The Master-Space Supergravity: Particle mechanics

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Abstract

This report aims to expose further the assertions made in a recent theory of *global Master space* (MS_p)-SUSY (Ter-Kazarian, 2023a, 2024a) by developing its local extension. The global MS_p-SUSY reviews the physical processes underlying the standard Lorenz code of motion and its deformation tested in experiments for ultra-high energy cosmic ray and TeV-γ photons observed. The local extension of MS_p-SUSY yields the gauge theory of *translations*. This as a corollary makes room for the theory of $\widetilde{MS}_p$-Supergravity, subject to certain rules. The superspace is a direct sum of background semi-Riemannian 4D-space and curved Master space $\widetilde{MS}_p \equiv \underline{V}_2$ (2D semi-Riemannian space), $V_4 \oplus \underline{V}_2$, with an inclusion of additional fermionic coordinates $\Theta(\underline{\theta}, \underline{\bar{\theta}})$ and $\bar{\Theta}(\underline{\theta}, \underline{\bar{\theta}})$, which are induced by the spinors $\underline{\theta}$ and $\underline{\bar{\theta}}$ referred to $\widetilde{MS}_p$. Being embedded in V_4, the $\widetilde{MS}_p$ is the *unmanifested* indispensable individual companion of a particle of interest devoid of any matter influence. While all the particles are living on V_4, their superpartners can be viewed as living on $\widetilde{MS}_p$. In this framework supersymmetry and general coordinate transformations are described in a unified way as certain diffeomorphisms. The action of simple $\widetilde{MS}_p$-SG includes the Hilbert term for a *fictitious* graviton (with spin 2) coexisting with a *fictitious* fermionic field of, so-called, gravitino (sparticle with spin 3/2) described by the Rarita-Scwinger kinetic term. They are the bosonic and fermionic states of a gauge particle in V_4 and $\widetilde{MS}_p$, respectively, or vice versa. A curvature of $\widetilde{MS}_p$ arises entirely due to the inertial properties of the Lorentz-rotated frame of interest. This refers to the particle of interest itself, without relation to other matter fields, so that this can be globally removed by appropriate coordinate transformations. The supervielbein, being an alogue of Cartan's local frame, is the dynamical variable of superspace formulation, which identifies the tetrad field and the Rarita-Schwinger fields. The spin connection is the second dynamical variable in this theory. The tetrad field plays the role of a gauge field (graviton) associated with local transformations. The gravitino is a gauge field associated with local supersymmetry. Within that context, we consider particle mechanics.

Keywords: *Supersymmetry–Supergravity–Particle mechanics*

1. Introduction

The question about the very nature of the *uniform motion* of a particle in free space, which is one of the cornerstones put into physics by hand, at first glance classified as an empirical term rather than as a notion of pure reason and, thus, it may not seem like a subject of perception. While in recent years, the violation of CPT and Lorentz invariance at high energies has become a major concern for physicists. The discovery of the possibility of spontaneously breaking of Lorentz symmetry in bosonic string field theory (Kostelecky & Potting, 1991, 1996), there has been vital interest in its spontaneously breaking in the framework of quantum field theory as introduction of a preferred frame, or its explicit breaking in non-commutative geometries, as well as in certain supersymmetric theories. Spontaneous Lorentz break was discussed in several early papers, e.g. (Bialynicki-Birula, 1963, Bjorken, 1963, Cho & Freund, 1975, Eguchi, 1976, Guralnik, 1964, Nambu, 1968). Although precursors go back at least to early work by Dirac (Dirac, 1951) and Heisenberg (Heisenberg, 1957) in an effort to interpret the photon as a Nambu-Goldstone boson. An important aspect of spontaneous breaking is that both the fundamental theory and the effective low energy theory remain invariant under observer Lorentz transformations (Colladay & Kostelecky, 1997, 1998).

Quantum gravity (QG) has now gradually become a physical theory. Indeed, astrophysical experiments performed at the ultra-high energies (at ultra-short distances) should measure effects due to a QG regime. This QG search attempts to assess the state of our present knowledge and understanding of the laws of

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physics on the Planck scale. Most ideas in QG about the violation of the standard Lorentz code (SLC) of motion have been tested in recent decades in ultra-high energy cosmic ray (UHECR) and astrophysical TeV-γ photons experiments. The UHECR- and TeV-γ threshold anomalies found in these experiments provide a wealth of invaluable modern tests of the origin of LIV and has a strong potential in providing competitive constraints on suggested scenarios. They reflect the expectation that the solutions to this mystery seemed to require new physics (see e.g. (Batista & et al., 2019, 2023, Mattingly, 2005) and references therein). It is a well-established fact that due to fundamental quantum uncertainties, the microstructure of spacetime should be viewed as a dynamical entity fluctuating within a two or three orders of magnitude of Planck length $\ell_P \approx 1.62 \times 10^{-33}$cm and Planck time t_P/c, at which the structure of space-time and quantum effects would become inextricably intertwined. This means that if such fluctuations are large enough to cause non-trivial deformations of classical smooth spacetime, the latter will develop a `foamy´ structure at the ultra-microscopic level and all sorts of geometric changes and topologically non-trivial structures will be formed (e.g. via quantum tunnelling (Garfinkle & Strominger, 1991)), evolving, interacting and lasting only a few Planck times. In phenomenological minimal-length models, foamy effects come from the presence of a minimal accessible length, which modifies the Heisenberg or Poincar´e algebra to accommodate a minimal uncertainty in position measurements at the Planck energy, $E_P = M_P c^2$ ($\sim 1.22 \times 10^{19}$ GeV). The minimal length appears as a kinematic feature, while the shape itself of the Hamiltonian may be deformed from the combined action of a modified position-momentum algebra and the choice of a relativity principle.

However, even thanks to the fruitful interplay between phenomenological analysis and high energy astronomical experiments, the scientific situation remains, in fact, more inconsistent to day. A systematic analysis of these properties happens to be surprisingly difficult by conventional theoretical methods. Up to now, there has been no conclusive evidence of violation of the laws of conventional physics, with the results instead yielding ever more stringent upper bounds on this task, thus confirming the related aspects with concomitant precision. This may be due to the fact that present instruments do not yet have the necessary sensitivity to detect Planck scale effects, or that some effects have not been taken into account in the available data. Of course, failure to find a violation of these laws in any one experiment or class of experiments does not give us a final proof, and even as the experimental limits move more closely towards the fundamental bounds of measurement uncertainty, new conceptual approaches to the task continue to appear. The tremendous importance of the question and the lure of what might be revealed by attaining the next decimal place of obtained experimental results are as strong a draw on this question as they are in any other aspect of precise tests of physical laws.

Our primary interest in the subject is to understand the underlying reality, in which the *uniform motion* would have well defined. In a recent paper Ter-Kazarian (2024a) we developed a microscopic theory of deformed Lorentz symmetry and deformed geometry induced by `foamy´ effects near the Planck scale and tested in ultra-high energy experiments. To this aim, we proposed a theory of MS$_p$-SUSY and derive SLC in a new perspective of global double MS$_p$-SUSY transformations. To the best of our knowledge no one has ever studied the very nature of the *uniform motion* and the physical processes underlying it. We just like to mention its key points as an example of the lines on which one should seek to make advances and that other people, we hope, will follow along those lines. A notable conceptual element is the concept of 2D MS$_p$($\equiv \underline{M}_2$), which is a physical structure with *intrinsic geometrical properties of its own* (see Appendix A). The MS$_p$ embedded in background 4D Minkowski space, M_4, is the *unmanifested* indispensable individual companion to the particle of interest as the intrinsic property devoid of any external influence. A comprehensive principle underlying the global MS$_p$-SUSY theory hinges on the following:

the particle perseveres in its permanent state of superoscillations between the spaces M_4 and $\underline{M}_2$, unless acted upon by some external force, i.e. the particle undergoes the SUSY - transformations at sequential transitions from M_4 to $\underline{M}_2$ and back ($M_4 \rightleftharpoons \underline{M}_2$).

We consider the spaces M_4 and $\underline{M}_2$ formally and, therefore, mathematical devoid of any sense of *physical space-time*. To derive the most important *relative inertial uniform motion*, it is necessary to impose specific conditions on the spinor transformation matrix M in $\underline{M}_2$. We are of course not limited within MS$_p$-SUSY to consider particular constant spinor $\underline{\theta}$ referred to $\underline{M}_2$, which yields the constant velocity $\vec{v}(\underline{\theta})$ (Appendix A), but can choose at will any other constant spinors $\underline{\theta}'$, $\underline{\theta}''$, ... yielding respectively the constant velocities $\vec{v}'(\underline{\theta}')$, $\vec{v}''(\underline{\theta}'')$, ... of inertial observers that move uniformly forever on rectilinear timelike worldlines, whose transformational law on the original spinor $\underline{\theta}$ is known (first condition):

$$\underline{\theta}'_\alpha = M_\alpha^{\ \beta}\,\underline{\theta}_\beta, \quad \bar{\underline{\theta}}_{\dot\alpha} = (M^*)_{\dot\alpha}^{\ \dot\beta}\,\bar{\underline{\theta}}_{\dot\beta}, \quad \dot\alpha, \dot\beta = 1, 2, \tag{1}$$

where $M \in SL(2, C)$ is the hermitian unimodular two-by-two matrix, the matrix M^* is related by a similarity

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transformation to $(M^{-1})^{\dagger}$, i.e. $(M^{\dagger})^{\beta}{}_{\alpha} = (M^{*})_{\alpha}{}^{\beta}$. The (1) gives the second founding property of SR that the bilinear combinations are $c := \underline{\theta}\,\bar{\underline{\theta}} = \underline{\theta}'\,\bar{\underline{\theta}}' = \cdots = const$, which yields a second postulate of SR (Einstein's postulate) (Appendix A). Therewith a quantity $\underline{e}_m\,(\underline{\theta}\,\sigma^{\underline{m}}\,\bar{\underline{\xi}})$ (where $\underline{e}_m$ is a basis vector, $\underline{\theta}$, $\underline{\xi}$ are Weyl spinors) is a Lorentz scalar if and only if the second condition holds too:

$$\tfrac{1}{2}\,Tr\left(\sigma^{\underline{m}}\,M\sigma^{\underline{n}}\,M^{\dagger}\right)\sigma^{n}_{\alpha\dot{\alpha}} = (M^{-1})_{\alpha}{}^{\beta}\sigma^{\underline{m}}_{\beta\dot{\beta}}(M^{-1})^{\dagger\dot{\beta}}{}_{\dot{\alpha}}, \tag{2}$$

where the map from $SL(2,C)$ to the Lorentz group is established through the $\underline{\sigma}$-matrices (Appendix A). The latter, according to the embedding map, can be written in terms of $\vec{\sigma}$-Pauli spin matrices. The (2) combined with (103) give the first founding property (106) of SR.

The `superspace´ is a direct sum extension of background double spaces $M_4 \oplus \underline{M}_2$, with an inclusion of additional fermionic coordinates induced by the spinors $(\underline{\theta}, \bar{\underline{\theta}})$ referred to $\underline{M}_2$. Thanks to the embedding of $\underline{M}_2$ in M_4, the spinors $(\underline{\theta}, \bar{\underline{\theta}})$, in turn, induce the spinors $\theta(\underline{\theta}, \bar{\underline{\theta}})$ and $\bar{\theta}(\underline{\theta}, \bar{\underline{\theta}})$ (see (73)), as to M_4. Then the net result of sequential atomic double transitions induce the inhomogeneous Lorentz group, or Poincaré group, and that the unitary linear transformation $|x,t> \rightarrow U(L,a)|x,t>$ on vectors in the physical Hilbert space. Thus, we achieve the desired goal to derive the SLC in terms of spinors $(\underline{\theta}, \bar{\underline{\theta}})$ and period (τ) of superoscillations referred to the master space $\underline{M}_2$ (Appendix A). This calls for a complete reconsideration of our standard ideas of Lorentz motion code, to be now referred to as the *individual code of a particle*, defined as its intrinsic property. This reveals the nature of the most important concept of *physical space-time*, which *turns out to be a direct consequence of the processes of particle motion*. That is, we derive the relative *temporal* ($x^0 = ct$) and *spatial* ($|\vec{x}|$) coordinates of *physical space-time* as a function of constant spinors $(\underline{\theta}, \bar{\underline{\theta}})$ and a period τ of superoscillations as follows:

$$\begin{aligned} &x^0 = ct = (\underline{\theta}\,\bar{\underline{\theta}})\,k\tau/\sqrt{2}, \quad k = 1, 2, 3, ... \\ &|\vec{x}| = (\underline{\theta}_1\,\bar{\underline{\theta}}_1 - \underline{\theta}_2\,\bar{\underline{\theta}}_2)\,k\tau/\sqrt{2}. \end{aligned} \tag{3}$$

While all the particles are living on M_4, their superpartners can be viewed as living on MS_p. Particular emphasis is given that the ground state of MS_p-SUSY model has a vanishing energy value and is nondegenerate (SUSY unbroken). The particles in M_4 themselves can be considered as excited states above the underlying quantum vacuum of background double spaces $M_4 \oplus \underline{M}_2$, where the zero point cancellation occurs at ground-state energy, provided that the natural frequencies are set equal ($q_0^2 \equiv \nu_b = \nu_f$), because the fermion field has a negative zero point energy while the boson field has a positive zero point energy.

This theory, among other things, actually explores the first part of the phenomenon of inertia, which refers to *inertial uniform motion along rectilinear timelike world lines*. This developments are in many ways exciting, yet mysteries remain, and some of deeper issues are still unresolved, such as those which relate the inertial effects, which comprises a second half of phenomenon of inertia. Perhaps the most striking phenomenon of inertia is a deep mystery in physics, representing the most incomprehensible problem in need of resolution. Governing the motions of planets both fundamental phenomena of nature the gravity and inertia reside at the very beginning of physics, so that understanding in depth mystery of the origin of the whole phenomenon of inertia consisting of two parts represents a tremendous opportunity for present-day theoretical physics. General Relativity (GR) was a great success in explaining the gravity, and it showed how, by departing from ideas suggested by classical mechanics, you could make an advance in a new direction. Despite the advocated success of GR, which was a significant landmark in the development of the field, the phenomenon of inertia stood one of the major unattained goals since the time traced back to the works developed by Galileo (Drake, 1978) and Newton (Newton, 1687). More than four centuries passed since the famous far-reaching discovery by Galileo (in 1602-1604) of Weak Principle of Equivalence (WPE) that all bodies fall at the same rate Drake (1978), which establishes the independence of free-fall trajectories of the internal composition and structure of bodies. This led to an early empirical version of the suggestion that gravitation and inertia may somehow result from a single mechanism. Besides describing these early gravitational experiments, Newton in *Principia Mathematica* Newton (1687) has proposed a comprehensive approach to studying the relation between the gravitational and inertial masses of a body. The *principle of inertia* they developed is one of the fundamental principles of classical mechanics. This governs the *uniform motion* of a body and describes how it is affected by applied forces. Ever since, there is an ongoing quest to understand the reason for the universality of the inertia, attributing to the WPE. In other words, WPE states that all bodies at the same spacetime point in a given gravitational field will undergo the same acceleration. However, the nature of the relationship of gravity and inertia continues to

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elude us and, beyond the WPE, there has been little progress in discovering their true relation. Today there is no known feasible way to account for credible explanation of this problem.

