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T-COIL TRANSFER FUNCTION USING THE TIME AND TRANSFER CONSTANT METHOD

The bridged T-coil (BTC) also called the T-coil circuit is often employed to extend the bandwidth of a wideband amplifier beyond the transition frequency f_T of the driver device. The transfer function of T-coil can be derived using the extra element theorem or the Δ-Y transformation. In this paper the time and transfer constant (TTC) method is used to derive the transfer function. An inductive proof of the TTC method is given which subsumes special cases, such as methods of zero and infinite value time constants.

Keywords: T-coil, Transfer function, time and transfer constant method, poles and zeros.

Introduction. Scaling of integrated circuit technology has continually increased the data rates in high-speed links. One way to increase the data rates is development of architectures that will be able to transfer information through serial links like high-speed Ser-Des protocols VSR (Very Short Reach) and PCIe (Prereferral component interconnect express) [1]. The advancement of process technologies, downscaling of device dimensions, lower supply voltages, increased the operating frequency making it very difficult to compensate the channel loss [2]. The low-pass characteristics of the channels greatly affect this process. The problem becomes more challenging as the specifications for operating the frequency band become wider. Therefore, degraded signal recovery in high-speed interfaces becomes a serious challenge [3]. There are many known approaches for restoring the corrupted data. The most well-known of these uses a continuous time linear equalizer (CTLE) [4].

Most common CTLE architectures consist of source degenerated common source amplifiers [5] (Fig. 1).

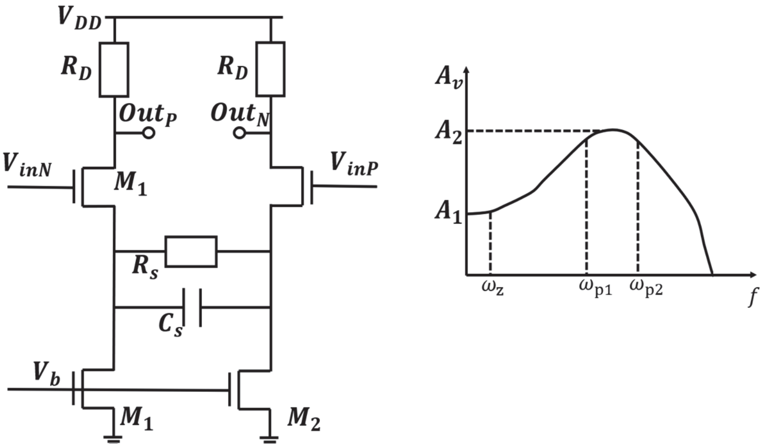

Fig. 1. CTLE circuit diagram and amplitude-frequency plot

In order to compensate the parasitic capacitances of the PAD, on-chip inductors and T-coils are a feature of modern wireline systems [6]. T-coils are also used as the load in high-speed amplifiers to enable high gain at the required frequency [7].

The T-coil circuit diagram is shown in Fig. 2. It consists of two mutually coupled inductors and a bridge capacitor.

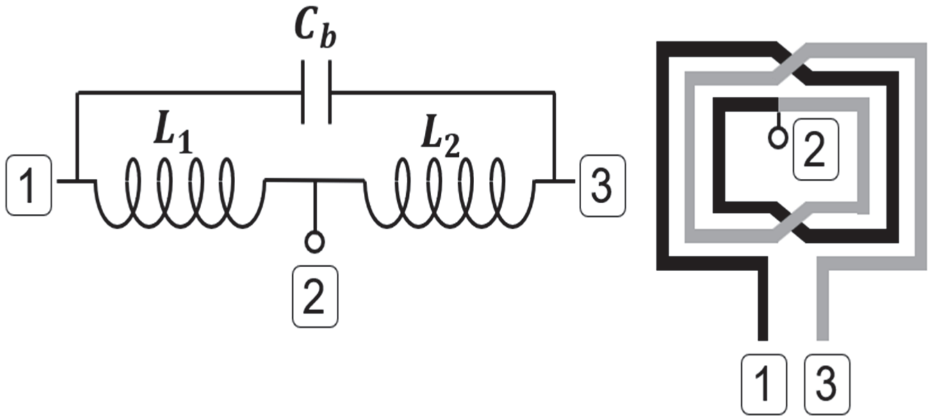

Fig. 2. Circuit diagram and layout of T-coil

For simplicity, a common source stage without degeneration is presented below Fig.3(a) with the usage of inductor(b) and T-coil(c). It is shown that T-coil can increase 3-dB bandwidth by 2.83 times, while inductor stage improves bandwidth by a factor of 1.8 [8].

