ՀԱՅԱՍՏԱՆԻ ԳԻՏՈՒԹՅՈՒՆՆԵՐԻ ԱԶԳԱՅԻՆ ԱԿԱԴԵՄԻԱՅԻ ՏԵՂԵԿԱԳԻՐ
ИЗВЕСТИЯ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Մեխանիկա 78, №2, 2025 Механика

УДК 539.3 DOI: 10.54503/0002-3051-2025.78.2-79

# SURFACE SH WAVES ON AN INTERFACE OF HOMOGENEOUS PIEZOELECTRIC AND BI- MATERIAL MULTILAYERED ELASTIC HALF- SPACES

**Ghazaryan K., Ghazaryan R., Terzyan S.**

**Key words:** Multilayered periodic structure, surface wave, Bleustein wave.

This paper investigates the existence of elastic SH wave surface modes near the interface between a bi-material periodic multilayer elastic half-space in perfect elastic contact with a homogeneous piezoelectric half-space. The results demonstrate that surface wave modes are present when the shear wave velocities of the elastic materials are lower than that of the piezoelectric material.

**Казарян К., Казарян Р., Терзян С.**

**Поверхностные SH волны на границе раздела однородного пьезоэлектрического полупространства и двухкомпонентного многослойного упругого полупространства**

**Ключевые слова:** Многослойная периодическая структура, поверхностная волна, волна Блюстейна.

Изучается проблема существования поверхностной SH волны на границе двухслойного упругого полупространства, находящегося в идеальном упругом контакте с однородным пьезоэлектрическим полупространством. Показано, что поверхностная волна существует только тогда, когда скорости сдвиговых волн упругих материалов меньше скорости сдвиговой волны пьезоэлектрического материала.

**Ղազարյան Կ., Ղազարյան Ռ., Թերզյան Ս.**

**Մակերևութային SH ալիքները համասեռ իեզոէլեկտրականկիսատարածության և երկբաղադրիչ բազմաշերտն առաձգական կիսատարածության բաժանման եզրին**

.

**Հիմնաբառեր.** բազմաշերտ պարբերական կառուցվածք, մակերևութային ալիք, Բլյուստեյնի ալիք:

Ուսումնասիրվում է մակերևութային SH ալիքի գոյության խնդիրը երկշերտ առաձգական կիսատարածության և համասեռ պիեզոէլեկտրական կիսատարածության իդեալական առաձգական կոնտակտի եզրում: Ցույց է տրված, որ մակերևութային ալիքը գոյություն ունի միայն այն դեպքում, երբ առաձգական նյութերի սահքի ալիքների արագությունը փոքր է պիեզոէլեկտրական նյութի սահքի ալիքի արագությունից:

## Introduction

Surface acoustic waves in phononic structures have garnered significant scholarly interest over the past several decades. The existence and properties of one-dimensional transverse acoustic waves in semi-infinite, periodic bi-material media has investigated in [1,2].

The propagation of surface waves was examined [3] in a semi-infinite superlattice composed of periodic bi-material piezoelectric-metallic layers, topped with a piezoelectric layer.

The paper [4] discusses surface acoustic waves propagating in half-infinite one-dimensional piezoelectric phononic crystals with general anisotropy.

The propagation of shear horizontal polarization acoustic waves in infinite and semi-infinite superlattices composed of two piezoelectric media is examined using a Green's function method [5].

In [6] an analytical method is presented for studying shear horizontal surface acoustic waves in semi-infinite piezoelectric/metal superlattices.

The existence of surface acoustic waves is discussed semi-infinite one-dimensional piezoelectric phononic structure consisting of perfectly bonded generally anisotropic layers, which are arranged in a symmetric unit cell.

The surface wave modes on traction-free or clamped surfaces have been identified in a cubic symmetry piezoelectric homogeneous half-space with periodically arranged, non-equidistant electrodes, as reported in [8]. In [9] shear elastic surface wave modes are investigated in a semi-infinite medium with periodically oriented stacks of interfaces of elastic imperfect bonded contact. It has been shown that this periodic structure supports the propagation of surface waves.

The studies in [10] examine various aspects of surface wave propagation in piezoelectric composites with both perfect and imperfect electro-elastic interfaces.