With this perspective in sight, the necessity of *local* theory of MS$_p$-SUSY is twofold. First, the general idea behind the success of any global theory was the need to promote the symmetries to local symmetries. Second, these features deserve careful study, because the theory of global MS$_p$-SUSY provides valuable theoretical clue for a complete revision of our standard ideas about the Lorentz code of motion to be now referred to as the *intrinsic* property of a particle. This is a result of the first importance for a really comprehensive theory of inertia. In this respect, it is important to recall once more that GR was designed to incorporate Mach's principle of *relativity of inertia*, i.e. this is the gravitational influence of the entire Universe which creates inertia. Of course, GR enters with its own multifacets, and pretty well it over about many years has been on these lines, and physicists have gone a long way in its development. Viewed from the perspective of GR theory, the fictitious forces (i.e. pseudo-forces) - the forces that result from the acceleration of the reference frame itself and not from any physical force acting on the body, are attributed to geodesic motion in spacetime. But nothing is reliable and such efforts do not make sense. The fact that the theory conforms GR does not prove that it is correct. Indeed, as Einstein emphasized later Bondi (1952), Sciama (1953), GR is failed to account for the inertial properties of matter, so that an adequate theory of inertia is still lacking.

A local extension of the MS$_p$-SUSY algebra leads to the gauge theory of *translations*. The overall purpose of the present article, therefore, is to conceive a local MS$_p$-SUSY as the theory of $\widetilde{MS}_p$-SG, an early version of which is given in (Ter-Kazarian, 2023b). As this solution is in use throughout the subsequent papers, much more will be done to make the early results and formulations complete, clear and rigorous. The action of simple $\widetilde{MS}_p$-SG includes the Hilbert term for a *fictitious* graviton coexisting with a *fictitious* fermionic field of gravitino described by the Rarita-Scwinger kinetic term. Whereas a coupling of supergravity with matter superfields no longer holds. Instead, a deformation of MS$_p$ is the origin of these fields. Finally, we consider a particle mechanics.

We proceed according to the following structure. To start with, in Section 2 we discuss the idea of what is a local $\widetilde{MS}_p$-SUSY. Section 3 is devoted to the non-trivial linear representation of the $\widetilde{MS}_p$-SUSY algebra. In Section 4 we turn to a simple $(N = 1)$ $\widetilde{MS}_p$ - SG without auxiliary fields. On these premises, in Section 5 we derive particle mechanics and discuss the velocity and acceleration in M_4. The ideas underlying our theoretical framework and concluding remarks are described in section 6. Whereas we highlight a few points and discuss issues to be studied further. Such approach is more transparent at any step but needs some technical details, definitions and algebraic operations. In Appendix A, therefore, we briefly revisit the global `double space´- or MS$_p$-SUSY without going into the subtleties as a guiding principle to make the rest of paper understandable. For brevity, whenever possible undotted and dotted spinor indices often can be ruthlessly suppressed without ambiguity. Unless indicated otherwise, the natural units, $h = c = 1$ are used throughout.

2. The local $\widetilde{MS}_p$-SUSY

One might guess that the condition for the parameter $\partial_{\hat{\mu}}\epsilon = 0$ of a global MS$_p$-SUSY theory (Ter-Kazarian, 2024a) should be relaxed for the accelerated particle motion, so that a global SUSY will be promoted to a local SUSY in which the parameter $\epsilon = \epsilon(X^{\hat{\mu}})$ depends explicitly on $X^{\hat{\mu}} = (\widetilde{x}^{\mu}, \widetilde{\underline{x}}^{\underline{\mu}}) \in V_4 \oplus \underline{V}_2$, where $\widetilde{x}^{\mu} \in V_4$ and $\widetilde{\underline{x}}^{\underline{\mu}} \in \underline{V}_2$, with V_4 and $\underline{V}_2$ are the 4D and 2D semi-Riemannian spaces. This extension will address the *accelerated motion* and *inertia effects*. To trace a maximal resemblance in outward appearance to the theory of MS$_p$-SUSY, we here accept all its conventions and notations unless otherwise noted. A smooth embedding map, generalized for curved spaces, becomes $\widetilde{f}: \ \underline{V}_2 \longrightarrow V_4$ defined to be an immersion (the embedding which is a function that is a homeomorphism onto its image):

$$\widetilde{\underline{e}}_{\underline{0}} = \widetilde{e}_0, \quad \widetilde{\underline{x}}^{\underline{0}} = \widetilde{x}^0, \quad \widetilde{\underline{e}}_{\underline{1}} = \vec{\widetilde{n}}, \quad \widetilde{\underline{x}}^{\underline{1}} = |\vec{\widetilde{x}}|, \tag{4}$$

where $\vec{\widetilde{x}} = \widetilde{e}_i\widetilde{x}^i = \vec{\widetilde{n}}|\vec{\widetilde{x}}|$ $(i = 1, 2, 3)$ (the middle letters of the Latin alphabet $(i, j, ...)$ will be reserved for space indices). On the premises of (Ter-Kazarian, 2024a), we review the accelerated motion of a particle in a new perspective of local $\widetilde{MS}_p$-SUSY transformations (see Fig. 1) that a *creation* of a particle in $\underline{V}_2$ means its transition from initial state defined on V_4 into intermediate state defined on $\underline{V}_2$, while an *annihilation* of a particle in $\underline{V}_2$ means vice versa. The same interpretation holds for the *creation* and *annihilation* processes

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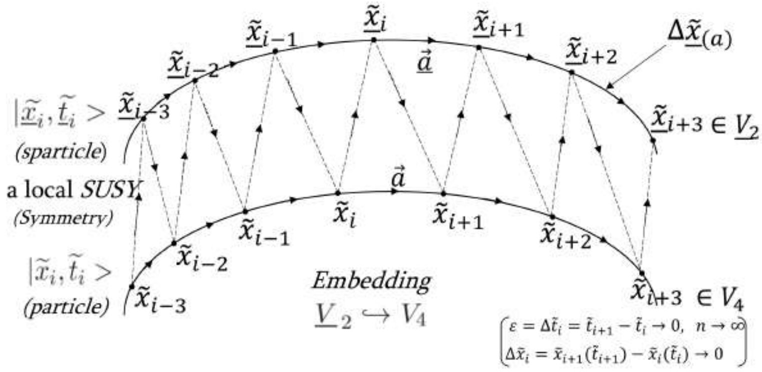

Figure 1. The extended Schwinger transformation function geometry. The net result of each atomic double transition of a particle $V_4 \rightleftharpoons V_2$ is as if we had operated with *a local space-time translation* with acceleration, $\vec{a}$, on the original space V_4. In the same way the acceleration, $\vec{\underline{a}}$, refers to $\underline{V}_2$ at $\underline{V}_2 \rightleftharpoons V_4$. The *atomic displacement*, $\Delta\underline{\widetilde{x}}_{(a)}$, is caused in V_2 by double transition of a particle, $V_2 \rightleftharpoons V_4$. All the particles are living on V_4, while their superpartners can be viewed as living on *unmanifested* Master space, V_2.

in V_4. The net result of each atomic double transition of a particle $V_4 \rightleftharpoons \underline{V}_2$ to $\underline{V}_2$ and back is as if we had operated with *a local space-time translation* with acceleration, $\vec{a}$, on the original space V_4. Accordingly, the acceleration, $\vec{\underline{a}}$, holds in $\underline{V}_2$ at $\underline{V}_2 \rightleftharpoons V_4$. So, the accelerated motion of boson $A(\widetilde{x})$ in V_4 is a chain of its sequential transformations to the Weyl fermion $\underline{\chi}(\widetilde{\underline{x}})$ defined on $\underline{V}_2$ (accompanied with the auxiliary fields $\underline{\widetilde{F}}$) and back,

$$\to A(\widetilde{x}) \to \underline{\chi}^{(F)}(\widetilde{\underline{x}}) \to A(\widetilde{x}) \to \underline{\chi}^{(F)}(\widetilde{\underline{x}}) \to, \tag{5}$$

and the same interpretation holds for fermion $\chi(\widetilde{x})$. A useful guide in the construction of local superspace is that it should admit rigid superspace as a limit. The reverse is also expected, since if one starts with a constant parameter ϵ and performs a local Lorentz transformation, then this parameter will in general become space-time dependent as a result of this Lorentz transformation.

The mathematical structure of a local theory of $\widetilde{MS}_p$-SUSY has much in common with those used in the geometrical framework of standard supergravity theories. Such a local SUSY can already be read off from the algebra of a global MS$_p$-SUSY (Ter-Kazarian, 2024a) in the form

$$[\epsilon(X)Q,\, \bar{Q}\bar{\epsilon}(X)] = 2\epsilon(X)\sigma^{\hat{\mu}}\bar{\epsilon}(X)\widetilde{p}_{\hat{\mu}}, \tag{6}$$

which says that the product of two supersymmetry transformations corresponds to a translation in 6D X-space of which the momentum

$$\widetilde{p}_{\hat{\mu}} = i\widetilde{\partial}_{\hat{\mu}} = \frac{\partial}{\partial X^{\hat{\mu}}} = (\frac{\partial}{\partial \widetilde{x}^{\mu}}, \frac{\partial}{\partial \widetilde{\underline{x}}^{\underline{\mu}}})$$

is the generator. We expect the notion of a general coordinate transformation should be

$$[\delta_{\epsilon_1(X)},\, \delta_{\epsilon_2(X)}]V = \tfrac{1}{2}\bar{\epsilon}_2(X)\sigma^{\hat{\mu}}\epsilon_1(X)\,\widetilde{\partial}_{\hat{\mu}}V. \tag{7}$$

Then for the local $\widetilde{MS}_p$-SUSY to exist it requires the background spaces (V_4, $\underline{V}_2$) to be curved. In this case, in order to become on the same footing with $\underline{V}_2$, the V_4 refers to the accelerated proper reference frame of a particle without relation to other matter fields. This leads us to extend the concept of differential forms to superspace. Points in curved superspace are then identified by the generalized coordinates

$$\begin{aligned} z^M &= (X^{\hat{\mu}},\, \Theta^{\hat{\alpha}},\, \bar{\Theta}_{\hat{\dot{\alpha}}}) = z^{(V_4)} \oplus z^{(\underline{V}_2)} \\ &= (\widetilde{x}^{\mu},\, \theta^{\alpha},\, \bar{\theta}_{\dot{\alpha}}) \oplus (\widetilde{\underline{x}}^{\underline{\mu}},\, \underline{\theta}^{\alpha},\, \underline{\bar{\theta}}_{\dot{\alpha}}), \end{aligned} \tag{8}$$

and differential elements

$$\begin{aligned} dz^M &= (dX^{\hat{\mu}},\, d\Theta^{\hat{\alpha}},\, d\bar{\Theta}_{\hat{\dot{\alpha}}}) = dz^{(V_4)} \oplus dz^{(\underline{V}_2)} \\ &= (d\widetilde{x}^{\mu},\, d\theta^{\alpha},\, d\bar{\theta}_{\dot{\alpha}}) \oplus (d\widetilde{\underline{x}}^{\underline{\mu}},\, d\underline{\theta}^{\alpha},\, d\underline{\bar{\theta}}_{\dot{\alpha}}), \end{aligned} \tag{9}$$

where $M \equiv (\hat{\mu}, \alpha, \dot{\alpha})$. Throughout we will use the 'two-in-one' notation of a theory MS$_p$-SUSY ((Appendix A), implying that any tensor (W) or spinor (Θ) with indices marked by 'hat' denote

$$\begin{aligned} &W^{\hat{\mu}_1\cdots\hat{\mu}_m}_{\hat{\nu}_1\cdots\hat{\nu}_n} := W^{\mu_1\cdots\mu_m}_{\nu_1\cdots\nu_n} \oplus W^{\underline{\mu}_1\cdots\underline{\mu}_m}_{\underline{\nu}_1\cdots\underline{\nu}_n}, \\ &\Theta^{\hat{\alpha}} := \theta^{\alpha} \oplus \underline{\theta}^{\alpha}, \quad \bar{\Theta}_{\hat{\dot{\alpha}}} := \bar{\theta}_{\dot{\alpha}} \oplus \underline{\bar{\theta}}_{\dot{\alpha}}. \end{aligned} \tag{10}$$

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This corresponds to the action of supercharge operators $Q \equiv$ (either q or $\underline{q}$) (see (88), (89)), which is due to the fact that the framework of $\widetilde{MS}_p$-SG combines bosonic and fermionic states in V_4 and $\underline{V}_2$ on the same base rotating them into each other under the action of operators $(q, \underline{q})$. The α are all upper indices, while $\dot\alpha$ is a lower index. Elements of superspace obey the following multiplication law: $z^M z^N = (-1)^{i(N)j(M)} z^N z^M$. Here $i(N)$ is a function of N and $j(M)$ is a function of M. These functions take the values zero or one, depending on whether N and M are vector or spinor indices. Exterior products in superspace are defined in complete analogy to ordinary space:

$$\begin{array}{l} dz^M \wedge dz^N = -(-1)^{i(N)j(M)} dz^N \wedge dz^M, \\ dz^M z^N = (-1)^{i(N)j(M)} z^N \wedge dz^M. \end{array} \tag{11}$$

