4 at a value of $\gamma = 0.1$, which can be realised in astrophysical objects with a high degree of hydrogen ionisation.

Another important quantity determining multiple scattering of radiation in spectral line frequencies is the average residence time of quanta in the medium before they either leave the medium or are thermalised in it. In the problem under consideration here, there are only two possible ways for

doi: https://doi.org/10.52526/25792776-25.72.1-106

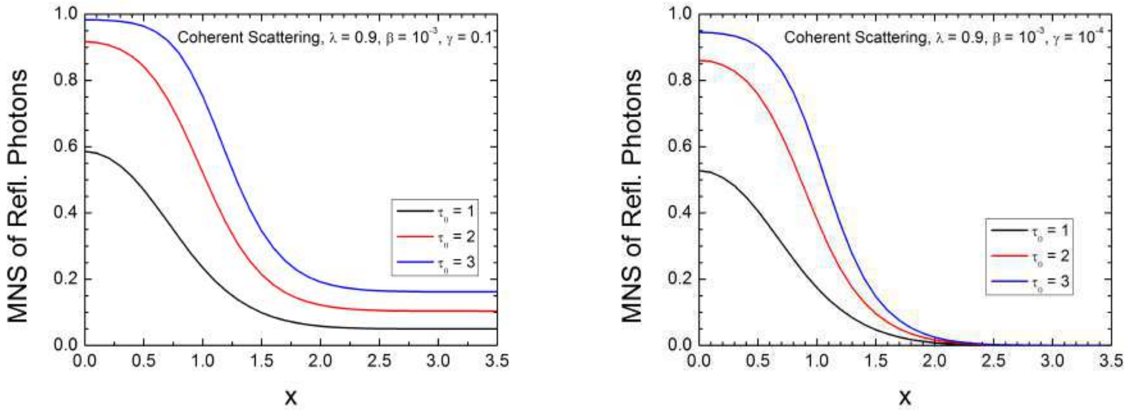

Figure 4. Average numbers of scattering acts correspondingly for photons reflected from the medium for $\gamma = 0.1$ (left panel) and $\gamma = 10^{-4}$ (right panel).

converting radiant energy into heat: either through non-radiative transitions in the line or in the continuous spectrum (as is often referred to as "absorption in flight"). The expression for the average residence time in the medium, taking into account the latter of these possibilities, is determined by the formula obtained by the author (Nikoghossian, 1986).

$$\langle T \rangle = -\frac{\partial \ln I}{\partial \beta} \tag{4}$$

Consequently, for the total number of thermalised quanta, in particular, we can write

$$(1-\lambda)\langle N \rangle + \lambda\beta \langle T \rangle = N_{term} \tag{5}$$

On the other hand, for the average values of the quanta residence time in the medium we may write separately for the reflected (Ω_ρ) and transmitted (Ω_q) quanta

$$\langle \Omega_\rho \rangle = \lambda \frac{\partial \ln \rho}{\partial \lambda}(t_1 + t_2) - \frac{\partial \ln \rho}{\partial \beta} t_2 \tag{6}$$

$$\langle \Omega_q \rangle = \lambda \frac{\partial \ln q}{\partial \lambda}(t_1 + t_2) - \frac{\partial \ln q}{\partial \beta} t_2 \tag{7}$$

where $t_1 \approx 10^{-7}$ sec is approximate time of the atom staying in the exited state (Sobolev, 1956). The accepted numerical value roughly corresponds to relatively strong lines in which multiple scattering plays an important role. The value of t_2 in the problem is determined by the method of specifying the optical thickness, $\tau_0 = nk_0L$, where k_0 is the coefficient of absorption for an atom/ion in the center of the line and L is the geometrical size of the medium and defines the overflight time of a given medium.