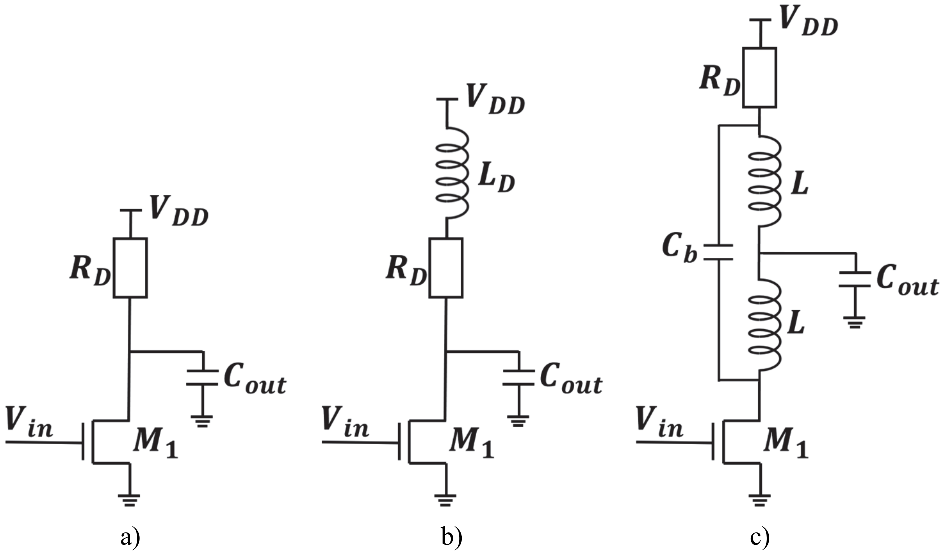

Fig. 3. Common-Source with (a) a resistive load (b) inductor (c)T-coil

The transfer function of common source stage with T-coil can be derived by finding its output impedance simply multiplying the result by transconductance (g_m) of the input device.

In this paper, the transfer function of T-coil is derived by the TTC method [9]. By the mentioned method, the transfer function to any degree of accuracy can be determined. Section 1 introduces the TTC method. Section 2 presents the T-coil transfer function derivation by using the TTC method.

Section 1. Today's sensitive and high-speed integrated circuits AC analysis help to find the transfer function of the designed circuit. But it is often necessary to derivate the transfer function by analytical methods that can be helpful for the circuit for design objectives. More importantly, the analytical methods do not need to carry the analysis to its end to be able to obtain useful information about the circuit dynamics. The transfer function in circuit design usually relates the current or voltage at one port to the current or voltage at another port. On the other hand, if the input, x, is the current of a current source driving a given port of the circuit, while the output y is the voltage across the same port, the transfer function, Z(s) ≡ v_1 (s)/ i_1 (s) would correspond to the impedance looking at that port.

The transfer function of a linear system with lumped elements can be written by:

$$H(s) = \frac{a_0+a_1s+a_2s^2+\cdots+a_ms^m}{1+b_1s+b_2s^2+\cdots+b_ns^n}, \tag{1}$$

where all a_i and b_j coefficients are real, and s represents the complex frequency [9].

Based on the fundamental theorem of algebra, equation (1) can be factored as:

$$H(s) = \frac{\left(1-\frac{s}{z_1}\right)\left(1-\frac{s}{z_2}\right)\ldots\left(1-\frac{s}{z_m}\right)}{\left(1-\frac{s}{p_1}\right)\left(1-\frac{s}{p_2}\right)\ldots\left(1-\frac{s}{p_n}\right)}, \tag{2}$$

where pole and zero frequencies, given by p_i and z_i respectively, are the real or complex conjugate pairs.

Knowing the coefficients of (1) as poles and zeros, we can predict circuit dynamics. By the TTC method we can determine the transfer function of an Nth order system to the desired level of accuracy using low frequency calculations of port resistances and low-frequency values of the transfer functions (transfer constants) for different combinations of shorting and opening of other elements. The system network with N energy-storing (reactive) elements can be represented as a system with N external ports with no frequency-dependent elements inside and each reactive element (namely inductors and capacitors) attached to one of the ports, as shown in Fig. 4 [9].