With this definition, differential forms have a standard extension to superspace. We shall drop the symbol $\wedge$ for exterior multiplication (unless indicated otherwise) and forbear to write the details out of a standard theory. They can be seen in, e.g. (Wess & Bagger, 1983, West, 1987, van Nieuwenhuizen, 1981). The multiplication of two local sequential supersymmetric transformations induces the motion

$$\begin{array}{l} g(0,\, \epsilon(X),\, \bar\epsilon(X))\, \Omega(X^{\hat\mu},\, \Theta^{\hat\alpha},\, \bar\Theta_{\hat{\dot\alpha}} \longrightarrow \\ (X^{\hat\mu} + i\, \Theta^{\hat\alpha}\, \sigma^{\hat\mu}\, \bar\epsilon(X) - i\, \epsilon(X)\, \sigma^{\hat\mu}\, \bar\Theta_{\hat{\dot\alpha}},\, \Theta + \epsilon(X), \bar\Theta + \bar\epsilon(X)), \end{array} \tag{12}$$

which gives

$$\begin{array}{l} g_q(0,\, \xi(\widetilde{x}),\, \bar\xi(\widetilde{x}))\, \Omega_q(\widetilde{x},\, \theta,\, \bar\theta) \;\rightarrow \\ (\widetilde{x}^m + i\, \theta\, \sigma^m\, \bar\xi(\widetilde{x}) - i\, \xi(\widetilde{x})\, \sigma^m\, \bar\theta,\;\; \theta + \xi(\widetilde{x}),\, \bar\theta + \bar\xi(\widetilde{x})), \\ g_{\underline{q}}(0,\, \underline{\xi}(\underline{\widetilde{x}}),\, \underline{\bar\xi}(\underline{\widetilde{x}}))\, \Omega_{\underline{q}}(\underline{\widetilde{x}},\, \underline{\theta},\, \underline{\bar\theta}) \;\rightarrow \\ (\underline{\widetilde{x}}^{\underline{m}} + i\, \underline{\theta}\, \sigma^{\underline{m}}\, \underline{\bar\xi}(\underline{\widetilde{x}}) - i\, \underline{\xi}(\underline{\widetilde{x}})\, \sigma^{\underline{m}}\, \underline{\bar\theta},\;\; \underline{\theta} + \underline{\xi}(\underline{\widetilde{x}}),\, \underline{\bar\theta} + \underline{\bar\xi}(\underline{\widetilde{x}})). \end{array} \tag{13}$$

In its simplest version, supergravity was conceived as a quantum field theory whose action included the gravitation field term, where the graviton coexists with a fermionic field called gravitino, described by the Rarita-Scwinger kinetic term. The two fields differ in their spin: 2 for the graviton, 3/2 for the gravitino. The different 4D $N = 1$ supergravity multiplets all contain the graviton and the gravitino, but differ by their systems of auxiliary fields. These fields would transform into each other under local supersymmetry. In this framework supersymmetry and general coordinate transformations are described in a unified way as certain diffeomorphisms. The motion (12) generates certain coordinate transformations:

$$z^M \;\longrightarrow\; z'^M = z^M - \zeta^M(z), \tag{14}$$

where $\zeta^M(z)$ arbitrary functions of z. The dynamical variables of superspace formulation are the frame field $E^A(z)$ and connection Ω. Using the analogue of Cartan's local frame, the superspace $(z^M,\, \Theta,\, \bar\Theta)$ has at each point a tangent superspace spanned by the frame field defined as a 1-form over superspace

$$E^A(z) = dz^M E_M^{\;A}(z), \tag{15}$$

with coefficient superfields, generalizing the usual frame, namely supervierbien $E_M^{\;A}(z)$. Here, we use the first half of capital Latin alphabet $A, B, \ldots$ to denote the anholonomic indices related to the tangent superspace structure group, which is taken to be just the Lorentz group. The inverse vielbein $E_A^{\;M}(z)$ is defined by the relations

$$E_M^{\;A}(z) E_A^{\;N}(z) = \delta_M^{\;N}, \quad E_A^{\;M}(z) E_M^{\;B}(z) = \delta_A^{\;B}, \tag{16}$$

where

$$\delta_M^{\;N} = \begin{pmatrix} \delta_{\hat m}^{\;\hat n} & 0 & 0 \\ 0 & \delta_{\hat\mu}^{\;\hat\nu} & 0 \\ 0 & 0 & \delta_{\hat{\dot\mu}}^{\;\hat{\dot\nu}} \end{pmatrix}, \tag{17}$$

The formulation of supergravity in superspace provides a unified description of the vierbein and the Rarita-Schwinger fields, which are identified in a common geometric object, the local frame $E^A(z)$ of superspace. They are manifestly coordinate independent. The upper index A is reserved for the structure group, for which we take the Lorentz group. This is because we would like to recover supersymmetric flat space as a solution to our dynamical theory. With this choice, the reference frame defined by the vielbein is locally Lorentz covariant.

$$\delta E^A = E^B L_B^{\;A}(z), \quad \delta E_M^{\;A} = E_M^{\;B} L_B^{\;A}(z). \tag{18}$$

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The indices transforming under the structure group will be called Lorentz indices. The Lorentz generators $L_B^{\ A}(z)$ have three irreducible components: $L_{\hat b}^{\ \hat a}, L_\beta^{\ \alpha}$ and $L^{\dot\beta}_{\ \dot\alpha}$. The vielbein forms $E^{\hat a} = dz^M E_M^{\hat a}$, $E^{\hat\alpha} = dz^M E_M^{\hat\alpha}$, and $E_{\hat\alpha} = dz^M E_{M\hat\alpha}$ are coordinate-independent irreducible Lorentz tensors.

To formulate covariant derivatives one must introduce a connection form

$$\phi = dz^M \phi_M, \quad \phi_M = \phi_{MA}^{\ \ B}, \tag{19}$$

transforming as follows under the structure group:

$$\delta\phi = \phi L - L\phi - dL. \tag{20}$$

Connections are Lie algebra valued one-forms

$$\phi = dz^M \phi_M^r(z) i T^r, \tag{21}$$

with the following transformation law:

$$\phi' = X^{-1}\phi X - X^{-1} dX, \tag{22}$$

where r runs over the dimension of the algebra. The connection is the second dynamical variable in this theory. The $\phi_{MA}^{\ \ B}$ is Lie algebra valued in its two Lorentz indices:

$$\phi_{MAB} = -(-1)^{ab}\phi_{MBA}. \tag{23}$$

Covariant derivatives with respect to local Lorentz transformations are constructed by means of the spin connection (or Lorentz connection), which is 1-form assuming values in the Lie algebra of the Lorentz group:

$$\omega_M(z) = \tfrac{1}{2}\omega^{AB}_{\ \ M}(z) S_{AB}, \tag{24}$$

with Lorentz generators S_{AB} of a given representation Y. The covariant derivative of the vielbein is called torsion:

$$T^A = dE^A + E^B \phi_B^{\ A}. \tag{25}$$

In flat space it is possible to transform the vielbein into the global reference frame: $E^A = e^A$. It is defined up to rigid Lorentz transformations. In this frame the connection vanishes: $\phi = 0$. The torsion, however, is non-zero because of the following non-zero components:

$$T_{\hat\alpha\hat{\dot\beta}}^{\ \ \ \hat c} = T_{\hat{\dot\beta}\hat\alpha}^{\ \ \ \hat c} = 2i\sigma_{\hat\alpha\hat{\dot\beta}}^{\ \ \ \hat c}. \tag{26}$$

The curvature tensor is defined in terms of the connection:

$$R = d\phi + \phi\phi. \tag{27}$$

It is a Lie algebra valued two-form:

$$R_A^{\ B} = \tfrac{1}{2} dz^M dz^N R_{NMA}^{\ \ \ \ B}. \tag{28}$$

Covariant derivatives with respect to local Lorentz transformations are constructed by means of the spin connection Ω, which is a 1-form in superspace. Supergauge transformations are constructed from the general coordinate and structure group transformations of superspace:

$$L_B^{\ A} = -\zeta^C \phi_{CB}^{\ \ A}. \tag{29}$$

They amount to a convenient reparametrization of these transformations. Supergauge transformations map Lorentz tensors into Lorentz tensors and reduce to supersymmetry transformations in the limit of flat space. The parameter ζ characterizes infinitesimal changes in coordinates. Whereas, either ζ^A or ζ^M may be chosen as the field-independent transformation parameter. Its companion then depends on the fields through the vielbein. Since we would like Lorentz tensors to transform into Lorentz tensors, we shall choose ζ^A to be field-independent. Supergauge transformations consist of a general coordinate transformation with field-independent parameter ζ^A followed by a structure group Lorentz transformation with field-dependent parameter (39). It is among this restricted class of transformations that we shall find the gauged supersymmetry transformations.

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The super-vielbein $E_M{}^A$ and spin-connection Ω contain many degrees of freedom. Although some of these are removed by the tangent space and supergeneral coordinate transformations, there still remain many degrees of freedom. There is no general prescription for deducing necessary covariant constraints which if imposed upon the superfields of super-vielbein and spin-connection will eliminate the component fields. However, some usual constraints can be found using tangent space and supergeneral coordinate transformations of the torsion and curvature covariant tensors, given in appropriate super-gauge. The transformation parameters ζ^A and $L_{\hat{a}\hat{b}}$ are functions of superspace. Their lowest components characterize general coordinate transformations in six-dimensional X-space $[\zeta^{\hat{a}}(X^{\hat{m}})]$, gauged supersymmetry transformations $[\zeta^{\hat{\alpha}}(X)], \zeta_{\hat{\dot{\alpha}}}(X)]$, and local Lorentz transformations $L_{\hat{a}\hat{b}}(X)$. We will use their higher components to transform away certain $\Theta = \bar{\Theta} = 0$ components of the vielbein and the connection.

Let us consider the vielbein. Its transformation law may be written as a super-gauge transformation together with an additional Lorentz transformation $L_B{}^A$:

$$\delta_\zeta E_M{}^A = -\mathcal{D}_M \zeta^A - \zeta^B T_{BM}{}^A + E_M{}^B L_B{}^A. \tag{30}$$

The lowest component of this equation gives the transformation property of $E_M{}^A|_{\Theta=\bar{\Theta}=0}$. The $\Theta = \bar{\Theta} = 0$ components of ζ^α and $\bar{\zeta}_{\dot{\alpha}}$ parametrize gauged supersymmetry transformations:

$$\begin{array}{l} \zeta^{\hat{a}}(z)|_{\Theta=\bar{\Theta}=0} = 0, \quad \zeta^\alpha(z)|_{\Theta=\bar{\Theta}=0} = \zeta^\alpha(X), \\ \bar{\zeta}_{\dot{\alpha}}(z)|_{\Theta=\bar{\Theta}=0} = \bar{\zeta}_{\dot{\alpha}}(X), \quad L_{AB}(z) = |_{\Theta=\bar{\Theta}=0} = 0. \end{array} \tag{31}$$

Higher components of ζ^A enter $\delta E_M{}^A|$ through the covariant derivatives $\mathcal{D}_\alpha \zeta^A$ and $\bar{\mathcal{D}}^{\dot{\alpha}} \zeta^A$. One may use these higher components to transform super-vielbein to the final form, where the minimum number of independent component fields are the graviton, $e_{\hat{\mu}}{}^{\hat{a}}(X)$, and the gravitino, $\Psi_{\hat{\mu}}{}^\alpha(X), \bar{\Psi}_{\hat{\mu}\dot{\alpha}}(X)$. Since, by virtue of (10),

$$E_A{}^M(z)\big|_{\Theta=\bar{\Theta}=0} = E_a{}^\mu(z^{(V_4)})\big|_{\theta=\bar{\theta}=0}, \oplus\, E_{\underline{a}}{}^{\underline{\mu}}(z^{\underline{V}_2})\Big|_{\underline{\theta}=\underline{\bar{\theta}}=0} \tag{32}$$

accordingly, we find

$$E_a{}^\mu(z^{(V_4)})\big|_{\theta=\bar{\theta}=0} = \begin{pmatrix} e_\mu{}^a(x) & \frac{1}{2}\psi_\mu{}^\alpha(x) & \frac{1}{2}\bar{\psi}_{\mu\dot{\alpha}}(x) \\ 0 & \delta^\alpha_\gamma & 0 \\ 0 & 0 & \delta^{\dot{\gamma}}_{\dot{\alpha}} \end{pmatrix}, \tag{33}$$

and

$$E_{\underline{a}}{}^{\underline{\mu}}(z^{\underline{V}_2})\Big|_{\underline{\theta}=\underline{\bar{\theta}}=0} = \begin{pmatrix} e_{\underline{\mu}}{}^{\underline{a}}(\underline{\widetilde{x}}) & \frac{1}{2}\psi_{\underline{\mu}}{}^\alpha(\underline{\widetilde{x}}) & \frac{1}{2}\bar{\psi}_{\underline{\mu}\dot{\alpha}}(\underline{\widetilde{x}}) \\ 0 & \delta^\alpha_\gamma & 0 \\ 0 & 0 & \delta^{\dot{\gamma}}_{\dot{\alpha}} \end{pmatrix}. \tag{34}$$

The fields of graviton and gravitino cannot be gauged away. Provided, we have

$$\begin{array}{l} e_{\hat{a}}{}^{\hat{\mu}} e_{\hat{\mu}}{}^{\hat{b}} = \delta^{\hat{b}}_{\hat{a}}, \quad \Psi_{\hat{a}}{}^\gamma = e_{\hat{a}}{}^{\hat{\mu}} \Psi_{\hat{\mu}}{}^\alpha \delta^\gamma_\alpha, \\ \bar{\Psi}_{\hat{a}\dot{\gamma}} = e_{\hat{a}}{}^{\hat{\mu}} \bar{\Psi}_{\hat{\mu}\dot{\alpha}} \delta_{\dot{\gamma}\dot{\alpha}}. \end{array} \tag{35}$$

The tetrad field $e_{\hat{\mu}}{}^{\hat{a}}(X)$ $(= e_\mu{}^a(x) \oplus e_{\underline{\mu}}{}^{\underline{a}}(\underline{\widetilde{x}}))$ plays the role of a gauge field associated with local transformations. The Majorana type field $\Psi_{\hat{\mu}}{}^\alpha(X)$ $(= \psi_\mu{}^\alpha(x) \oplus \psi_{\underline{\mu}}{}^\alpha(\underline{\widetilde{x}}))$ is the gauge field related to local supersymmetry. These two fields belong to the same supergravity multiplet which also accommodates auxiliary fields so that the local supersymmetry algebra closes. Under infinitesimal transformations of local supersymmetry, they transformed as

$$\begin{array}{l} \delta e_{\hat{\mu}}{}^{\hat{a}} = i(\Psi_{\hat{\mu}} \sigma^{\hat{a}} \zeta - \zeta \sigma^{\hat{a}} \bar{\Psi}_{\hat{\mu}}), \\ \delta \Psi_{\hat{\mu}} = -2\mathcal{D}_{\hat{\mu}} \zeta^\alpha + i e_{\hat{\mu}}{}^{\hat{c}} \{\frac{1}{3} M (\varepsilon \sigma_{\hat{c}} \bar{\zeta})^\alpha + b_{\hat{c}} \zeta^\alpha + \frac{1}{3} b^{\hat{d}} (\zeta \sigma_{\hat{d}} \bar{\sigma}_{\hat{c}})\}, \end{array} \tag{36}$$

etc., where $M(X) = -6R(z)|_{\Theta=\bar{\Theta}=0}$ and $b_{\hat{a}}(X) = -3G(z)|_{\Theta=\bar{\Theta}=0}$ are the auxiliary fields, and

$$\begin{array}{l} \zeta^\alpha(z) = \zeta^\alpha(X), \quad \bar{\zeta}^\alpha(z) = \bar{\zeta}^\alpha(X), \\ \zeta^{\bar{a}}(z) = 2i[\Theta \sigma^{\hat{a}} \bar{\zeta}(X) - \zeta(X) \sigma^{\hat{a}} \bar{\Theta}]. \end{array} \tag{37}$$

These auxiliary fields are not restricted by any differential equations in X-space. We cut short further description of the unitary supersymmetry representations that give rise to the concept of supermultiplets, since they are so well known.