$$\frac{L}{c} = \frac{\tau_0}{nk_0c} = \tau_0 t_2, \tag{8}$$

i.e., t_2 is the overflight time of the optical thickness unit distance for a given line. Thus, the value t_2 appears implicitly in the equations of radiation transfer, hence in the following discussion, the time variables will be expressed in units of this value. At the same time, the formal presence of t_1 in this model problem can be considered valid only when its values are relatively small compared to t_2. A typical example of the average values of the residence time of reflected and transmitted quanta is shown in Fig.5. The curves shown in the left panel indicate a significantly larger range of variation in the average residence time of photons in the wings of the line during reflection. The sharp increase in the wings of the line is associated with the low probability of reflection from the medium of photons located at great depths. As a result, the curves corresponding to different optical thicknesses merge, which is characteristic of monochromatic scattering. At the same time, the relatively narrow range

doi: https://doi.org/10.52526/25792776-25.72.1-106

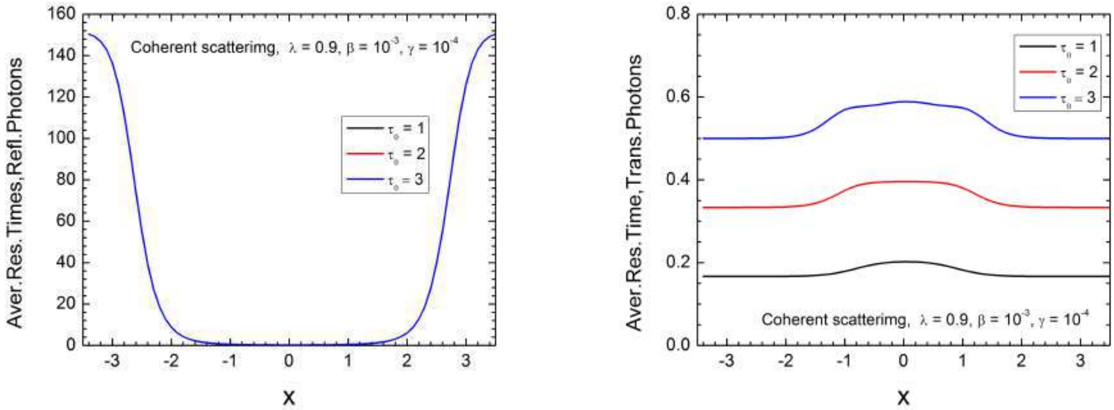

Figure 5. Average residence time of the lines formed as a result of reflection from the medium (left panel) and those resulting from transmission through medium (right panel).

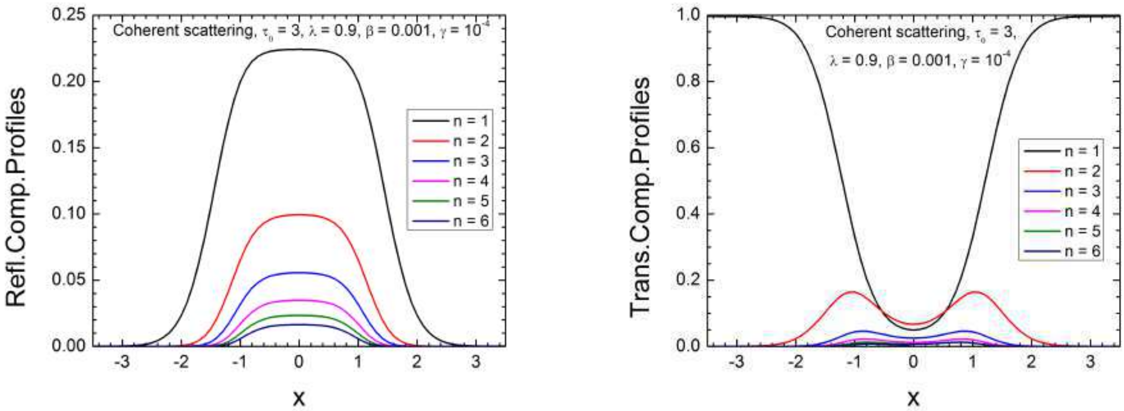

Figure 6. Components of the Neumann series of the reflection coefficient (left panel) and transmission coefficient (right panel).

of variation in the same average residence time in the medium for photons passing through it (right panel) is striking.

The profiles of the components of the above quantities for fixed values of the number of scattering events, i.e. the components of the corresponding Neumann series, are also important for interpreting the observational data.