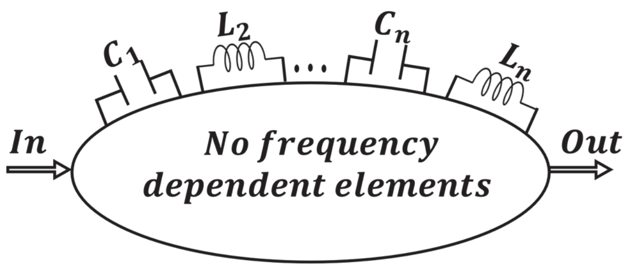

Fig. 4. A system with N energy-storing (reactive) elements

In [9] research proved, generalized, and discussed the TTC method with its several important and useful implications.

a_m and b_n we are able to determine by

$$b_n = \sum_i^{1\le i<j} \ \sum_j^{j<k} \ \ldots \sum_{k\ldots}^{\ldots\le N} \ \tau_i^0 \tau_j^i \tau_k^{ij} \ldots, \tag{3}$$

where τ_k^{ij} corresponds to the time constant due to the reactive element at port k and the low frequency resistance seen at port k when ports whose indexes are in the superscript $(i, j, \ldots)$ are infinite valued (shorted capacitors and opened inductors).

So, the time constants will have one of the following forms depending on whether there is an inductor, or a capacitor connected to port k:

for capacitor, C_i:

$$\tau_i^{jk\ldots} = C_i R_i^{jk\ldots}; \tag{4}$$

for inductor, L_t:

$$\tau_t^{mn\dots} = \frac{L_t}{R_l^{mn\dots}}, \tag{5}$$

where $R_i^{jk\dots}(R_l^{mn\dots})$ is the resistance seen at port $k(n)$ with the reactive elements at port $i, j, \dots (l, m, \dots)$ at their infinite values.

And the a_m defined as:

$$a_m = \sum_i^{1\le i<j} \sum_j^{j<k} \dots \sum_{k\dots}^{\dots \le N} \tau_i^0 \tau_j^i \tau_k^{ij} \dots H^{ijk\dots}, \tag{6}$$

where $H^{ijk\dots}$ is the Nth-order transfer constant evaluated with the energy storing elements at ports $i, j, \dots$ at their infinite values (opened inductors and shorted capacitors) and all others are zero valued (opened capacitors and shorted inductors).

Section 2. The small signal model of T-coil is shown below, where L_3 is the mutual inductance (Fig.5).

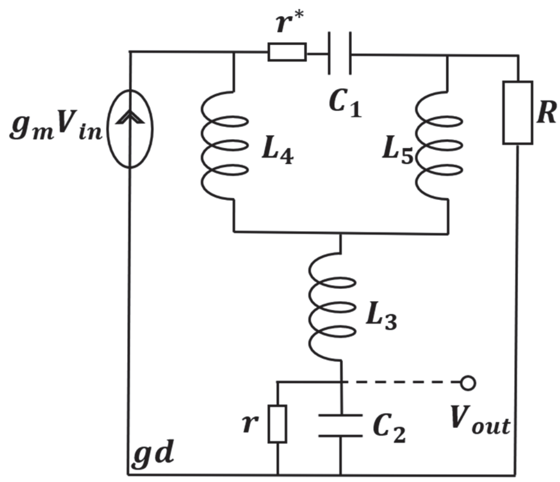

Fig. 5. Circuit diagram for calculating

Because we have infinite and zero values in transfer constants, r^* and r resistors are added where $r \to \infty$ and $r^* \to 0$.

The time constants are shown below:

$\tau_1^0 = C_1 r^*$	$\tau_2^0 = C_2(R\,\|\|r)$	$\tau_3^0 = \frac{L_3}{R+r}$	$\tau_4^0 = \frac{L_4}{2r}$
$\tau_5^0 = \frac{L_5}{R+r}$	$\tau_2^1 = C_2(R\,\|\|r)$	$\tau_3^1 = \frac{L_3}{R+r}$	$\tau_4^1 = \frac{L_4}{r^*}$
$\tau_5^1 = \frac{L_5}{r^*}$	$\tau_3^2 = \frac{L_3}{R}$	$\tau_4^2 = \frac{L_4}{R+r}$	$\tau_5^2 = \frac{L_5}{R}$
$\tau_4^3 = \frac{L_4}{r}$	$\tau_5^3 = \frac{L_5}{R+r}$	$\tau_5^4 = \frac{L_5}{R+r}$	$\tau_3^{12} = \frac{L_3}{R}$

$\tau_4^{12} = \frac{L_4}{r^*}$ $\tau_5^{12} = \frac{L_5}{r^*}$ $\tau_4^{13} = \frac{L_4}{r^*}$ $\tau_5^{13} = \frac{L_5}{r^*}$