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3. Non-trivial linear representation of the $\widetilde{MS}_p$-SUSY algebra

With these guidelines to follow, we start by considering a simplest example of a supersymmetric theory in six dimensional background curved spaces $V_4 \oplus \underline{V}_2$ as a $\widetilde{MS}_p$-generalization of flat space MS_p-SUSY model. We consider the chiral superfield, which is instructive because it contains the essential elements of the $\widetilde{MS}_p$-SUSY. The chiral superfields are defined as $\bar{\mathcal{D}}_{\dot{\alpha}}\Phi = 0$, which reduces to $\bar{D}_{\dot{\alpha}}\Phi = 0$ in flat space. To obtain a feeling for this model we may consider first example of non-trivial linear representation $(\widehat{\chi}, \mathcal{A}, \mathcal{F})$, of the $\widetilde{MS}_p$-SUSY algebra. This has $N = 1$ and $s_0 = 0$, and contains two Weyl spinor states $\widehat{\chi}(\chi,\, \underline{\chi})$, two complex scalar fields $\mathcal{A}(A,\, \underline{A})$, and two more real scalar degrees of freedom in the complex auxiliary fields $\mathcal{F}(F,\, \underline{F})$, which provide in a supersymmetry theory the fermionic and bosonic degrees of freedom to be equal off-shell as well as on-shell, and are eliminated when one goes on-shell. The component multiplets, $(\widehat{\chi}, \mathcal{A}, \mathcal{F})$, are called the chiral or scalar multiplets. We could define the component fields as the coefficient functions of a power series expansion in Θ and $\bar{\Theta}$. This decomposition, however, is coordinate-dependent. It is, therefore, more convenient to define them as

$$\mathcal{A} = \Phi|_{\Theta=\bar{\Theta}=0}\,, \quad \widehat{\chi}_\alpha = \tfrac{1}{\sqrt{2}}\mathcal{D}_\alpha\, \Phi|_{\Theta=\bar{\Theta}=0}\,, \quad \mathcal{F} = -\tfrac{1}{4}\mathcal{D}^\alpha\mathcal{D}_\alpha\, \Phi|_{\Theta=\bar{\Theta}=0}\,, \tag{38}$$

which carry Lorentz indices. They are related to the Θ and $\bar{\Theta}$ expansion coefficients through a transformation which depends on the supergravity multiplet. The transformation laws of the component fields are found from the transformation law of the superfield Φ: $\delta\Phi = -\zeta^A\mathcal{D}_A\Phi$, provided, the parameters ζ^A are specified as (37). Under infinitesimal transformations of local supersymmetry, the transformation law of the chiral multiplet, incorporating with embedding map $\underline{V}_2 \hookrightarrow V_4$ (4), and the transformation law $\underline{A}(\underline{\widetilde{x}}) = A(\widetilde{x})$ for spin-zero scalar field, give

$$\begin{aligned}
&\delta\underline{A} = -\sqrt{2}\zeta^\alpha\chi_\alpha, \\
&\delta\chi_\alpha = -\sqrt{2}\zeta_\alpha F - i\sqrt{2}\sigma_{\alpha\dot{\beta}}{}^{a}\bar{\zeta}^{\dot{\beta}}\widehat{D}_a\underline{A}, \\
&\delta F = -\tfrac{\sqrt{2}}{3}M^*\zeta^\alpha\chi_\alpha + \bar{\zeta}^{\dot{\alpha}}\left(\tfrac{1}{6}\sqrt{2}b_{\alpha\dot{\alpha}}\chi^\alpha - i\sqrt{2}\widehat{D}_{\alpha\dot{\alpha}}\chi^\alpha\right),
\end{aligned} \tag{39}$$

where

$$\begin{aligned}
&\widehat{D}_a\underline{A} \equiv e_a{}^\mu\left[\left(\tfrac{\partial\widetilde{x}^0}{\partial\widetilde{x}^\mu}\right)\underline{\widetilde{\partial}_0}\,\underline{A} + \left(\tfrac{\partial|\vec{\widetilde{x}}|}{\partial\widetilde{x}^\mu}\right)\underline{\widetilde{\partial}_1}\,\underline{A} - \tfrac{1}{\sqrt{2}}\Psi_\mu{}^\beta\chi_\beta\right], \\
&\widehat{D}_a\chi_\alpha = e_a{}^\mu\left(\mathcal{D}_\mu\chi_\alpha - \tfrac{1}{\sqrt{2}}\Psi_{\mu\alpha}F - \tfrac{i}{\sqrt{2}}\bar{\Psi}_\mu{}^{\dot{\beta}}\widehat{D}_{\alpha\dot{\beta}}\underline{A}\right).
\end{aligned} \tag{40}$$

In the same way, we should define the spinor $\underline{\chi}$ as the field into which $A(x)$ transforms. In this case, the infinitesimal supersymmetry transformations for $Q = \underline{q}$ read

$$\begin{aligned}
&\delta A = -\sqrt{2}\underline{\zeta}^\alpha\underline{\chi}_\alpha, \\
&\delta\underline{\chi}_\alpha = -\sqrt{2}\underline{\zeta}_\alpha\underline{F} - i\sqrt{2}\sigma_{\alpha\dot{\beta}}{}^{\underline{a}}\bar{\underline{\zeta}}^{\dot{\beta}}\widehat{D}_{\underline{a}}A, \\
&\delta\underline{F} = -\tfrac{\sqrt{2}}{3}\underline{M}^*\underline{\zeta}^\alpha\underline{\chi}_\alpha + \bar{\underline{\zeta}}^{\dot{\alpha}}\left(\tfrac{1}{6}\sqrt{2}\underline{b}_{\alpha\dot{\alpha}}\underline{\chi}^\alpha - i\sqrt{2}\widehat{D}_{\alpha\dot{\alpha}}\underline{\chi}^\alpha\right),
\end{aligned} \tag{41}$$

where

$$\begin{aligned}
&\widehat{D}_{\underline{a}}A \equiv \underline{e}_{\underline{a}}{}^{\underline{\mu}}\left[\left(\tfrac{\partial\widetilde{x}^0}{\partial\underline{\widetilde{x}}^{\underline{\mu}}}\right)\widetilde{\partial}_0\,A + \left(\tfrac{\partial\widetilde{x}^i}{\partial\underline{\widetilde{x}}^{\underline{\mu}}}\right)\widetilde{\partial}_i\,A - \tfrac{1}{\sqrt{2}}\underline{\Psi}_{\underline{\mu}}{}^\beta\underline{\chi}_\beta\right], \\
&\widehat{D}_{\underline{a}}\chi_\alpha = \underline{e}_{\underline{a}}{}^{\underline{\mu}}\left(\mathcal{D}_{\underline{\mu}}\underline{\chi}_\alpha - \tfrac{1}{\sqrt{2}}\underline{\Psi}_{\underline{\mu}\alpha}\underline{F} - \tfrac{i}{\sqrt{2}}\underline{\bar{\Psi}}_{\underline{\mu}}{}^{\dot{\beta}}\widehat{D}_{\alpha\dot{\beta}}A\right).
\end{aligned} \tag{42}$$

The graviton and the gravitino form thus the basic multiplet of local $\widetilde{MS}_p$-SUSY, and one expects the simplest locally supersymmetric model to contain just this multiplet.

4. The simple $(N = 1)$ $\widetilde{MS}_p$ - SG without auxiliary fields

An essential difference arisen between the standard supergravity theories and some rather unusual properties of a $\widetilde{MS}_p$-SG theory is as follows. In the framework of the standard supergravity theories, as in GR, a curvature of the space-time acts on all the matter fields. The source of graviton is the energy-momentum tensor of matter fields, while the source of gravitino is the spin-vector current of supergravity. The gauge action of simple $\widetilde{MS}_p$-SG is the sum of the Hilbert action for the tetrad field - *fictitious* graviton, and the Rarita-Schwinger action for the *fictitious* gravitino field. Instead we argue that they refer to the particle of

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interest itself, without relation to other matter fields, so that these fields can be globally removed by appropriate coordinate transformations. The $\widetilde{MS}_p$-SG theory is so constructed as to make these two particles just as being the two bosonic and fermionic states in the curved background spaces V_4 and $\underline{V}_2$, respectively, or vice versa. Whereas, in order to become on the same footing with $\underline{V}_2$, the V_4 refers only to the accelerated proper reference frame of a particle. With these physical requirements, a standard coupling of supergravity with matter superfields evidently no longer holds. We are now looking for an alternative way of implications of $\widetilde{MS}_p$-SG for the model of accelerated motion and inertial effects.

We will use the techniques of (van Nieuwenhuizen, 1981) extended in a plausible fashion to the $\widetilde{MS}_p$-SG. The generalized Poincaré superalgebra for the simple ($N=1$) $\widetilde{MS}_p$-SG reads:

$$\begin{aligned}
&[P_{\hat a},P_{\hat b}]=0, \quad [S_{\hat a\hat b},P_{\hat c}]=(\eta_{\hat a\hat c}P_{\hat b}-\eta_{\hat b\hat c}P_{\hat a}),\\
&[S_{\hat a\hat b},S_{\hat c\hat d}]=i(\eta_{\hat a\hat c}S_{\hat b\hat d}-\eta_{\hat b\hat c}S_{\hat a\hat d}+\eta_{\hat b\hat d}S_{\hat a\hat c}-\eta_{\hat a\hat d}S_{\hat b\hat c}),\\
&[S_{\hat a\hat b},Q^{\alpha}]=\tfrac{1}{2}(\gamma_{\hat a\hat b})^{\alpha}_{\beta}Q^{\beta},\\
&[P_{\hat a},Q^{\beta}]=0, \quad [Q_{\alpha},\bar Q_{\dot\beta}]=\tfrac{1}{2}(\gamma^{\hat a})_{\alpha\dot\beta}P_{\hat a}.
\end{aligned} \tag{43}$$

with $(S_{\hat a\hat b})^{\hat c}_{\hat d}=i(\delta^{\hat c}_{\hat a}\eta_{\hat b\hat d}-\delta^{\hat c}_{\hat b}\eta_{\hat a\hat d})$ (24) a given representation of the Lorentz generators. Using (43) and a general form for gauge transformations on B^A,

$$\delta B=\mathcal{D}\lambda=d\lambda+[B,\lambda], \tag{44}$$

with

$$\lambda=\rho^{\hat a}P_{\hat a}+\tfrac{1}{2}\kappa^{\hat a\hat b}S_{\hat a\hat b}+\bar Q\varepsilon, \tag{45}$$

we obtain that the $(e^{\hat a},\omega^{\hat a\hat b},\Psi)$ transform under Poincaré translations as

$$\delta e^{\hat a}=\mathcal{D}\rho^{\hat a}, \quad \delta\omega^{\hat a\hat b}=0, \quad \delta\Psi=0; \tag{46}$$

under Lorentz rotations as

$$\delta e^{\hat a}=\kappa^{\hat a}_{\hat b}\delta e^{\hat b}, \quad \delta\omega^{\hat a\hat b}=-\mathcal{D}\kappa^{\hat a\hat b}, \quad \delta\Psi=\tfrac{1}{4}\kappa^{\hat a\hat b}\gamma_{\hat a\hat b}\Psi; \tag{47}$$

and under supersymmetry transformation as

$$\delta e^{\hat a}=\tfrac{1}{2}\bar\varepsilon\gamma^{\hat a}\Psi, \quad \delta\omega^{\hat a\hat b}=0, \quad \delta\Psi=\mathcal{D}\varepsilon. \tag{48}$$

In first-order formalism, the gauge fields $(e^{\hat a},\omega^{\hat a\hat b},\,\Psi)$, (with $\Psi=(\psi,\underline{\psi})$ a two-component Majorana spinor) are considered as an independent members of multiplet in the adjoint representation of the Poincaré supergroup of $D=6$ $((3+1),(1+1))$ simple ($N=1$) $\widetilde{MS}_p$-SG with the generators $(P_{\hat a},S_{\hat a\hat b},Q^{\alpha})$. Unless indicated otherwise, henceforth the world indices are kept implicit without ambiguity. The operators carry Lorentz indices not related to coordinate transformations. The Yang-Mills connection for the Poincare´ supergroup is given by

$$B=B^AT_A=e^{\hat a}P_{\hat a}+\tfrac{1}{2}i\omega^{\hat a\hat b}S_{\hat a\hat b}+\Psi\bar Q. \tag{49}$$

The field strength associated with connection B is defined as the Poincaré Lie superalgebra-valued curvature two-form R^A. Splitting the index A, and taking the $\Theta=\bar\Theta=0$ component of R^A, we obtain

$$\begin{aligned}
&R^{\ \hat a\hat b}(\omega)=d\omega^{\hat a\hat b}-\omega^{\hat a}{}_{\hat c}\omega^{\hat c\hat d},\\
&\tilde T^{\hat a}=T^{\hat a}-\tfrac{1}{2}\bar\Psi\gamma^{\hat a}\Psi, \quad \rho=\mathcal{D}\Psi,
\end{aligned} \tag{50}$$

where $\gamma^{\hat a}=(\gamma^a,\sigma^{\underline{a}})$, $R^{\ \hat a\hat b}(\omega)$ is the Riemann curvature in terms of the spin connection $\omega^{\hat a\hat b}$, and the generalized Weyl lemma (see App./(4)) requires that the, so-called, supertorsion $\tilde T^{\hat a}$ be inserted. The solution $\omega(e)$ satisfies the tetrad postulate that the completely covariant derivative of the tetrad field vanishes, therefore $R^{\hat a\hat b}(\omega)=R(\omega)e^{\hat a}e^{\hat b}$.