Various methods and models of imperfectly bonded interfaces in elastic and electro-magneto-elastic composite structures, as well as problems based on these models, are discussed in [11-15].

This paper presents an analytical and numerical investigation of surface shear waves at the interface between a homogeneous elastic bi-metal half-space with periodic elastic sublayers and a homogeneous half-space composed from hexagonal piezoelectric crystals.

**Statement of the problem**

In Cartesian coordinate system $(x, y, z)$ we consider bi-material layered elastic half-space in contact with homogeneous piezoelectric half-space. Layered elastic half- space $(x \in (0, \infty), |y| < \infty, |z| < \infty)$ is constituted by an infinite number of repeated two alternative sub-layers of widths $d_1, d_2$ made from different elastic metal materials $A_1, A_2$.

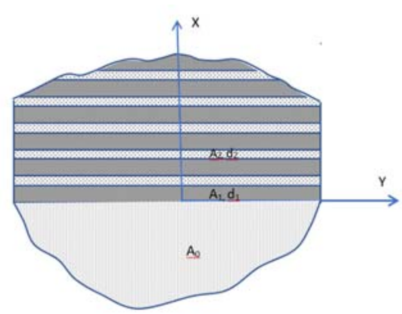


*Fig.1. Bi-material layered elastic half-space in contact with homogeneous piezoelectric half-space.*

We consider also the semi-space $\left(x \in (-\infty, 0), |y| < \infty, |z| < \infty\right)$ from the homogeneous piezoelectric material $A_0$ of hexagonal $6mm$ symmetry, the polling axis of the piezoelectric crystals is aligned along. At the interface $x = 0$, the semi-spaces are perfectly bonded, ensuring continuity of both elastic displacements and stresses.

**Multi-layered bi-material elastic half-space**

Consider SH wave propagation in a multi-layered elastic bi-material half-space constituted by an infinite number of repeated two sub-layers consisting from different elastic materials. Each of these sub-layers of widths $d_1, d_2$ and materials are labelled by the index $(s) = 1, (s) = 2$ within the elementary unit cell labelled by the index $n(n = 1, 2, 3\ldots)$, $x \in \left((n-1)d, nd\right)$, $d = d_1 + d_2$. Each of the two sub-layers is assumed to be perfectly bonded to the adjoining sub-layers.

The elastic displacements and stresses obey to the anti-plane equations of motion and Hooke's law.

$$\frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{yz}}{\partial y} = \rho \frac{\partial^2 U_z}{\partial t^2},$$

$$\sigma_{xz} = G \frac{\partial U_z}{\partial x}, \sigma_{yz} = G \frac{\partial U_z}{\partial y} \tag{1}$$

Here $U_z(x, y, t)$ is the displacement in $z$ – direction, $\sigma_{xz}(x, y, t), \sigma_{yz}(x, y, t)$ are the shear stresses.

We consider harmonic wave travelling along the $y$ direction,

$$U_z(x, y, t) = U(x) \exp\left[i(ky - \omega t)\right], \tag{2}$$

where $\omega$ is the wave angular frequency, $k$ is the wave number.

The solutions of (1) for functions $U_n^{(s)}(x)$ within each material $A_1, A_2$ domains of the sub-layer material can be found as

$$U_n^{(s)}(x) = \alpha_{ni}^{(s)} \exp(iq_s x) + \alpha_{nr}^{(s)} \exp(-iq_s x) \tag{3}$$

Here materials are labelled by the index $(s) = 1, (s) = 2$ within the elementary unit cell are labelled by the index $n(n = 1, 2, 3\ldots)$, $x \in \left((n-1)d, nd\right)$, $d = d_1 + d_2$, $q_s = k\sqrt{\frac{\omega^2}{k^2 c_s^2} - 1}, c_s = \sqrt{\frac{G_s}{\rho_s}}$ is the material shear wave velocity, $G_s$ is the shear modulus, $\rho_s$ is the bulk density of materials, $\alpha_{ni}^{(s)}, \alpha_{nr}^{(s)}$ are the complex amplitudes of plane waves.