$\tau_5^{14} = \frac{L_5}{R+r}$ $\tau_4^{23} = \frac{L_4}{r}$ $\tau_5^{23} = \frac{L_5}{r}$ $\tau_5^{24} = \frac{L_5}{R}$

$\tau_5^{34} = \frac{L_5}{R}$ $\tau_4^{123} = \frac{L_4}{r^*}$ $\tau_5^{123} = \frac{L_5}{r}$ $\tau_5^{124} = \frac{L_5}{R}$

$\tau_5^{134} = 0$ $\tau_5^{234} = 0$

The number of poles in the circuits is represented by the number of independent reactive elements, i.e. initial conditions of capacitors and inductors that can be defined independently. The initial condition for capacitor corresponds to voltage and for inductor is current. The four initial conditions of T-coil (C_1, C_2, L_4, L_5) imply four poles. The number of zeros in a circuit is defined by the maximum number of reactive elements for which setting infinite values (shorting capacitors and open circuits for inductors) will lead to a non-zero output. The derivation of T-coil poles and zeros defined by (3), (6) expressions are shown below:

$$b_1 = \sum_{i=1}^{5} \tau_i^0 = \tau_1^0 + \tau_2^0 + \tau_3^0 + \tau_4^0 + \tau_5^0 =$$
$$= C_1 r^* + C_2(R\,||r) + \frac{L_3}{R+r} + \frac{L_4}{2r} + \frac{L_5}{R+r} = C_2 R\,, \tag{7}$$

$$b_2 = \sum_{i=1}^{1\le i<j} \sum_{j=2}^{j\le 5} \tau_i^0 \tau_j^i = \tau_1^0\tau_2^1 + \tau_1^0\tau_3^1 + \tau_1^0\tau_4^1 + \tau_1^0\tau_5^1 +$$
$$+\tau_2^0\tau_3^2 + \tau_2^0\tau_4^2 + \tau_2^0\tau_5^2 + \tau_3^0\tau_4^3 + \tau_3^0\tau_5^3 + \tau_4^0\tau_5^4 =$$
$$= C_2(R\,||r)(\frac{L_3}{R} + \frac{L_5}{R}) + C_2 r^* \left(\frac{L_4}{r^*} + \frac{L_5}{r^*}\right) = C_2(L_3 + L_5) + C_1(L_4 + L_5)\,, \tag{8}$$

$$b_3 = \sum_{i=1}^{1\le i<j} \sum_{j=2}^{j<k} \sum_{k=3}^{k\le 5} \tau_i^0 \tau_j^i \tau_k^{ij} = \tau_1^0\tau_2^1\left(\tau_3^{12} + \tau_4^{12} + \tau_5^{12}\right) +$$
$$+\tau_1^0\tau_3^1\left(\tau_4^{13} + \tau_5^{13}\right) + \tau_1^0\tau_4^1(\tau_5^{14}) + \tau_2^0\tau_3^2\left(\tau_4^{23} + \tau_5^{23}\right) + \tau_2^0\tau_4^2\left(\tau_5^{24}\right) + \tau_3^0\tau_4^3\tau_5^{34} =$$
$$= \frac{L_4}{r^*} r^* C_1 R C_2 + \frac{L_5}{r^*} r^* C_1 R C_2 = C_1 C_2 R(L_4 + L_5)\,, \tag{9}$$

$$b_4 = \sum_{i=1}^{1\le i<j} \sum_{j=2}^{j<k} \sum_{k=3}^{k<e} \sum_{e=4}^{e\le 5} \tau_i^0 \tau_j^i \tau_k^{ij} \tau_e^{ijk} =$$
$$= \tau_1^0\tau_2^1\tau_3^{12}\left(\tau_4^{123} + \tau_5^{123}\right) + \tau_1^0\tau_3^1\tau_4^{12}\tau_5^{124} + \tau_1^0\tau_3^1\tau_4^{13}\tau_5^{134} + \tau_2^0\tau_3^2\tau_4^{23}\tau_5^{234} =$$
$$= r^* C_1 R C_2 \frac{L_3}{r^*}\left(\frac{L_4}{r^*} + \frac{L_5}{r^*}\right) + r^* C_1 R C_2 \frac{L_4}{r^*}\frac{L_5}{R} = C_1 C_2 L_3(L_4 + L_5) + C_1 C_2 L_4 L_5\,. \tag{10}$$

The transfer constants for Fig.5 are shown below:

$H^0 = R$ $\quad H^1 = R$ $\quad H^2 = 0$ $\quad H^3 = (r + R)||r$

$H^4 = 0$ $\quad H^5 = r$ $\quad H^{12} = 0$ $\quad H^{13} = r$

$H^{14} = R$ $\quad H^{15} = R$ $\quad H^{23} = 0$ $\quad H^{24} = 0$

$H^{25} = 0$ $\quad H^{34} = r$ $\quad H^{35} = 0$ $\quad H^{45} = 0$

$$a_1 = \sum_{i=1}^{5} \tau_i^0 H^i = \tau_1^0 H^1 + \tau_2^0 H^2 + \tau_3^0 H^3 + \tau_4^0 H^4 + \tau_5^0 H^5 = \tau_5^0 H^5 = L_5\,, \quad (11)$$

$$a_2 = \sum_{i=1}^{1\le i<j} \sum_{j=2}^{j\le 5} \tau_i^0 \tau_j^i H^{ij} = \tau_1^0 \tau_2^1 H^{12} + \tau_1^0 \tau_3^1 H^{13} + \tau_1^0 \tau_4^1 H^{14} + \tau_1^0 \tau_5^1 H^{15} +$$
$$+\tau_2^0 \tau_3^2 H^{23} + \tau_2^0 \tau_4^2 H^{24} + \tau_2^0 \tau_5^2 H^{25} + \tau_3^0 \tau_4^3 H^{34} + \tau_3^0 \tau_5^3 H^{35} + \tau_4^0 \tau_5^4 H^{45} =$$
$$= RC_1(L_4 + L_5)\,, \quad (12)$$

$$H(s) = \frac{1+a_1 s+a_2 s^2}{1+b_1 s+b_2 s^2+b_3 s^3+b_4 s^4}\,, \quad (13)$$

Conclusion. Estimating the bandwidth and transfer function is essential in SERDES systems. T-coils are an operational way to overcome bandwidth issues. The Paper presents an efficient analysis of T-coil transfer function and output impedance. The transfer function of T-coil is derived using the TTC method. Time and transfer constants are calculated using just low frequency calculations for various combinations of shorted and opened energy-storing components.

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T-ԱՁԵՎ ԻՆԴՈՒԿՏՈՐԻ ՓՈԽԱՆՑՄԱՆ ՖՈՒՆԿՑԻԱՅԻ ԴՈՒՐՍԲԵՐՈՒՄԸ ԺԱՄԱՆԱԿԻ ԵՎ ՓՈԽԱՆՑՄԱՆ ՀԱՍՏԱՏՈՒՆՆԵՐԻ ՄԵԹՈԴՈՎ

Կամրջակային T-աձև ինդուկտորը, որն անվանում են նաև T-աձև ինդուկտոր, հաճախ օգտագործվում է լայնաշերտ ուժեղացուցչի թողունակությունը մեծացնելու համար՝ մինչև հաղորդչի անցումային f_T հաճախություն: T-աձև ինդուկտորի փոխանցման ֆունկցիան կարող է դուրս բերվել՝ օգտագործելով լրացուցիչ տարրերի թեորեմը կամ Δ-Y փոխակերպումը: Աշխատանքում ժամանակի և փոխանցման հաստատունների(ԺՓՀ) մեթոդն օգտագործվում է փոխանցման ֆունկցիան դուրս բերելու համար: Ներկայացված է ԺՓՀ մեթոդը, որն ընդգրկում է որոշակի դեպքեր, ինչպիսին է զրոյական և անսահման արժեքի հաստատուններ պարունակող մոտեցումը:

Առանցքային բառեր□ T-աձև ինդուկտոր, փոխանցման ֆուկցիա, ժամանակի և փոխանցման հաստատունի մեթոդ, բևեռներ և զրոներ:

Г.А. ПЕТРОСЯН, С.К. ГУЛЯН

ФУНКЦИЯ ПЕРЕДАЧИ T-ОБРАЗНОЙ КАТУШКИ С ИСПОЛЬЗОВАНИЕМ МЕТОДА ВРЕМЕНИ И ПЕРЕДАЧНОЙ ПОСТОЯННОЙ

Мостовая T-образная катушка, также называемая T- образной катушкой, часто используется для расширения полосы пропускания широкополосного усилителя до пределов частоты перехода f_T драйверного устройства. Передаточная функция

T- образной катушки может быть получена с использованием теоремы о дополнительных элементах или Δ-Y преобразования. В данной статье для получения передаточной функции используется метод времени и передаточной константы (ВПК). Дано индуктивное доказательство метода ВПК, выделены частные случаи, такие как методы нулевых и бесконечных значений постоянных времени.

Ключевые слова: T- образная катушка, передаточная функция, метод времени и передаточной постоянной, полюсы и нули.