For the bosonic part of the gauge action (graviton of spin 2) of simple $\widetilde{MS}_p$-SG it then seems appropriate to take the generalized Hilbert action with $e=det\,e^{\hat a}{}_{\mu}(X)$. While the fermionic part of the standard gauge action (garvitino of spin 3/2), which has positive energy, is the Rarita-Schwinger action. The full nonlinear gravitino action in curved space then should be its extension to curved space, which can be achieved by inserting the Lorentz covariant derivative $\mathcal{D}\Psi=d\Psi+\frac{1}{2}\omega^{\ \hat a\hat b}\gamma_{\hat a\hat b}\Psi$. In both parts, the spin connection is

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considered a dependent field, otherwise in the case of an independent spin connection ω, the action will be invariant under diffeomorphism, and under local Lorentz rotations, but it will be not invariant under the neither the Poincaré translations nor the supersymmetry. In the case if spin connection is independent, we should have under the local Poincaré translations

$$\delta\hat{\mathcal{L}}_{pt} = \delta\left(\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}e^{\hat{a}}e^{\hat{b}}R^{\hat{c}\hat{d}} + 4\bar{\Psi}\gamma_{\hat{5}}e^{\hat{a}}\gamma_{\hat{a}}\mathcal{D}\Psi\right) = 2\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}R^{\hat{a}\hat{b}}\tilde{T}^{\hat{c}}\rho^{\hat{d}} + \text{surf. term}, \tag{51}$$

and under local supersymmetry transformations

$$\delta\hat{\mathcal{L}}_{SUSY} = -4\bar{\varepsilon}\gamma_{\hat{5}}\gamma_{\hat{a}}\mathcal{D}\Psi\tilde{T}^{\hat{a}} + \text{surf. term}. \tag{52}$$

The invariance of the action then requires the vanishing of the supertorsion $\tilde{T}^{\hat{a}} = 0$, which means that the connection is no longer an independent variable. So that the starting point of our approach is the action of a simple $\widetilde{MS}_p$-SG theory written in 'two in one'-notation (10), which is invariant under the local supersymmetry transformation (48), where the Poincaré superalgebra closes off shell without the need for any auxiliary fields:

$$\mathcal{L}_{MS-SG} = \varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}e^{\hat{a}}e^{\hat{b}}R^{\hat{c}\hat{d}}(\omega) + 4\bar{\Psi}\gamma_{\hat{5}}e^{\hat{a}}\gamma_{\hat{a}}\mathcal{D}\Psi. \tag{53}$$

This is the sum of bosonic and fermionic parts with the same spin connection, where $\gamma_{\hat{a}} = (\gamma_a \oplus \sigma_{\underline{a}})$, $\gamma_{\hat{5}} = (\gamma_5 \oplus \gamma_{\underline{5}})$, $\gamma_{\underline{5}} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is given in the chiral or Weyl representations, i.e. in the irreducible 2-dimensional spinor representations $(\frac{1}{2}, 0)$ and $(0, \frac{1}{2})$, since two-component formalism works for a Weyl fermion. This is indispensable in order to solve algebraical constraints in superspace because they can be used as building blocks of any fermion field (van Nieuwenhuizen, 1981). In this representation, action of projection matrices $L = (1/2)(1 + \gamma_{\underline{5}})$ and $R = (1/2)(1 - \gamma_{\underline{5}})$ on a Dirac fermion leads to zero two lower components of the left-handed spinor and zero two upper components of the right-handed spinor, respectively. The two-component notation described above essentially does away with the vanishing components explicitly and deals only with the non-trivial ones. Taking into account that $g_{\hat{\mu}\hat{\nu}} = \eta_{\hat{a}\hat{b}}e_{\hat{\mu}}{}^{\hat{a}}e_{\hat{\nu}}{}^{\hat{b}}$ and $\gamma_{\hat{\mu}} = e_{\hat{\mu}}{}^{\hat{a}}\gamma_{\hat{a}}$, with $\eta_{\hat{a}\hat{b}} = (\eta_{ab} \oplus \underline{\eta}_{\underline{ab}})$ related to the tangent space, where $\eta_{ab} = diag(+1, -1, -1, -1)$ and $\underline{\eta}_{\underline{ab}} = diag(+1, -1)$, we can recast the generalized bosonic and fermionic actions given in (53), respectively, in the forms

$$\mathcal{L}^{(2)} = -\tfrac{1}{4}\sqrt{g}R(g, \Gamma) = -\tfrac{1}{4}eR(e, \omega), \tag{54}$$

and

$$\mathcal{L}^{(3/2)} = 4\varepsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\bar{\Psi}_{\hat{\mu}}\gamma_{\hat{5}}\gamma_{\hat{\nu}}\mathcal{D}_{\hat{\rho}}\Psi_{\hat{\sigma}}. \tag{55}$$

Here we taken into account that $\mathcal{D}_{\hat{\rho}}\Psi_{\hat{\sigma}}$ is the curl due to the ε-symbol, and as far as $\varepsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}$ is the density (which always equals $0, \pm 1$), so there is no need to put the density e in front of fermionic part. The variation of action (53) with respect to $(e^{\hat{a}}, \omega^{\hat{a}\hat{b}}, \bar{\Psi})$ leads to the following equations for $\widetilde{MS}_p$ - SG:

$$\begin{aligned} &\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}e^{\hat{b}}R^{\hat{c}\hat{d}} + 2\bar{\Psi}\gamma_{\hat{5}}\gamma_{\hat{a}}\mathcal{D}\Psi = 0; \\ &\varepsilon_{\hat{a}\hat{b}\hat{c}\hat{d}}e^{\hat{c}}\tilde{T}^{\hat{d}} = 0; \ \gamma_{\hat{5}}e^{\hat{a}}\gamma_{\hat{a}}\mathcal{D}\Psi = 0, \end{aligned} \tag{56}$$

which can be rewritten as well in the form

$$\begin{aligned} &R^{\hat{\tau}\hat{\mu}} - \tfrac{1}{2}g^{\hat{\tau}\hat{\mu}}R + 2\varepsilon^{\hat{\lambda}\hat{\mu}\hat{\nu}\hat{\rho}}\bar{\Psi}_{\hat{\lambda}}\gamma_{\hat{5}}\gamma^{\hat{\tau}}\mathcal{D}_{\hat{\nu}}\Psi_{\hat{\rho}} = 0; \\ &\tilde{T}^{\hat{\lambda}}{}_{\hat{\mu}\hat{\nu}} = T^{\hat{\lambda}}{}_{\hat{\mu}\hat{\nu}} - \tfrac{1}{2}\bar{\Psi}_{\hat{\mu}}\gamma^{\hat{\lambda}}\Psi_{\hat{\nu}}; \\ &\varepsilon^{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}}\gamma_{\hat{5}}\gamma_{\hat{\nu}}\mathcal{D}_{\hat{\rho}}\Psi_{\hat{\sigma}} = 0. \end{aligned} \tag{57}$$

This, according to (10), gives the following equations for the Rarita-Schwinger fields $\psi(x)$ and $\underline{\psi}(\underline{x})$ defined, respectively, on the $x \in V_4$ and $\underline{x} \in \underline{V}_2$:

$$\varepsilon^{\mu\nu\rho\sigma}\gamma_5\gamma_\nu\mathcal{D}_\rho\psi_\sigma = 0, \quad \varepsilon^{\underline{\mu\nu\rho\sigma}}\gamma_{\underline{5}}\sigma_{\underline{\nu}}\underline{\mathcal{D}}_{\underline{\rho}}\underline{\psi}_{\underline{\sigma}} = 0. \tag{58}$$

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5. Particle mechanics in the 4D Minkowski space-time: Velocity and acceleration

From embedding map (4), we obtain the components of velocity of a particle

$$\begin{array}{l} \underline{\widetilde{v}}^{(\pm)} = \frac{d\underline{x}^{(\pm)}}{d\underline{\widetilde{x}}^{\underline{0}}} = \frac{1}{\sqrt{2}}(\underline{\widetilde{v}}^{\underline{0}} \pm \underline{\widetilde{v}}^{\underline{1}}), \\ \underline{\widetilde{v}}^{\underline{1}} = \frac{d\underline{\widetilde{x}}^{\underline{1}}}{d\underline{\widetilde{x}}^{\underline{0}}} = |\vec{\widetilde{v}}| = |\frac{d\vec{\widetilde{x}}}{d\widetilde{x}^{0}}|, \end{array} \tag{59}$$

so that

$$\begin{array}{l} \underline{\widetilde{u}} = \underline{\widetilde{e}}_{\underline{m}}\underline{\widetilde{v}}^{\underline{m}} = (\vec{\underline{\widetilde{v}}}_{\underline{0}}, \vec{\underline{\widetilde{v}}}_{\underline{1}}), \\ \vec{\underline{\widetilde{v}}}_{\underline{0}} = \underline{\widetilde{e}}_{0}\underline{\widetilde{v}}^{\underline{0}}, \quad \vec{\underline{\widetilde{v}}}_{\underline{1}} = \underline{\widetilde{e}}_{\underline{1}}\underline{\widetilde{v}}^{\underline{1}} = \vec{\widetilde{n}}|\vec{\widetilde{v}}| = \vec{\widetilde{v}}, \end{array} \tag{60}$$

therefore, $\underline{\widetilde{u}} = (\vec{\underline{\widetilde{v}}}_{\underline{0}}, \vec{\underline{\widetilde{v}}}_{\underline{1}}) = \widetilde{u} = (\widetilde{e}_0, \vec{\widetilde{v}})$. Thence, the components of the acceleration vector, $\dot{a}^{\hat{\rho}} = (a^{\rho}, \underline{a}^{\underline{\rho}})$, satisfy the following embedding relations

$$\underline{a}^{\underline{0}} = a^{0}, \quad \underline{a}^{\underline{1}} = |\vec{a}|. \tag{61}$$

The accelerated motion of a particle is described by the parameter $\epsilon = \epsilon(X^{\hat{\mu}})$ in (12) of local SUSY, which depends explicitly on $X^{\hat{\mu}} = (\widetilde{x}^{\mu}, \underline{\widetilde{x}}^{\underline{\mu}})$, where $\widetilde{x}^{\mu} \in V_4$ and $\underline{\widetilde{x}}^{\underline{\mu}} \in \underline{V}_2$. To be specific, let us focus for the motion (37) on the simple case of a peculiar anticommuting spinors $(\underline{\xi}(\underline{\widetilde{x}}), \bar{\underline{\xi}}(\underline{\widetilde{x}}))$ and $(\xi(\widetilde{x}), \bar{\xi}(\widetilde{x}))$ defined as

$$\begin{array}{ll} \underline{\xi}^{\alpha}(\underline{\widetilde{x}}) = i\,\frac{\underline{\tau}(\underline{\widetilde{x}})}{2}\,\underline{\theta}^{\alpha}, & \bar{\underline{\xi}}_{\dot{\alpha}}(\underline{\widetilde{x}}) = -i\,\frac{\underline{\tau}^{*}(\underline{\widetilde{x}})}{2}\,\bar{\underline{\theta}}_{\dot{\alpha}}, \\ \xi^{\alpha}(\widetilde{x}) = i\frac{\tau(\widetilde{x})}{2}\theta^{\alpha}, & \bar{\xi}_{\dot{\alpha}}(\widetilde{x}) = -i\frac{\tau^{*}(\widetilde{x})}{2}\bar{\theta}_{\dot{\alpha}}. \end{array} \tag{62}$$

Here the real parameter $\tau(\widetilde{x}) = \tau^{*}(\widetilde{x}) = \underline{\tau}(\underline{\widetilde{x}}) = \underline{\tau}^{*}(\underline{\widetilde{x}})$ can physically be interpreted as the *atomic duration time* of double transition of a particle $V_4 \rightleftharpoons \underline{V}_2$ (Fig. 1), i.e. the period of superoscillations. In this case, the *atomic displacement* caused by double transition, according to (60), reads

$$\Delta\underline{\widetilde{x}}_{(a)} = \underline{\widetilde{e}}_{\underline{m}}\Delta\underline{\widetilde{x}}^{\underline{m}}_{(a)} = \underline{\widetilde{u}}\tau(\underline{\widetilde{x}}), \tag{63}$$

where, according to the motion (100), the components $\Delta\underline{\widetilde{x}}^{\underline{m}}_{(a)}$ are written

$$\Delta\underline{\widetilde{x}}^{\underline{m}}_{(a)} = \underline{\widetilde{v}}^{\underline{m}}\,\tau(\underline{\widetilde{x}}) = i\underline{\theta}\,\sigma^{\underline{m}}\,\bar{\underline{\xi}}(\underline{\widetilde{x}}) - i\underline{\xi}(\underline{\widetilde{x}})\,\sigma^{\underline{m}}\,\bar{\underline{\theta}}. \tag{64}$$

The corresponding acceleration reads

$$a^{(\pm)} = i\underline{\theta}\,\sigma^{(\pm)}\,\frac{d^2\bar{\underline{\xi}}}{d\underline{\widetilde{s}}^2} - i\frac{d^2\underline{\xi}}{d\underline{\widetilde{s}}^2}\,\sigma^{(\pm)}\,\bar{\underline{\theta}}, \tag{65}$$

where $\sigma^{(\pm)} = \frac{1}{\sqrt{2}}(\sigma^{\underline{0}} \pm \sigma^{\underline{1}}) = \frac{1}{\sqrt{2}}(\sigma^{0} \pm \sigma^{3})$ and $d\underline{\widetilde{s}}^2 = d\underline{\widetilde{x}}^{(+)}d\underline{\widetilde{x}}^{(-)}$. By virtue of (62), the (65) is reduced to

$$a^{(\pm)} = v_c^{(\pm)}\,\frac{d^2\tau}{d\underline{\widetilde{s}}^2}, \tag{66}$$

where $v_c^{(\pm)} \equiv (\underline{\theta}\,\sigma^{(\pm)}\bar{\underline{\theta}})$.