Fig. 6 shows the profiles of the first six components of the reflection and transmission coefficients, which give an idea of the changes in the magnitude and shape of the profiles with increasing n. Significant changes in the shape of the absorption profiles occur in the case of radiation transmitted through the medium (left panel). We have shown in Nikoghossian (2024) that at various late stages of the development of non-stationary phenomena, the shapes of the observed spectral lines are determined by radiation regimes in which multiply scattered photons play a major role. The transformation of lines in the absorption spectra into weak, sometimes double-peaked profiles can occur after the cessation of external or internal energy sources. Fig. 7 depicts the frequency distribution of the average values of the time spent in the medium by reflected (left panel) and transmitted (right panel) photons as a function of n. The profiles shown have a double-peaked shape, with the exception of a single component in the right panel corresponding to photons freely transmitted through the medium. The latter, in the wings of the lines, in accordance with what was indicated in the previous section, tends towards the value $\tau_0 = 3$. The graphs also show that intermediate frequency quanta in the core of the spectral line wander the longest, and this applies to all components of both reflection and transmission functions.

doi: https://doi.org/10.52526/25792776-25.72.1-106

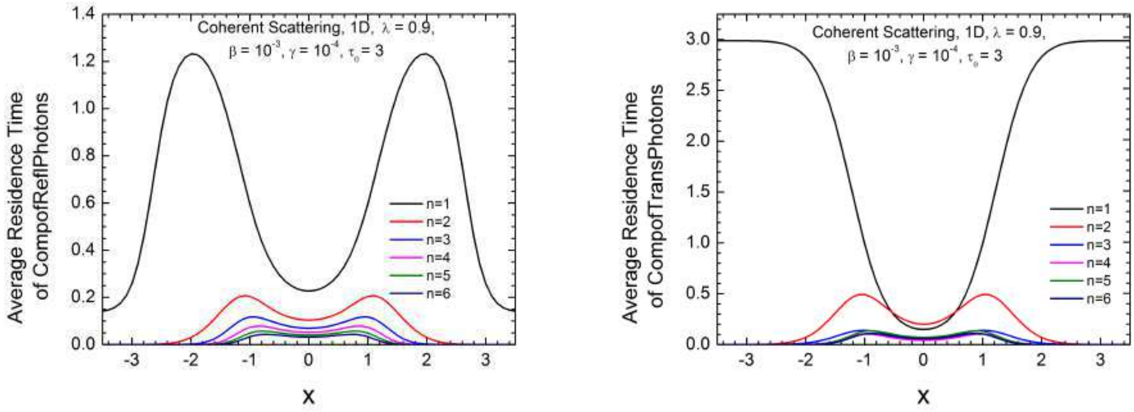

Figure 7. Average residence time of the lines formed as a result of reflection from the medium (left panel) and those resulting from transmission through medium (right panel).

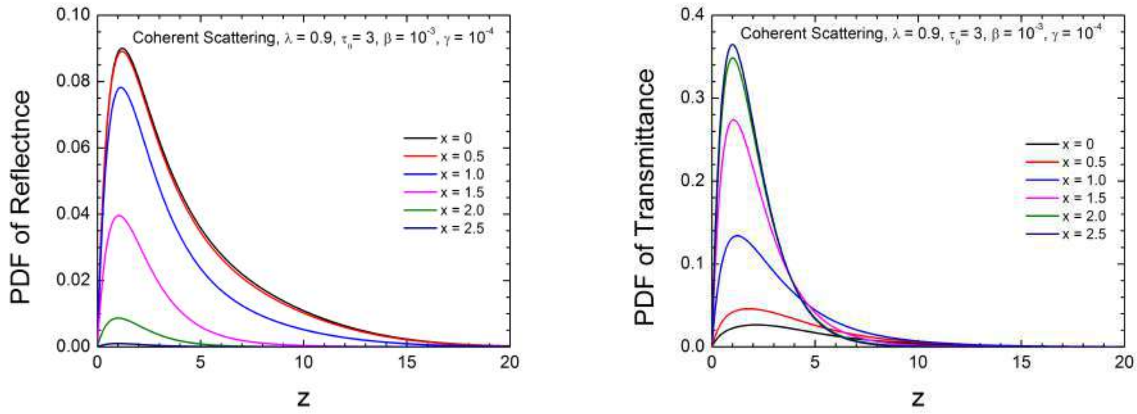

Figure 8. Probability density functions of reflectance (left panel) and transmittance (right panel).

4. Time-dependent problem

As we know, after constructing Neumann series, it is easy to determine the distribution functions of the probability densities of the quantities we are interested in. Recall that each of the components of these series corresponds to a specific form of a law describing changes over time, which is determined by the Erlang distribution, a special case of the gamma distribution (see, for example, Nikoghossian, 2022)

$$F_n\left(z\right) = \mathrm{e}^{-z} \frac{z^{2n-1}}{(2n-1)!}, \tag{9}$$

where z - a time variable defined in units of t_2.

