Quantum Gravitational States of Ultracold Hydrogen near Liquid Helium Surface

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Abstract. We study quantum gravitational states of ultracold hydrogen atoms in a combined potential consisting of H – ^{4}He surface interaction potential and gravitational potential of the Earth. The key feature of this system is the dramatic separation of spatial (~10^4) and energy (~10^{-9}) scales between potential components. We show that due to this separation, the influence of H – ^{4}He surface interaction on gravitational states can be described with high accuracy by a boundary condition at zero, expressed through the scattering length. Our main result is the analysis of the near-resonant case, where the scattering length becomes large, revealing pronounced effects on gravitational states and highlighting the novel physical regimes accessible in this system.

Keywords: quantum states, ultracold hydrogen, gravitational potential, helium surface, scattering length

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1. Introduction

Let us consider the quantum-mechanical description of a hydrogen atom of mass m in the gravitational field of the Earth above a perfectly reflecting surface. Unlike a classical particle, which inevitably falls and bounces off the surface, a quantum system exhibits a fundamentally different picture [1, 9]: the Earth's gravitational attraction and the hard reflection from the ideal mirror form a triangular potential that supports a discrete spectrum of stationary states localized near the surface.

If the mirror is located at height $z=0$, the Schrödinger equation in this potential takes the form:

$$-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}\psi(z)+mgz\psi(z)=E\psi(z) \tag{1}$$

with the boundary condition $\psi(z=0)=0$, where $\psi(z)$ is the coordinate-space wave function of the hydrogen atom.

It is convenient to introduce characteristic length and energy scales for the quantum gravitational states of a particle of mass m:

$$l_g=\left(\frac{\hbar^2}{2m^2g}\right)^{1/3} \tag{2}$$

$$E_g=mgl_g \tag{3}$$

which serve as natural units for the problem.

For atomic hydrogen, the numerical values are:

$$l_g \approx 6\mu m, \quad E_g \approx 0.6 peV \tag{4}$$

With these notations, the general solution of the equation can be expressed in terms of Airy functions:

$$\psi(z) = C_1 \cdot Ai\left(z/l_g - E/E_g\right) + C_2 \cdot Bi\left(z/l_g - E/E_g\right) \tag{5}$$

where $Ai(z)$ and $Bi(z)$ are the Airy functions of the first and second kind, respectively, and C_1, C_2 are arbitrary constants.

From the normalizability condition of the wave function as $z \to \infty$, it follows that $C_2 = 0$. The boundary condition at $z = 0$ leads to energy quantization:

$$Ai\left(-\frac{E_n}{E_g}\right) = 0 \tag{6}$$

Thus, the wave functions and energies of the n-th gravitational level are given by:

$$\psi_n(z) = C_n Ai\left(\frac{z}{l_g} - \frac{E_n}{E_g}\right), \tag{7}$$

$$E_n = \lambda E_g \tag{8}$$

where λ_n are the negative zeros of the Airy function satisfying $Ai(-\lambda_n) = 0$.

2. Interaction of a Hydrogen Atom with the Surface of Liquid Helium

A hydrogen atom near the surface of ^{4}He experiences a complex interaction, which can conveniently be decomposed into two fundamentally different components [2, 4]:

1. Short-range adsorption potential $V_{ads}(z)$ is characterized by a depth of about 5K and a characteristic spatial scale of approximately 10Å. This component forms a single adsorption level: the atom “sticks” to the surface without penetrating into the liquid. The binding energy of this level is approximately -1K.

2. Van der Waals–Casimir–Polder potential $V_{vdW}(z)$ dominates at distances of tens to hundreds of angstroms compared to $V_{ads}(z)$ and determines the long-range characteristics of the interaction.

A key physical factor is the large separation in both energy and spatial scales between the adsorption and gravitational levels: their energy ratio is on the order of 10^9, and the spatial ratio is about 10^{-4}. This extreme scale separation makes it possible to replace the complex short-range potential with an effective boundary condition at the surface. The real part of this boundary condition determines the overall shift of all gravitational levels, while the imaginary part accounts for their finite widths.