In Van der Warden notations for the Weyl two-component formalism $(\bar{\underline{\theta}}_{\dot{\alpha}})^{*} = \underline{\theta}_{\alpha}$ and $\bar{\underline{\theta}}_{\dot{\alpha}} = (\underline{\theta}_{\alpha})^{*}$, the (66) gives

$$\begin{array}{l} \underline{\widetilde{a}} = \sqrt{2}(a^{(+)}a^{(-)})^{1/2} = \sqrt{2}\underline{v}_c\,\frac{d^2\tau}{d\underline{\widetilde{s}}^2}, \\ \underline{v}_c = (v_c^{(+)}v_c^{(-)})^{1/2} = \sqrt{2}(\underline{\theta}_1\,\bar{\underline{\theta}}_1\underline{\theta}_2\,\bar{\underline{\theta}}_2)^{1/2}, \end{array} \tag{67}$$

with $v_c^{(+)} = \sqrt{2}(\underline{\theta}_1\,\bar{\underline{\theta}}_1)$ and $v_c^{(-)} = \sqrt{2}(\underline{\theta}_2\,\bar{\underline{\theta}}_2)$. The acceleration will generally remain a measure of the velocity variation over proper time $(\underline{\widetilde{s}})$. The (66) and (67) yield

$$\begin{array}{l} v^{(\pm)} = v_c^{(\pm)}\left(\frac{d\tau}{d\underline{\widetilde{s}}} + 1\right), \\ \underline{\widetilde{v}} = \sqrt{2}(v^{(+)}v^{(-)})^{1/2} = \sqrt{2}\underline{v}_c\left(\frac{d\tau}{d\underline{\widetilde{s}}} + 1\right). \end{array} \tag{68}$$

The spinors $\theta(\underline{\theta}, \bar{\underline{\theta}})$ and $\bar{\theta}(\underline{\theta}, \bar{\underline{\theta}})$ satisfy the embedding map (4), namely $\Delta\underline{\widetilde{x}}^{\underline{0}} = \Delta\widetilde{x}^{0}$ and $(\Delta\underline{\widetilde{x}}^{\underline{1}})^2 = (\Delta\vec{\widetilde{x}})^2$, so from (100) we obtain

$$\begin{array}{l} \underline{\theta}\,\sigma^{0}\,\bar{\underline{\xi}} - \underline{\xi}\,\sigma^{0}\,\bar{\underline{\theta}} = \theta\,\sigma^{0}\,\bar{\xi} - \xi\,\sigma^{0}\,\bar{\theta}, \\ (\underline{\theta}\,\sigma^{3}\,\bar{\underline{\xi}} - \underline{\xi}\,\sigma^{3}\,\bar{\underline{\theta}})^2 = (\theta\,\vec{\sigma}\,\bar{\xi} - \xi\,\vec{\sigma}\,\bar{\theta})^2. \end{array} \tag{69}$$

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Denote

$$\begin{aligned} \underline{v}^{0}_{(c)} &= \tfrac{1}{\sqrt{2}}\left(v_c^{(+)} + v_c^{(-)}\right) = (\underline{\theta}\,\underline{\bar\theta}), \\ \underline{v}^{1}_{(c)} &= \tfrac{1}{\sqrt{2}}\left(v_c^{(+)} - v_c^{(-)}\right) = (\underline{\theta}_1\,\underline{\bar\theta}_1 - \underline{\theta}_2\,\underline{\bar\theta}_2), \end{aligned} \tag{70}$$

then both relations in map (69) are reduced to

$$\theta\bar\theta = \underline{v}^{0}_{(c)}, \quad \theta\theta\bar\theta\bar\theta = -\tfrac{2}{3}(\underline{v}^{1}_{(c)})^2. \tag{71}$$

Here we have used the following spinor algebra relations (Wess & Bagger, 1983):

$$(\theta\sigma^m\,\bar\theta)(\theta\sigma^n\,\bar\theta) = \tfrac{1}{2}\,\theta\theta\bar\theta\bar\theta\,g^{mn}. \tag{72}$$

By virtue of relations $\theta_\alpha\theta_\beta = \frac{1}{2}\varepsilon_{\alpha\beta}\theta\theta$ and $\bar\theta_{\dot\alpha}\bar\theta_{\dot\beta} = -\frac{1}{2}\varepsilon_{\dot\alpha\dot\beta}\bar\theta\bar\theta$, where the antisymmetric tensors $\varepsilon_{\alpha\beta}$ and $\varepsilon^{\alpha\beta}$ ($\varepsilon_{21} = \varepsilon^{12} = 1, \varepsilon_{12} = \varepsilon^{21} = -1, \varepsilon_{11} = \varepsilon_{22} = 0$), and that of the inner product of two spinors $\theta\theta = \theta^\alpha\theta_\alpha$ and $\bar\theta\bar\theta = \bar\theta_{\dot\alpha}\bar\theta^{\dot\alpha}$, are invariant under Lorentz transformations because of unimodular matrix M (Lahanas & Nanopoulos, 1987, Wess & Bagger, 1983), we obtain from (71): $\theta_1^2 + \theta_2^2 = \underline{v}^{0}_{(c)}$, and $\theta_1\theta_2 = \frac{1}{\sqrt{6}}\underline{v}^{1}_{(c)}$, which yield

$$\begin{aligned} \theta_1(\underline{\theta},\,\underline{\bar\theta}) &= \tfrac{1}{2}\left[\left(\underline{v}^{0}_{(c)} + \sqrt{\tfrac{2}{3}}\underline{v}^{1}_{(c)}\right)^{1/2} + \left(\underline{v}^{0}_{(c)} - \sqrt{\tfrac{2}{3}}\underline{v}^{1}_{(c)}\right)^{1/2}\right], \\ \theta_2(\underline{\theta},\,\underline{\bar\theta}) &= \tfrac{1}{2}\left[\left(\underline{v}^{0}_{(c)} + \sqrt{\tfrac{2}{3}}\underline{v}^{1}_{(c)}\right)^{1/2} - \left(\underline{v}^{0}_{(c)} - \sqrt{\tfrac{2}{3}}\underline{v}^{1}_{(c)}\right)^{1/2}\right]. \end{aligned} \tag{73}$$

The dynamical aspects of particle mechanics involve derivatives with respect to proper time along the particle worldline. A worldline C of a particle, parametrized by proper time as $C(s) = X^{\hat\mu}(s)$, will have as six-velocity the vector of components $u^{\hat\mu} = dX^{\hat\mu}/ds$ and $u^{\hat a} = \dot e^{\hat a}_{\hat\mu}u^{\hat\mu}$, which are the particle velocity along this curve respectively in the holonomic and anholonomic bases in the X-space.

6. Concluding remarks

In this section we highlight a few points and discuss issues to be studied further. To innovate the solution to the problems involved, in this paper we develop on the theory of $\widetilde{MS}_p$-SG, which is a *local* extension of a global MS$_p$-SUSY theory (Ter-Kazarian, 2024a).

(I) We emphasize that the MS$_p$-SUSY (which is about the *inertial motion*, first part of inertia), together with the $\widetilde{MS}_p$-SG (which is about the *acceleration and inertia effects*, second part of inertia), provide valuable theoretical clue for a complete revision of our ideas about the Lorentz code of motion, as well as the acceleration and inertia effects, to be now referred to as the *intrinsic* property of a particle of interest devoid of any matter influence. This is a result of the first importance for a really comprehensive entire theory of inertia, which radically contradicts Mach's principle of *relativity of inertia*.

(II) We consider the accelerated motion of a particle in a new perspective of local $\widetilde{MS}_p$-SUSY transformations, whereas a *creation* of a particle in $\underline{V}_2$ means its transition from initial state defined on V_4 into intermediate state defined on $\underline{V}_2$, while an *annihilation* of a particle in $\underline{V}_2$ means vice versa (Fig. 1). The same interpretation holds for the *creation* and *annihilation* processes in V_4.

(III) The local-SUSY is conceived as a theory of $\widetilde{MS}_p$-SG, which can only be implemented if $\underline{V}_2$ and V_4 are curved (deformed). The $\widetilde{MS}_p$, being embedded in the V_4, is the *unmanifested* indispensable individual companion of a particle of interest devoid of any matter influence. The superspace $(z^M,\,\Theta,\,\bar\Theta)$ is a direct sum extension of background double spaces $V_4 \oplus \underline{V}_2$, with an inclusion of additional fermionic coordinates $(\Theta(\underline{\theta},\theta),\,\bar\Theta(\underline{\bar\theta},\bar\theta))$ induced by the spinors $(\underline{\theta},\underline{\bar\theta})$, which refer to $\underline{V}_2$. Thanks to the embedding $\underline{V}_2 \hookrightarrow V_4$, the spinors $(\underline{\theta},\underline{\bar\theta})$, in turn, induce the spinors $\theta(\underline{\theta},\underline{\bar\theta})$ and $\bar\theta(\underline{\theta},\underline{\bar\theta})$, as to V_4. While all the particles are living on V_4, their superpartners can be viewed as living on $\widetilde{MS}_p$. In this framework supersymmetry and general coordinate transformations are described in a unified way as certain diffeomorphisms. The action of simple $\widetilde{MS}_p$-SG includes the Hilbert term for a *fictitious* graviton coexisting with a *fictitious* fermionic field of gravitino described by the Rarita-Scwinger kinetic term. A coupling of supergravity with matter superfields no longer holds. The different 4D $N = 1$ supergravity multiplets all contain the graviton and the gravitino, but differ by their systems of auxiliary fields.

(IV) The accelerated motion of a particle is described by the parameter $\epsilon = \epsilon(X^{\hat\mu})$ of local SUSY, which depends explicitly on $X^{\hat\mu} = (\widetilde{x}^\mu,\,\widetilde{\underline{x}}^\mu)$, where $\widetilde{x}^\mu \in V_4$ and $\widetilde{\underline{x}}^\mu \in \underline{V}_2$. Here the real parameter $\tau(x) = \underline{\tau}(\underline{x})$ is

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interpreted as the *atomic duration time*, i.e. period of superoscillations, of double transition of a particle $V_4 \rightleftharpoons \underline{V}_2$.

(VI) Further studies on the $\widetilde{MS}_p$-SG are warranted with special emphasis on Palatini's formalism, the flat $\widetilde{MS}_p$-SG theory with Weitzenböck torsion as a $\widetilde{MS}_p$-Teleparallel SG theory, a general deformation of MS_p induced by external force exerted on a particle and inertial effects, the hypothesis of locality, which will essentially improve the framework of present paper. Actually,

a) using Palatini's formalism extended in a plausible fashion to the $\widetilde{MS}_p$ - SG, one will reinterpret a flat $\widetilde{MS}_p$-SG theory with Weitzenböck torsion as a $\widetilde{MS}_p$-Teleparallel SG theory, having the gauge *translation* group in tangent bundle. Whereas the Hilbert action vanishes and the gravitino action loses its spin connections, so one finds torsion induced by gravitinos. The accelerated reference frame has Weitzenböck torsion. The spin connection represents only *inertial effects*, but not gravitation at all. The action of a $\widetilde{MS}_p$-TSG theory will be invariant under the Poincaré supergroup and under diffeomorphisms.

b) The Weitzenböck connection ($\dot{\Gamma}$), which defines the Fock-Ivanenko derivative ($\dot{\mathcal{D}}_{\hat{\mu}}$) written in terms of covariant derivative ($\dot{\nabla}_{\hat{\mu}}$), defines the acceleration too. By means of it, one will derive a force equation, with torsion (or contortion) playing the role of force. The connection ($\dot{\Gamma}$) will be considered a kind of dual of the Levi-Civita connection (Γ), which is a connection with vanishing torsion (T), and non-vanishing *fictitious* curvature (R).

c) This allows one to complement a theory of $\widetilde{MS}_p$-TSG with implications for special cases. In particular, one will discuss the Newtonian limit, and describe the homogeneous acceleration field.

d) As we emphasized already essential difference arisen between the standard supergravity theories and some rather unusual properties of a $\widetilde{MS}_p$-SG theory is as follows. In the framework of the standard supergravity theories, as in GR, a curvature of the space-time acts on all the matter fields. The source of graviton is the energy-momentum tensor of matter fields, while the source of gravitino is the spin-vector current of supergravity. The gauge action of simple $\widetilde{MS}_p$-SG is the sum of the Hilbert action for the tetrad field - *fictitious* graviton, and the Rarita-Schwinger action for the *fictitious* gravitino field. Instead we argue that a deformation of MS_p is the origin of these fields. They refer to the particle of interest itself, without relation to other matter fields, so that these fields can be globally removed by appropriate coordinate transformations. With these physical requirements, a standard coupling of supergravity with matter superfields evidently no longer holds. We, therefore, would work out the theory of a general deformation of MS_p induced by external force exerted on a particle, in order to show that in the $\widetilde{MS}_p$-TSG theory the occurrence of the *absolute* and *inertial* accelerations, and the *inertial* force are obviously caused by this. In the same time, the *relative* acceleration (in Newton's terminology) (both magnitude and direction), to the contrary, has nothing to do with a deformation of $\underline{M}_2$ and, thus, it cannot produce the inertia effects. One will determine the most important period of superoscillations as a function of proper time for given deformed MS_p.

e) In standard framework of the construction of reference frame of an accelerated observer, the hypothesis of locality holds for huge proper acceleration lengths and that represents strict restrictions, because it approximately replaces a noninertial frame of reference $\widetilde{S}_{(2)}$, which is held stationary in the deformed space $\mathcal{M}_2 \equiv \underline{V}_2^{(\varrho)}\ (\varrho \neq 0)$, where $\underline{V}_2$ is the 2D semi-Riemannian space, with a continuous infinity set of the inertial frames $\{S_{(2)}, S'_{(2)}, S''_{(2)}, ...\}$ given in the flat $\underline{M}_2\ (\varrho = 0)$. In this situation the use of the hypothesis of locality is physically unjustifiable. In this study, therefore, it is worthwhile to take into account a deformation $\underline{M}_2 \longrightarrow \underline{V}_2^{(\varrho)}$, which will essentially improve the standard framework.

All the above mentioned problems (d,e) will become separate topics for research in subsequent papers (Ter-Kazarian, 2024c,b).

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Appendices

Appendix A A glimpse on the global MS$_p$-SUSY

For a benefit of the reader, as a guiding principle to make the rest of paper understandable, in this section we necessarily recount some of the highlights behind of *global* MS$_p$-SUSY (Ter-Kazarian, 2023a, 2024a), on which the *local* $\widetilde{MS}_p$-SUSY is based.