Introducing field vector

$$\mathbf{U}_n^{(s)}(x) = \begin{pmatrix} U_n^{(s)}(x) \\ \sigma_{xzn}^{(s)}(x) \end{pmatrix}, \tag{4}$$

we can establish the link between $U_n^{(s)}((n-1)d), U_n^{(s)}(nd)$ at the elementary unit cell edges as [16],

$$U_n^{(2)}(nd) = \mathbf{M}U_n^{(1)}((n-1)d) \tag{5}$$

Herein $\mathbf{M}$ is the unimodal propagator matrix for SH wave field, which links the field vectors at the at edges of elementary unit cell $n$ -*th* cell.

The explicit expressions of the unimodal propagator matrix $\mathbf{M}$ elements be derived as [16]

$$\begin{aligned}
m_{11} &= \cos(d_1 q_1)\cos(d_2 q_2) - \frac{q_1 G_1}{q_2 G_2}\sin(d_1 q_1)\sin(d_2 q_2), \\
m_{12} &= \frac{\cos(d_1 q_1)\sin(d_2 q_2)}{q_2 G_2} + \frac{\cos(d_2 q_2)\sin(d_1 q_1)}{q_1 G_1}, \\
m_{21} &= -q_2 G_2 \cos(d_1 q_1)\sin(d_2 q_2) - q_1 G_1 \cos(d_2 q_2)\sin(d_1 q_1) \\
m_{22} &= \cos(d_1 q_1)\cos(d_2 q_2) - \frac{q_2 G_2}{q_1 G_1}\sin(d_1 q_1)\sin(d_2 q_2).
\end{aligned} \tag{6}$$

Using formality of the Floquet theory [16] we have

$$\begin{aligned}
U_n^{(2)}(nd) &= \lambda U_n^{(1)}((n-1)d) \\
U_n^{(2)}(nd) &= \lambda^n U_n^{(1)}(0)
\end{aligned} \tag{7}$$

where $\lambda = \exp(ipd)$, $p$ is the Bloch-Floquet wave number.

In the case of the real $\lambda$ (complex $p$) if $|\lambda| < 1$ according to (7) we have the exponentially damped solution corresponding to the surface waves.

Taking into account (7) the following matrix equation can be obtained

$$(\mathbf{M} - \lambda \mathbf{I})U_n^{(1)}((n-1)d) = 0 \tag{8}$$

Where $\mathbf{I}$ is the identity matrix.

Using this equation for the $n = 1$ unit cell, we come to the matrix equation

$$\begin{pmatrix} m_{11} - \lambda & m_{12} \\ m_{21} & m_{22} - \lambda \end{pmatrix} \begin{pmatrix} U_1^{(1)}(0) \\ \sigma_{xzn}^{(1)}(0) \end{pmatrix} = 0 \tag{9}$$

In the case of the traction free surface $\sigma_{xzn}^{(s)}\left(0\right)=0$, from the non-trivial solutions of (9) $U_1^{(1)}\left(0\right)\neq 0$ one can find

$$m_{21}=0;\ m_{11}-\lambda=0 \tag{10}$$

The equation

$$m_{21}\left(\omega,k\right)=0 \tag{11}$$

for a given wave number $k$ defines the countable set solutions of $\omega_{j0}\left(k\right),\ j=1,2,\ldots$, determining the surface wave frequencies if

$$\lambda=\left|m_{11}\left(\omega\left(k\right)\right)\right|<1 \tag{12}$$

Considering (11) we can obtain that

$$\lambda=\frac{\cos\left(d_1q_1\right)}{\cos\left(d_2q_2\right)}$$

in full accordance with results of [2].

**Homogeneous semi-infinite piezoelectric half space**

We consider the anti-plane problem #for hexagonal $6mm$ symmetry piezoelectric material. In quasi- static set of Maxwell equations [10,17] we have the following equations and constitutive relations

$$V_z=V_z\left(x,y\right);\varphi=\varphi\left(x,y\right)$$

$$\frac{\partial D_x}{\partial x}+\frac{\partial D_y}{\partial y}=0,\frac{\partial\sigma_{xz}}{\partial x}+\frac{\partial\sigma_{yz}}{\partial y}=\rho\frac{\partial^2V_z}{\partial t^2}$$

$$\sigma_{xz}=\frac{\partial}{\partial x}\left(c_{44}V_z+e_{15}\varphi\right)\ ,\ \sigma_{yz}=\frac{\partial}{\partial y}\left(c_{44}V_z+e_{15}\varphi\right) \tag{13}$$

$$D_x=\frac{\partial}{\partial x}\left(-\varepsilon_{11}\varphi+e_{15}V_z\right)\ ;D_y=\frac{\partial}{\partial y}\left(-\varepsilon_{11}\varphi+e_{15}V_z\right)$$