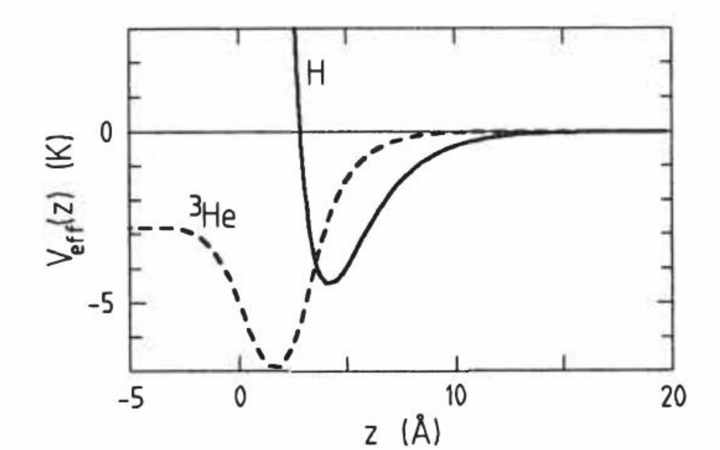

Figure 1: Effective potential of a hydrogen atom above helium [2]

The adsorption potential is traditionally described by the Morse model [4, 5], and at distances of about 10Å it is smoothly matched to $V_{vdW}(z)$:

$$V_{ads}(z)=\begin{cases} D_e\left[e^{-2\beta(z-z_e)}-2e^{-\beta(z-z_e)}\right], & z<z_c \\ V_{vdW}^{He}(z), & z\geq z_c \end{cases} \tag{9}$$

The Van der Waals potential exhibits a characteristic asymptotic behavior determined by the relationship between the distance and the characteristic wavelength of atomic transitions:

$$V_{vdW}(z)=\begin{cases} -C_3/z^3, & z\ll\lambda_A \\ -C_4/z^4, & z\gg\lambda_A \end{cases} \tag{10}$$

where $\lambda_A\approx 121\,\mathrm{nm}$ is the characteristic wavelength of transitions in the hydrogen atom. For the H–He interaction: $C_3^{He}=0.0045,\ C_4^{He}=1.55$ (in atomic units).

It has been shown that over the entire range of distances, the Van der Waals potential can be approximated with high accuracy by the expression [8, 10]:

$$V_{vdW}(z)\approx\frac{-W(1+0.22y)}{y^3\left(1+0.95y+0.22y^2\right)} \tag{11}$$

where

$$W=\frac{C_3^4}{C_4^3}\qquad y=z\frac{C_3}{C_4} \tag{12}$$

3. Boundary Conditions for Gravitational States

The concept of long-lived gravitational states of quantum particles was first experimentally confirmed in the work of V.V. Nesvizhevsky and co-authors through the observation of quantum gravitational states of neutrons [1]. These studies demonstrated that light quantum particles can be bound by a combination of the gravitational potential and the particle–surface interaction potential, possessing sufficiently long lifetimes to be observed experimentally.

Subsequent theoretical works predicted the existence of long-lived gravitational quantum states of (anti)hydrogen, whose characteristics are practically independent of the short-range atom–surface interaction due to the phenomenon of quantum reflection [3]. The extreme separation of scales between the gravitational potential ($l_g \approx 6\mu m$) and the atom–surface potential ($l_{vdW} \approx 500 a.u. \approx 26 nm$) allows one to describe the interaction of the atom with the surface via a boundary condition in the intermediate region $l_{vdW} \ll z_0 \ll l_g$. This boundary condition connects the asymptotic behavior of the wave function in the region dominated by the atom–surface interaction with that in the region dominated by the gravitational potential.