The flat MS$_p$ is the 2D composite space

$$MS_p \equiv \underline{M}_2 = \underline{R}^1_{(+)} \oplus \underline{R}^1_{(-)}, \tag{74}$$

with Lorentz metric. The ingredient 1D-space $\underline{R}^1_{\underline{m}}$ is spanned by the coordinates $\eta^{\underline{m}}$. The following notational conventions are used throughout this paper: all quantities related to the space $\underline{M}_2$ will be underlined. In particular, the underlined lower case Latin letters $\underline{m}, \underline{n}, ... = (\pm)$ denote the world indices related to $\underline{M}_2$.

Suppose the position of the particle is specified by the coordinates $x^m(s)$ $(x^0 = t)$ in the basis e_m (m=0,1,2,3) at given point in the background M_4 space. Consider a smooth (injective and continuous) embedding $\underline{M}_2 \hookrightarrow M_4$. That is, a smooth map $f: \underline{M}_2 \longrightarrow M_4$ is defined to be an immersion (the embedding which is a function that is a homeomorphism onto its image):

$$\underline{e}_{\underline{0}} = e_0, \quad \underline{x}^{\underline{0}} = x^0, \quad \underline{e}_{\underline{1}} = \vec{n}, \quad \underline{x}^{\underline{1}} = |\vec{x}|, \tag{75}$$

where $\vec{x} = e_i x^i = \vec{n}|\vec{x}|$ $(i = 1, 2, 3)$. Given the inertial frames $S_{(4)}, S'_{(4)}, S''_{(4)}, ...$ in unaccelerated uniform motion in M_4, we may define the corresponding inertial frames $\underline{S}_{(2)}$, $\underline{S}'_{(2)}$, $\underline{S}''_{(2)}$,... in $\underline{M}_2$, which are used by the non-accelerated observers for the positions $\underline{x}^{\underline{r}}, \underline{x}'^{\underline{r}}, \underline{x}''^{\underline{r}}, ...$ of a free particle in flat $\underline{M}_2$. According to (75), the time axes of the two systems $\underline{S}_{(2)}$ and $S_{(4)}$ coincide in direction, and the time coordinates are taken the same. For the case at hand,

$$\underline{v}^{(\pm)} = \frac{d\eta^{(\pm)}}{d\underline{x}^{\underline{0}}} = \frac{1}{\sqrt{2}}(\underline{v}^{\underline{0}} \pm \underline{v}^{\underline{1}}), \quad \underline{v}^{\underline{1}} = \frac{d\underline{x}^{\underline{1}}}{d\underline{x}^{\underline{0}}} = |\vec{v}| = |\frac{d\vec{x}}{dx^0}|, \tag{76}$$

and that

$$\underline{u} = \underline{e}_{\underline{m}} \underline{v}^{\underline{m}} = (\vec{\underline{v}}_{\underline{0}}, \vec{\underline{v}}_{\underline{1}}), \quad \vec{\underline{v}}_{\underline{0}} = \underline{e}_{\underline{0}} \underline{v}^{\underline{0}}, \quad \vec{\underline{v}}_{\underline{1}} = \underline{e}_{\underline{1}} \underline{v}^{\underline{1}} = \vec{n}|\vec{v}| = \vec{v}, \tag{77}$$

therefore, $\underline{u} = u = (e_0, \vec{v})$. To explain why MS$_p$ is two dimensional, we note that only 2D real *null vectors* are allowed as the basis at given point in MS$_p$, which is embedded in M_4. Literally speaking, the $\underline{M}_2$ can be viewed as 2D space living on the 4D world sheet.

The elementary act of particle motion at each time step (t_i) through the infinitely small spatial interval $\triangle x_i = (x_{i+1} - x_i)$ in M_4 during the time interval $\triangle t_i = (t_{i+1} - t_i) = \varepsilon$ is probably the most fascinating challenge for physical research. Since this is beyond our perception, it appears legitimate to consider extension to the infinitesimal Schwinger transformation function, $F_{ext}(x_{i+1}, t_{i+1}; x_i, t_i)$, in fundamentally different aspect. We hypothesize that

in the limit $n \to \infty (\varepsilon \to 0)$, the elementary act of motion consists of an `annihilation´ of a particle at point $(x_i, t_i) \in M_4$, which can be thought of as the transition from initial state $|x_i, t_i>$ into unmanifested intermediate state, so-called, `motion´ state, $|\underline{x}_i, \underline{t}_i>$, and of subsequent `creation´ of a particle at infinitely close final point $(x_{i+1}, t_{i+1}) \in M_4$, which means the transition from `motion´ state, $|\underline{x}_i, \underline{t}_i>$, into final state, $|x_{i+1}, t_{i+1}>$. The motion state, $|\underline{x}_i, \underline{t}_i)>$, should be defined on unmanifested `master´ space, $\underline{M}_2$, which includes the points of all the atomic elements, $(\underline{x}_i, \underline{t}_i) \in \underline{M}_2$ $(i = 1, 2, ...)$.

This furnishes justification for an introduction of *unmanifested master space*, $\underline{M}_2$.

The fields of spin-zero $(\vec{S} = \vec{K} = 0)$ scalar field $A(x)$ and spin-one $A^n(x)$, corresponding to the $(1/2, 1/2)$ representation, transform under a general Lorentz transformation as follows:

$$\begin{array}{ll} \underline{A}(\underline{\eta}) \equiv A(x), & (\text{spin} \quad 0); \\ \underline{A}^{\underline{m}}(\underline{\eta}) = \Lambda^{\underline{m}}_{\ n} A^n(x), & (\text{spin} \quad 1). \end{array} \tag{78}$$

The map from $SL(2, C)$ to the Lorentz group is established through the $\vec{\sigma}$-Pauli spin matrices, $\sigma^m = (\sigma^0, \sigma^1, \sigma^2, \sigma^3) \equiv (I_2, \vec{\sigma})$, $\bar{\sigma}^m \equiv (I_2, -\vec{\sigma})$, where I_2 is the identity two-by-two matrix.

According to embedding map (75), the $\underline{\sigma}$-matrices are

$$\sigma^{\underline{m}} = \sigma^{(\pm)} = \frac{1}{\sqrt{2}}(\sigma^{\underline{0}} \pm \sigma^{\underline{1}}) = \frac{1}{\sqrt{2}}(\sigma^0 \pm \sigma^3). \tag{79}$$

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The matrices $\sigma^{\underline{m}}$ form a basis for two-by-two complex matrices $\underline{P}$:

$$\underline{P} = (p_{\underline{m}}\sigma^{\underline{m}}) = (p_{(\pm)}\sigma^{(\pm)}) = (p_{\underline{0}}\sigma^{\underline{0}} + p_{\underline{1}}\sigma^{\underline{1}}), \tag{80}$$

provided $p_{(\pm)} = i\partial_{\underline{\eta}^{(\pm)}}$, $p_{\underline{0}} = i\partial_{\underline{x}^0}$ and $p_{\underline{1}} = i\partial_{\underline{x}^1}$. The real coefficients $p'_{\underline{m}}$ and $p_{\underline{m}}$, like p'_m and p_m, are related by a Lorentz transformation $p'_{\underline{m}} = \Lambda^{\underline{n}}_{\underline{m}}\, p_{\underline{n}}$, because the relations $det(\sigma^{\underline{m}}\, p_{\underline{m}}) = p^2_{\underline{0}} - p^2_{\underline{1}}$ and $det\, M = 1$ yield ${p'_{\underline{0}}}^2 - {p'_{\underline{1}}}^2 = p^2_{\underline{0}} - p^2_{\underline{1}}$. Correspondence of $p_{\underline{m}}$ and $\underline{P}$ is uniquely: $p_{\underline{m}} = \frac{1}{2}Tr(\sigma^{\underline{m}}\,\underline{P})$, which combined with (82) yields

$$\Lambda^{\underline{m}}_{\ \underline{n}}(M) = \tfrac{1}{2}\, Tr\left(\sigma^{\underline{m}}\, M\sigma^{\underline{n}}\, M^{\dagger}\right). \tag{81}$$

Thus, both hermitian matrices P and P' or $\underline{P}$ and $\underline{P}'$ have expansions, respectively, in σ or $\underline{\sigma}$:

$$(\sigma^m\, p'_m) = M(\sigma^m\, p_m)M^{\dagger}, \quad (\sigma^{\underline{m}}\, p'_{\underline{m}}) = M(\sigma^{\underline{m}}\, p_{\underline{m}})M^{\dagger}, \tag{82}$$

where $M(M \in SL(2,C))$ is unimodular two-by-two matrix.

A two-component $(1/2,0)$ Weyl fermion, $\chi_\beta(x)$, therefore, transforms under Lorentz transformation to yield $\underline{\chi}_\alpha(\underline{\eta})$:

$$\chi_\beta(x) \longrightarrow \underline{\chi}_\alpha(\underline{\eta}) = (M_R)_\alpha^{\ \beta}\, \chi_\beta(x), \quad \alpha, \beta = 1, 2, \tag{83}$$

where the orthochronous Lorentz transformation, corresponding to a rotation by the angles ϑ_3 and ϑ_2 about, respectively, the axes n_3 and n_2, is given by rotation matrix

$$M_R = e^{i\frac{1}{2}\sigma_2\vartheta_2} e^{i\frac{1}{2}\sigma_3\vartheta_3}. \tag{84}$$

There with the rotation of an hermitian matrix P is

$$p_{\underline{m}}\sigma^{\underline{m}} = M_R\, p_m\sigma^m\, M_R^{\dagger}, \tag{85}$$

where p_m and $p_{\underline{m}}$ denote the momenta $p_m \equiv m(ch\beta,\ sh\beta \sin\vartheta_2 \cos\vartheta_3,\ sh\beta \sin\vartheta_2 \sin\vartheta_3,$ $sh\beta \cos\vartheta_2)$, and $p_{\underline{m}} \equiv m(ch\underline{\beta},\ 0,\ 0,\ sh\underline{\beta})$.

A two-component $(0,1/2)$ Weyl spinor field is denoted by $\bar{\chi}^{\dot\beta}(x)$, and transforms as

$$\bar{\chi}^{\dot\beta}(x) \longrightarrow \underline{\bar{\chi}}^{\dot\alpha}(\underline{\eta}) = (M_R^{-1})^{\dagger\dot\alpha}_{\ \ \ \dot\beta}\, \bar{\chi}^{\dot\beta}(x), \quad \dot\alpha, \dot\beta = 1, 2. \tag{86}$$

The so-called `dotted´ indices have been introduced to distinguish the $(0,1/2)$ representation from the $(1/2,0)$ representation. The `bar´ over the spinor is a convention that this is the $(0,1/2)$-representation. We used the Van der Waerden notations for the Weyl two-component formalism: $(\underline{\bar{\chi}}_{\dot\alpha})^* = \underline{\chi}_\alpha$ and $\underline{\bar{\chi}}_{\dot\alpha} = (\underline{\chi}_\alpha)^*$.

The odd part of the supersymmetry algebra is composed entirely of the spin-1/2 operators $Q_\alpha^{\ i},\ Q_\beta^{\ j}$. In order to trace a maximal resemblance in outward appearance to the standard SUSY theories, here we set one notation $\hat{m} = (m \quad \text{if} \quad Q = q$, or $\underline{m} \quad \text{if} \quad Q = \underline{q})$, and as before the indices α and $\dot\alpha$ run over 1 and 2.

If that is the case as above, a *creation* of a particle in $\underline{M}_2$ means its transition from initial state defined on M_4 into intermediate state defined on $\underline{M}_2$, while an *annihilation* of a particle in $\underline{M}_2$ means vice versa. The same interpretation holds for the *creation* and *annihilation* processes in M_4. All the fermionic and bosonic states taken together form a basis in the Hilbert space. The basis vectors in the Hilbert space composed of $H_B \otimes H_F$ is given by

$$\{|\underline{n}_b > \otimes |0>_f,\ |\underline{n}_b > \otimes f^{\dagger}\, |0>_f\},$$

or

$$\{|n_b > \otimes |\underline{0}>_f,\ |n_b > \otimes \underline{f}^{\dagger}\, |\underline{0}>_f\},$$

where we consider two pairs of creation and annihilation operators $(b^\dagger,\ b)$ and $(f^\dagger,\ f)$ for bosons and fermions, respectively, referred to the background space V_4, as well as $(\underline{b}^\dagger,\ \underline{b})$ and $(\underline{f}^\dagger,\ \underline{f})$ for bosons and fermions, respectively, as to background master space $\underline{V}_2$. The boson and fermion number operators are $N_b = b^\dagger b$ or $\underline{N}_b = \underline{b}^\dagger \underline{b}$, where $N_b|n_b >= n_b|n_b >$ and $\underline{N}_b|\underline{n}_b >= \underline{n}_b|\underline{n}_b > (= 0, 1, ..., \infty)$, and $N_f = f^\dagger f$ or $\underline{N}_f = \underline{f}^\dagger \underline{f}$, provided $N_f|n_f >= n_f|n_f >$ and $\underline{N}_f|\underline{n}_f >= \underline{n}_f|\underline{n}_f > (= 0, 1)$. Taking into account the action of $(b, b^\dagger)$ or $(\underline{b}, \underline{b}^\dagger)$ upon the eigenstates $|n_b >$ or $|\underline{n}_b >$, we may construct the quantum operators, $(q^\dagger,\ \underline{q}^\dagger)$ and $(q,\ \underline{q})$ as

$$\begin{aligned} q &= q_0, \underline{b}\, f^\dagger, \quad q^\dagger = q_0\, \underline{b}^\dagger\, f, \\ \underline{q} &= q_0\, b\underline{f}^\dagger, \quad \underline{q}^\dagger = q_0\, b^\dagger\, \underline{f}. \end{aligned} \tag{87}$$

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which replace bosons by fermions and vice versa:

$$\begin{array}{l} q\,|\underline{n}_b,\,n_f>=q_0\sqrt{\underline{n}_b}\,|\underline{n}_b-1,\,n_f+1>,\\ q^\dagger\,|\underline{n}_b,\,n_f>=q_0\sqrt{\underline{n}_b+1}\,|\underline{n}_b+1,\,n_f-1>, \end{array} \tag{88}$$

and that

$$\begin{array}{l} \underline{q}\,|n_b,\,\underline{n}_f>=q_0\sqrt{n_b}\,|n_b-1,\,\underline{n}_f+1>,\\ \underline{q}^\dagger\,|n_b,\,\underline{n}_f>q_0\sqrt{n_b+1}\,|n_b+1,\,\underline{n}_f-1>. \end{array} \tag{89}$$