In (13) $\sigma_{xz},\sigma_{yz},D_x,D_y,V_z,\varphi$ are shear stresses, electric displacements, elastic displacement and electric potential, respectively and $c_{44},e_{15},\varepsilon_{11}$ and $\rho$ are the shear modulus, piezoelectric constant, permittivity and bulk density of the material, respectively.

Taking solutions (13) in the form of harmonic wave travelling along the $y$ direction (2), for the displacement and the electric field potential decaying at $x\to-\infty$

$$V_0\left(x\right)\to 0,\varphi_0\left(x\right)\to 0 \tag{14}$$

we get solutions for $V_0\left(x\right),\varphi_0\left(x\right),\sigma_0\left(x\right)$ as

$$V_0\left(x\right)=C_1\exp\left(rx\right),$$

$$\varphi_0(x) = C_2 \exp(kx) + C_1 \frac{e_{15}}{\varepsilon_{11}} \exp(rx) \tag{15}$$

$$\sigma_{xz0}(0) = \sigma_0(x) = C_2 k e_{15} \mathbf{e}^{kx} + C_1 G_0 r \mathbf{e}^{rx}$$

Here $C_1, C_2$ are arbitrary constants, $r = k\sqrt{1-\eta^2}; G_0 = c_{44} + e_{15}^2 / \varepsilon_{11}$, $c_0 = \sqrt{G_0 / \rho}$; $\eta = \omega / kc_0$ , $\eta$ is the dimensionless phase velocity of electro-elastic vibrations.

Applying condition $\varphi_0(0) = 0$ at metalized surface we get $C_2 = -C_1 e_{15} / \varepsilon_{11}$

Finally, for the field vector at $x = 0$ we have

$$V_0(0) = \begin{pmatrix} V_0(0) \\ \sigma_0(0) \end{pmatrix} = C_1 \begin{pmatrix} 1 \\ kG_0\left(\sqrt{1-\eta^2} - \chi^2\right) \end{pmatrix} \tag{16}$$

Here $\chi = \dfrac{e_{15}}{\sqrt{G_0 \varepsilon_{11}}}$ is the electromechanical coupling constant.

In the case of the traction free surface $\sigma_{xz0}(0) = 0$ we get from (16) the equation defining the dimensionless phase velocity of the Bluestein wave [17]

$$\eta_0 = \sqrt{1-\chi^4}$$

**Bi- material layered elastic half- space in contact with homogeneous piezoelectric half- space.**

On the interface of bi-material layered elastic and homogeneous piezoelectric half-spaces $x = 0$ we have continuity condition for displacements and elastic stresses

$$\begin{pmatrix} U_1^{(1)}(0) \\ \sigma_{xz1}^{(1)}(x0) \end{pmatrix} = \begin{pmatrix} V_0(0) \\ \sigma_0(0) \end{pmatrix} \tag{17}$$

Considering (16) we obtain the following matrix equation

$$\begin{pmatrix} m_{11} - \lambda & m_{12} \\ m_{21} & m_{22} - \lambda \end{pmatrix} \begin{pmatrix} 1 \\ G_0 k\left(\sqrt{1-\eta^2} - \chi^2\right) \end{pmatrix} = 0 \tag{18}$$

From (18) the set of equations determining phase velocity $\eta$ and localization parameter $\lambda$ can be cast as

$$\lambda = m_{11} + k m_{12}\left(\sqrt{1-\eta^2} - \chi^2\right) G_0 \tag{19}$$

$$m_{21}-k\left(m_{11}-m_{22}\right)\left(\sqrt{1-\eta^2}-\chi^2\right)G_0+k^2m_{12}\left(-1+\eta^2+2\chi^2\sqrt{1-\eta^2}-\chi^4\right)G_0^2=0$$

The matrix elements (6) $m_{11}\left(q_s\right), m_{12}\left(q_s\right), m_{21}\left(q_s\right), m_{22}\left(q_s\right)$, are functions from $\eta$, since

$$q_s=k\sqrt{\frac{c_0^2\eta^2}{c_s^2}-1}\;; \tag{20}$$

If solutions of (19) exist, such as $\eta<1, \left|\lambda\right|<1$, these solutions describe coupled electro-elastic surface waves that decay exponentially at $x\to\pm\infty$ from the interface $x=0$.