For illustration purposes, we will limit ourselves to considering the case where the medium is illuminated by a non-stationary radiation flux in a continuous spectrum of the Dirac $\delta(t)$ function form. Fig. 8 demonstrates the temporal changes in the probability density functions of both the reflected quanta (left panel) and the quanta transmitted through the medium (right panel) at different frequencies of the spectral line. The curves shown contain complete information on the evolution of the spectral line with the given optical parameters. The curves in Fig. 9 describe changes in the average residence time of reflected (left panel) and transmitted (right panel) quanta of different frequencies in the medium for the non-stationary illumination under consideration. As we can see, changes in the values of the statistical averages under consideration occur in relatively small intervals.

doi: https://doi.org/10.52526/25792776-25.72.1-106

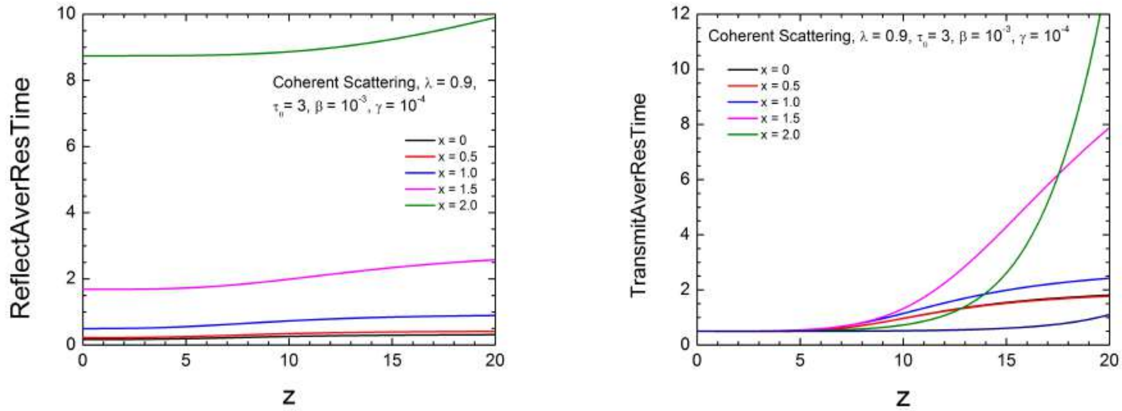

Figure 9. Probability density functions of the average residence time (ART) of reflected photons (left panel) and transmitted photons (right panel).

5. Concluding remarks

This work should be considered as a continuation of our previous series of works, which develops a theory devoted to the study of temporal changes in line spectra formed in the presence of various types of non-stationary processes in observed objects related to the active medium, its dynamics, energy sources and their localisation, etc. In order to understand the wide variety of data obtained, it is necessary to study the theoretical characteristics of the formation of spectral lines themselves, both absorption and emission. Most of the information about the observed objects and phenomena in the line spectrum comes mainly from strong lines, in which multiple scattering occurs. The greatest attention in the literature describing this process has been devoted to determining the average number of scatterings.

In addition to this value, other average statistical characteristics are also considered, the most important of which is the average time of formation of spectral lines, both emission and absorption. A general formula is derived for the average statistical value of the residence time of quanta in the medium at line frequencies, both those that are reflected and those that are transmitted by the medium. The formula obtained in this work is used both when considering the equilibrium regime and in the case of illumination of the medium by non-stationary energy sources.

References

Amrtsumian V. A., 1948, DAN ArmSSR, 8, 101

Nikoghossian A. G., 1986, Astrophysics, 24, 149

Nikoghossian A., 2022, Communications of the Byurakan Astrophysical Observatory, 69, 140

Nikoghossian A. G., 2023, Communications of the Byurakan Astrophysical Observatory, 70, 131

Nikoghossian A., 2024, Communications of the Byurakan Astrophysical Observatory, 71, 215

Sobolev V. V., 1956, Transfer of Radiative Energy in Stellar and Planetary Atmospheres . [in Russian], Gostechizdat, M.,

doi: https://doi.org/10.52526/25792776-25.72.1-106