This framework combines bosonic and fermionic states on the same footing, rotating them into each other under the action of operators q and $\underline{q}$. So, we may refer the action of the supercharge operators q and $q^\dagger$ to the background space M_4, having applied in the chain transformations of fermion χ (accompanied with the auxiliary field F as it will be seen later on) to boson $\underline{A}$, defined on $\underline{M}_2$:

$$\longrightarrow \chi^{(F)} \longrightarrow \underline{A} \longrightarrow \chi^{(F)} \longrightarrow \underline{A} \longrightarrow \chi^{(F)} \longrightarrow . \tag{90}$$

Respectively, we may refer the action of the supercharge operators $\underline{q}$ and $\underline{q}^\dagger$ to the $\underline{M}_2$, having applied in the chain transformations of fermion $\underline{\chi}$ (accompanied with the auxiliary field $\underline{F}$) to boson A, defined on the background space M_4:

$$\longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow A \longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow A \longrightarrow \underline{\chi}^{(\underline{F})} \longrightarrow . \tag{91}$$

The successive atomic double transitions of a particle $M_4 \rightleftharpoons \underline{M}_2$ is investigated within MS$_p$-SUSY, wherein all the particles are living on M_4, their superpartners can be viewed as living on MS$_p$. The underlying algebraic structure of MS$_p$-SUSY generators closes with the algebra of *translations* on the original space M_4 in a way that it can then be summarized as a non-trivial extension of the Poincaré group algebra those of the commutation relations of the bosonic generators of four momenta and six Lorentz generators referred to M_4. Moreover, if there are several spinor generators $Q_\alpha^{\;i}$ with $i = 1, ..., N$ - theory with $N-$extended supersymmetry, can be written as a graded Lie algebra of SUSY field theories, with commuting and anticommuting generators:

$$\begin{array}{l} \{Q_\alpha^{\;i},\,\bar{Q}^j{}_{\dot\alpha}\} = 2\delta^{ij}\,\sigma^{\hat m}_{\alpha\dot\alpha}\,p_{\hat m};\\ \{Q_\alpha^{\;i},\,Q_\beta^{\;j}\} = \{\bar{Q}^i{}_{\dot\alpha},\,\bar{Q}^j{}_{\dot\beta}\} = 0; \quad [p_{\hat m},\,Q_\alpha^{\;i}] = [p_{\hat m},\,\bar{Q}^j{}_{\dot\alpha}] = 0, \quad [p_{\hat m},\,p_{\hat n}] = 0. \end{array} \tag{92}$$

The anticommuting (Grassmann) parameters $\epsilon^\alpha(\xi^\alpha,\,\underline{\xi}^\alpha)$ and $\bar\epsilon^\alpha(\bar\xi^\alpha,\,\underline{\bar\xi}^\alpha)$:

$$\{\epsilon^\alpha,\,\epsilon^\beta\} = \{\bar\epsilon^\alpha,\,\bar\epsilon^\beta\} = \{\epsilon^\alpha,\,\bar\epsilon^\beta\} = 0, \quad \{\epsilon^\alpha,\,Q_\beta\} = \cdots = [p_{\hat m},\,\epsilon^\alpha] = 0, \tag{93}$$

allow us to write the algebra (92) for $(N = 1)$ entirely in terms of commutators:

$$[\epsilon Q,\,\bar Q\bar\epsilon] = 2\epsilon\sigma^{\hat m}\bar\epsilon p_{\hat m}, \quad [\epsilon Q,\,\epsilon Q] = [\bar Q\bar\epsilon,\,\bar Q\bar\epsilon] = [p^{\hat m},\,\epsilon Q] = [p^{\hat m},\,\bar Q\bar\epsilon] = 0. \tag{94}$$

For brevity, here the indices $\epsilon Q = \epsilon^\alpha Q_\alpha$ and $\bar\epsilon\bar Q = \bar\epsilon_{\dot\alpha}\bar Q^{\dot\alpha}$ will be suppressed unless indicated otherwise. This supersymmetry transformation maps tensor fields $\mathcal{A}(A,\,\underline{A})$ into spinor fields $\psi(\chi,\,\underline{\chi})$ and vice versa. From the algebra (94) we see that Q has mass dimension $1/2$. Therefore, as usual, fields of dimension ℓ transform into fields of dimension $\ell + 1/2$ or into derivatives of fields of lower dimension. It can be checked that the supersymmetry transformations close supersymmetry algebra:

$$(\delta_{\xi_1}\delta_{\xi_2} - \delta_{\xi_2}\delta_{\xi_1})\underline{A} = -2i(\xi_1\sigma^m\bar\xi_2 - \xi_2\sigma^m\bar\xi_1)(\delta^0_m\underline{\partial}_0 + \tfrac{1}{|\vec x|}x^i\delta_{im}\underline{\partial}_1)\underline{A}. \tag{95}$$

The guiding principle of MS$_p$-SUSY resides in constructing the superspace which is a 14D-extension of a direct sum of background spaces $M_4 \oplus \underline{M}_2$ (spanned by the 6D-coordinates $X^{\hat m} = (x^m,\,\underline{\eta}^{\underline{m}})$ by the inclusion of additional 8D-fermionic coordinates $\Theta^\alpha = (\theta^\alpha,\,\underline{\theta}^\alpha)$ and $\bar\Theta_{\dot\alpha} = (\bar\theta_{\dot\alpha},\,\underline{\bar\theta}_{\dot\alpha})$, as to $(q,\,\underline{q})$, respectively. Therewith thanks to the embedding $\underline{M}_2 \hookrightarrow M_4$, the spinors $(\underline{\theta},\underline{\bar\theta})$, in turn, induce the spinors $\theta(\underline{\theta},\underline{\bar\theta})$ and $\bar\theta(\underline{\theta},\underline{\bar\theta})$, as to M_4. These spinors satisfy the following relations:

$$\begin{array}{l} \{\Theta^\alpha,\,\Theta^\beta\} = \{\bar\Theta_{\dot\alpha},\,\bar\Theta_{\dot\beta}\} = \{\Theta^\alpha,\,\bar\Theta_{\dot\beta}\} = 0,\\ [x^m,\,\theta^\alpha] = [x^m,\,\bar\theta_{\dot\alpha}] = 0, \quad [\underline{\eta}^{\underline{m}},\,\underline{\theta}^\alpha] = [\underline{\eta}^{\underline{m}},\,\underline{\bar\theta}_{\dot\alpha}] = 0. \end{array} \tag{96}$$

and $\Theta^{\alpha *} = \bar\Theta^{\dot\alpha}$. Points in superspace are identified by the generalized coordinates

$$z^{(M)} = (X^{\hat m},\,\Theta^\alpha,\,\bar\Theta_{\dot\alpha}) = (x^m,\,\theta^\alpha,\,\bar\theta_{\dot\alpha}) \oplus (\underline{\eta}^{\underline{m}},\underline{\theta}^\alpha,\,\underline{\bar\theta}_{\dot\alpha}).$$

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We have then the one most commonly used `real´ or `symmetric´ superspace parametrized by

$$\Omega(X,\,\Theta,\,\bar\Theta)=e^{i(-X^{\hat m}p_{\hat m}+\Theta^\alpha Q_\alpha+\bar\Theta_{\dot\alpha}\bar Q^{\dot\alpha})}=\Omega_q(x,\,\theta,\,\bar\theta)\times\Omega_{\underline{q}}(\underline{\eta},\,\underline{\theta},\,\underline{\bar\theta}), \tag{97}$$

where we now imply a summation over $\hat m=(m,\underline{m})$. To study the effect of supersymmetry transformations, we consider

$$g(0,\,\epsilon,\,\bar\epsilon)\,\Omega(X,\,\Theta,\,\bar\Theta)=e^{i(\epsilon^\alpha Q_\alpha+\bar\epsilon_{\dot\alpha}\bar Q^{\dot\alpha})}\,e^{i(-X^{\hat m}p_{\hat m}+\Theta^\alpha Q_\alpha+\bar\Theta_{\dot\alpha}\bar Q^{\dot\alpha})}. \tag{98}$$

the transformation (98) induces the motion:

$$g(0,\,\epsilon,\,\bar\epsilon)\,\Omega(X^{\hat m},\,\Theta,\,\bar\Theta)\;\to(X^{\hat m}+i\,\Theta\,\sigma^{\hat m}\,\bar\epsilon-i\,\epsilon\,\sigma^{\hat m}\,\bar\Theta,\;\Theta+\epsilon,\,\bar\Theta+\bar\epsilon), \tag{99}$$

namely,

$$\begin{array}{l} g_q(0,\,\xi,\,\bar\xi)\,\Omega_q(x,\,\theta,\,\bar\theta)\;\to(x^m+i\,\theta\,\sigma^m\,\bar\xi-i\,\xi\,\sigma^m\,\bar\theta,\;\theta+\xi,\,\bar\theta+\bar\xi),\\ g_{\underline{q}}(0,\,\underline{\xi},\,\underline{\bar\xi})\,\Omega_{\underline{q}}(\underline{\eta},\,\underline{\theta},\,\underline{\bar\theta})\;\to(\underline{\eta}^{\underline{m}}+i\,\underline{\theta}\,\sigma^{\underline{m}}\,\underline{\bar\xi}-i\,\underline{\xi}\,\sigma^{\underline{m}}\,\underline{\bar\theta},\;\underline{\theta}+\underline{\xi},\,\underline{\bar\theta}+\underline{\bar\xi}). \end{array} \tag{100}$$

The spinors $\theta(\underline{\theta},\underline{\bar\theta})$ and $\bar\theta(\underline{\theta},\underline{\bar\theta})$ satisfy the embedding relations $\Delta\underline{x}^0=\Delta x^0$ and $\Delta\underline{x}^2=(\Delta\vec x)^2$, so from (100) we obtain

$$\underline{\theta}\,\sigma^0\,\underline{\bar\xi}-\underline{\xi}\,\sigma^0\,\underline{\bar\theta}=\theta\,\sigma^0\,\bar\xi-\xi\,\sigma^0\,\bar\theta,\quad(\underline{\theta}\,\sigma^3\,\underline{\bar\xi}-\underline{\xi}\,\sigma^3\,\underline{\bar\theta})^2=(\theta\,\vec\sigma\,\bar\xi-\xi\,\vec\sigma\,\bar\theta)^2, \tag{101}$$

which gives (73). The *atomic displacement* caused by double transition of a particle $M_4\rightleftharpoons\underline{M}_2$ reads

$$\Delta\underline{\eta}_{(a)}=\underline{e}_{\underline{m}}\Delta\underline{\eta}^{\underline{m}}_{(a)}=\underline{u}\tau, \tag{102}$$

where the components $\Delta\underline{\eta}^{\underline{m}}_{(a)}$ are written

$$\Delta\underline{\eta}^{\underline{m}}_{(a)}=(\underline{\theta}\,\sigma^{\underline{m}}\,\underline{\bar\theta})\tau. \tag{103}$$

In Van der Warden notations for the Weyl two-component formalism $\underline{\bar\theta}_{\dot\alpha}=(\underline{\theta}_\alpha)^*$, the (102) can be recast into the form

$$\Delta\underline{\eta}^2_{(a)}=\tfrac{1}{2}\left[(\Delta\underline{x}^{\underline{0}}_{(a)}q)^2-(\Delta\underline{x}^{\underline{1}}_{(a)})^2\right], \tag{104}$$

where $\Delta\underline{x}^{\underline{0}}_{(a)}=\underline{v}^{\underline{0}}\tau$, $\Delta\underline{x}^{\underline{1}}_{(a)}=\underline{v}^{\underline{1}}\tau$, and $\underline{v}^{(\pm)}=\frac{1}{\sqrt2}(\underline{v}^{\underline{0}}\pm\underline{v}^{\underline{1}})$. Hence the velocities of light in vacuum, $\underline{v}^{\underline{0}}=c$, and of a particle ,$\underline{\vec v}_{\underline{1}}=\underline{e}_{\underline{1}}\underline{v}^{\underline{1}}=\vec n|\vec v|=\vec v$ $(|\vec v|\le c)$, are

$$\begin{array}{l} \underline{v}^{\underline{0}}=\underline{\theta}\,\sigma^{\underline{0}}\,\underline{\bar\theta}=(\underline{\theta}_1\,\underline{\bar\theta}_1+\underline{\theta}_2\,\underline{\bar\theta}_2)=\underline{\theta}\,\underline{\bar\theta},\\ \underline{v}^{\underline{1}}=\underline{\theta}\,\sigma^{\underline{1}}\,\underline{\bar\theta}=(\underline{\theta}_1\,\underline{\bar\theta}_1-\underline{\theta}_2\,\underline{\bar\theta}_2). \end{array} \tag{105}$$

Thus we derive the first founding property (i) that *the atomic displacement* $\Delta\underline{\eta}_{(a)}$, caused by double transition of a particle $M_4\rightleftharpoons\underline{M}_2$, is an invariant:

$$\text{(i)}\quad\Delta\underline{\eta}_{(a)}=\Delta\underline{\eta}'_{(a)}=\cdots=inv. \tag{106}$$

The (105) gives the second (ii) founding property that the bilinear combination $\underline{\theta}\,\underline{\bar\theta}$ is a constant:

$$\text{(ii)}\quad c=\underline{\theta}\,\underline{\bar\theta}=\underline{\theta}'\,\underline{\bar\theta}'=\cdots=const. \tag{107}$$

The latter yields a second postulate of SR (Einstein's postulate) - *the velocity of light, c, in free space appears the same to all observers regardless the relative motion of the source of light and the observer*. The c is the maximum attainable velocity (105) for uniform motion of a particle in Minkowski background space, M_4. Equally noteworthy is the fact that (106) and (107) combined yield *invariance of the element of interval between two events* $\Delta x=k\Delta\underline{\eta}_{(a)}$ *(for given integer number k) with respect to the Lorentz transformation*:

$$\begin{array}{l} k^2\Delta\underline{\eta}^2_{(a)}=(c^2-\underline{v}^2_{\underline{1}})\Delta t^2=(c^2-\vec v^{\,2})\Delta t^2=(\Delta x^0)^2-(\Delta\vec x)^2\equiv(\Delta s)^2=(\Delta x'^0)^2-\\ (\Delta\vec x)'^2\equiv(\Delta s')^2=\cdots=inv., \end{array} \tag{108}$$

where $x^0=ct,\quad x^{0'}=ct',\ldots.$ We have here introduced a notion of physical relative finite *time intervals* between two events $\Delta t=k\tau/\sqrt2$, $\Delta t'=k\tau'/\sqrt2,\ldots.$

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