The positive $\lambda<1$ corresponds to the Bloch-Floquet wave numbers

$$pd=ip_0+2\pi m, p_0>0, m=0,1,2...\,, \tag{21}$$

the negative $\lambda>-1$ corresponds to the wave numbers

$$pd=ip_0+\pi\left(2m+1\right), p_0>0, m=0,1,2\,. \tag{22}$$

**Numerical results, discussions and results**

Numerical analysis of equations (19) shows that this equation have solutions corresponding to surface wave if the shear velocities of elastic materials are less than the shear velocity of piezoelectric material, $c_s<c_0$. This condition for the existence of surface waves is, in certain respects, analogous to the criteria for surface wave propagation described in the classic Love problem.

The numerical calculations have been carried out for piezoelectric material PZT-4 $\left(G_0=51\,\mathbf{GPa}, c=2600\,\mathbf{m/sec}\right)$ and metals: copper $\left(G_1=45\mathbf{GPa}, c_1=2240\mathbf{m/sec}\right)$, platinum $\left(G_2=61\mathbf{GP}a, c_2=1640\,\mathbf{m/sec}\right)$ where the shear velocities are lower than those of the piezoelectric material.

Numerical analysis of equations (19) shows that when $K<4$ the surface wave does not exist. For fixed value $\beta$ and $K$ there exist only one surface wave in range $\eta\in(0.94, 0.98)$ at $K=5$ and $\eta\in(0.82, 0.98)$ at $K=10$.

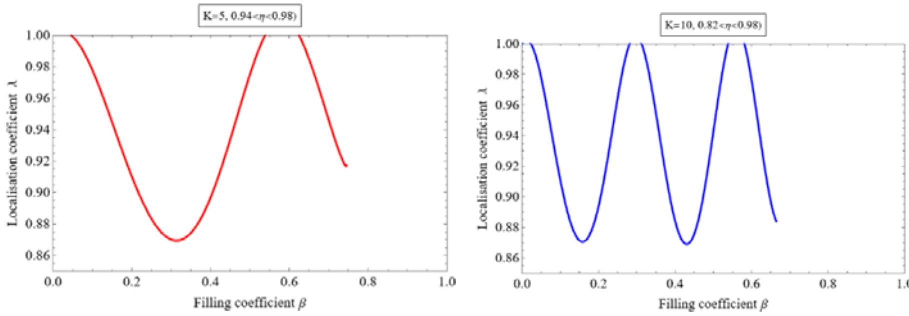


*Fig.2 Plots of the localisation coefficients versus the filling coefficient*

On Fig.2 the plots of localisation coefficients are presented versus filling coefficient $\beta = d_1 / d$ . As it follows from plots of Fiq.2 surface wave does not exist for some values of $\beta$ .

For the PZT-4 material the dimensionless phase velocity of the Bleustein wave velocity is equal $\eta_0 = 0.86$ .

The main results can be formulated as follows:

The phononic structure including a two-component, layered periodic bi-material elastic half-space and a homogeneous piezoelectric half-space, which are perfectly bonded at their interface, can support a coupled surface wave whose amplitudes attenuate with increasing distance from the interface. The existence of this wave is due to the condition that the shear velocities of elastic materials are less than the shear velocity of piezoelectric material.

**Acknowledgment**
The research was supported by the Higher Education and Science Committee of MESCS RA Research Project No 1-7/25RG-2C136.