In the atom–surface interaction region, the asymptotics of the wave function for $z_0 \gg l_{vdW}$ takes the form:

$$\frac{\psi}{d\psi / dz}\left(z = z_0\right) \sim z_0 - a \tag{13}$$

where a is the complex scattering length, fully determined by the atom–surface potential.
In the gravitational region, the wave function is described by the Airy function:

$$\psi_n\left(z\right) \sim Ai\left(\frac{z}{l_g} - \lambda_n\right) \tag{14}$$

Matching these solutions leads to the equation for the eigenvalues of gravitational states:

$$\frac{z_0 - a}{l_g} = \frac{Ai\left(\frac{z_0}{l_g} - \lambda_n\right)}{Ai'\left(\frac{z_0}{l_g} - \lambda_n\right)} \tag{15}$$

For typical scattering length values $a \ll l_g$, one can use the wave function expansion:

$$\frac{\psi}{d\psi / dz}\left(z = z_0\right) = z_0 - \frac{\Delta E}{mg} \tag{16}$$

where ΔE is the energy shift of the gravitational state due to the change in the boundary condition compared to the zero boundary condition at the origin.

For hydrogen, the corresponding scattering length is real (neglecting inelastic interactions with ripplons [6, 7]). For a helium film, numerical results show that the scattering length is:

$$a_H = -18.406 a.u. \tag{17}$$

and the gravitational level shift is:

$$\Delta E_H \sim 10^{-16} eV \tag{18}$$

The obtained results show that the shift of the gravitational levels is small and the same for all states for typical scattering length values for the $H / \bar{H}$–liquid He potential.

4. Zero-Energy Resonance Case

If it becomes possible to modify the helium surface potential in such a way that a bound state is tuned exactly to the threshold, a zero-energy resonance will occur. As is well known from scattering theory, in this situation the scattering length formally diverges:

$$|a(E)| \sim \frac{\hbar}{\sqrt{2m|E_b|}} \to \infty \tag{19}$$

where E_b is the binding energy of the near-threshold state.

To study the situation $|a| \gg l_g$, it is convenient to rewrite equation (15) in the form

$$Ai'\left(\frac{z_0}{l_g} - \lambda_n\right) = \frac{l_g Ai\left(\frac{z_0}{l_g} - \lambda_n\right)}{z_0 - a} \tag{20}$$

The requirement that the solution be independent of the matching point z_0 leads to the condition:

$$Ai'(-\lambda_n) = -\frac{l_g Ai(-\lambda_n)}{a} \tag{21}$$

From equation (20) it follows that in the case of a zero-energy resonance $\left(|a|/l_g \gg 1\right)$, the boundary condition for the eigenvalues of gravitational states takes the limiting form

$$Ai'(-\Lambda_n) = 0 \tag{22}$$

which means a complete restructuring of the gravitational spectrum. A comparison of the energy levels in the ordinary and resonant cases is given in the Table 1:

Level	Ordinary case λ_n	Resonant case Λ_n
1	2.338	1.407
2	4.088	2.461
3	5.521	3.324
4	6.787	4.086

Table 1: Comparison of the energy levels in the ordinary and resonant cases

Using the properties of the Airy function, we obtain an expression for the shift of eigenvalues near resonance in the form of an expansion in the small parameter l_g / a:

$$\Delta E_n' = \frac{mgl_g^2}{a\Lambda_n} \tag{23}$$

5. Conclusion

We have analyzed the quantum gravitational states of ultracold hydrogen atoms in the presence of a combined potential consisting of the Earth's gravitational field and the $H-He$ surface interaction. The extreme separation of spatial and energy scales between the two potential components allows the complex short-range atom–surface interaction to be replaced by an effective boundary condition characterized by a single complex scattering length. This approach yields a simple and accurate description of the gravitational spectrum and its small shifts induced by the surface interaction.

For hydrogen above liquid helium, the calculated scattering length leads to an energy shift on the order of 10^{-16} eV, which is the same for all gravitational states. In the case of antihydrogen, the scattering length acquires a significant imaginary part, leading to finite state lifetimes, but the magnitude of the energy shifts remains extremely small. Such negligible perturbations confirm that the gravitational states are robust and largely unaffected by the microscopic details of the adsorption potential, except in near-resonant cases where the scattering length becomes large.

Importantly, in near-resonant cases where the scattering length becomes large, the gravitational states are significantly affected, demonstrating a novel physical regime not previously analyzed. These results demonstrate that ultracold (anti)hydrogen above a liquid helium surface represents a promising system for precision measurements of gravitational quantum states, and potentially for tests of fundamental symmetries in physics.

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