**References**

1. Camley, R. E., Djafari-Rouhani, B., Dobrzynski, L., & Maradudin, A. A. (1983). Transverse elastic waves in periodically layered infinite and semi-infinite media. Physical Review B, *27*(12), 7318.
2. Djafari-Rouhani, B., Dobrzynski, L., Duparc, O. H., Camley, R. E., & Maradudin, A. A. (1983). Sagittal elastic waves in infinite and semi-infinite superlattices. Physical Review *B*, *28*(4), 1711.
3. Alami, M., El Boudouti, E. H., Djafari-Rouhani, B., El Hassouani, Y., & Talbi, A. (2018). Surface acoustic waves in one-dimensional piezoelectric-metallic phononic crystal: Effect of a cap layer. Ultrasonics, *90*, 80-97.
4. Darinskii, A. N., & Shuvalov, A. L. (2019). Existence of surface acoustic waves in one-dimensional piezoelectric phononic crystals of general anisotropy. Physical Review *B*, *99*(17), 174305.
5. Bousfia, A., El Boudouti, E. H., Bria, D., Nougaoui, A., Djafari-Rouhani, B., & Velasco, V. R. (2000). Surface and interface shear horizontal acoustic waves in piezoelectric superlattices. Journal of Applied Physics, 87(9), 4507-4513.

6. Chen, S., Lin, S., & Wang, Z. (2009). Shear horizontal surface acoustic waves in semi-infinite piezoelectrics/metal superlattices. Ultrasonics, *49*(4-5), 446-451.
7. Darinskii, A.N. , Shuvalov, A.L. (2019). Surface acoustic waves in one-dimensional piezoelectric phononic crystals with symmetric unit cell, Physical review B ,100, 184303\
8. Ghazaryan, K., Piliposyan, D., Jilavyan, S., & Piliposian, G. (2025). Localization of shear waves in cubic piezoelectric structure stratified with periodically arranged metallized interfaces. European Journal of Mechanics-A/Solids, 105448.
9. Ghazaryan, K., Ghazaryan, R., Terzyan, S. (2025). Surface shear horizontally wave propagation in a half-space periodically stratified with interfaces of imperfect elastic contact. In: Altenbach, H. (eds) Current Developments in Solid Mechanics and Their Applications. Advanced Structured Materials, vol 223. Springer, Cham. 233-242.
10. Avetisyan, A., (2023). Electroacoustic Waves in Piezoelectric Layered Composites. Springer International Publishing, p. 215.
11. Avetisyan, A., Khachatryan V, (2025) Finite-amplitude surface wave in linear electro-acoustic (piezoelectric) waveguide, In: Altenbach, H. (eds) Current Developments in Solid Mechanics and Their Applications. Advanced Structured Materials, vol 223. Springer, Cham.,17-29.
12. Avetisyan, A., Khachatryan V, Mkrtchyan M (2022). Formation of a hybrid of electroacoustic waves in piezoelectric layered composites. Journal of Physics: Conference Series 2231,012025(1-11).
13. Avetisyan, A.S. On formulating problems of contact-less surface control of electroacoustic wave propagation. Acoustical Physics, (2022)., 68(3), 227-234.
14. Agayan K., 2020, Diffraction of shear flat waves on a semi-infinite crack in a compound elastic half-space. Proceedings of NAS RA, Mechanics, 73(2), 22-34.
15. Aghayan K, Zakaryan G, (2025) Surface electro-elastic shear wave in piezo-electrical half-space with semi-infinite electrodes. Altenbach, H. (eds) Current Developments in Solid Mechanics and Their Applications. Advanced Structured Materials, vol 223, Chapter 1, Springer, Cham, 2-16.
16. Ghazaryan K., Ghazaryan R., Terzyan S., (2021) Localization of shear waves in inhomogeneous periodically stratified waveguide / Mechanics - Proceedings of National Academy of Sciences of Armenia, 74 (4), 56-67.
17. Bleustein, J.L. (1968). A new surface wave in piezoelectric materials. Applied Physics Letters, 13(12), 412-413.

**Information about the authors:**

**Ghazaryan K.** Institute of mechanics NAS of Armenia,
ghkarren@gmail.com, Phone 374 99 227395
**Ghazaryan R.** Institute of mechanics NAS of Armenia,
rafaelghazaryan52@gmail.com, Phone 374 99 396344
**Terzyan S.,** Institute of mechanics NAS of Armenia,
sat_and_21@yahoo.com, Phone 374 91 340432

Received 26 June 2025