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DOMAIN WALL BIANCHI TYPE VI_0 UNIVERSE IN $f(R, T)$ GRAVITY

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We consider the Bianchi type- VI_0 space time with domain walls in the framework of the modified $f(R, T)$ theory of gravitation. To solve the field equations, we assume that the shear scalar σ is proportional to the expansion scalar θ . We also consider the parametrization of the equation of the state parameter of barotropic fluid and discuss the effect on domain walls. It has been observed that the domain wall may behave like dark energy. Some physical parameters are also discussed in detail.

Keywords: *domain walls: Bianchi type- VI_0 space time: $f(R, T)$ gravity*

1. *Introduction.* Recent cosmological observation data indicate that the universe is undergoing an accelerated expansion phase. One of the possible solutions to explain the cosmic speed-up of the universe is the modification of the general theory of relativity. The simplest class of modification is replacing $f(R)$, i.e., the function of Ricci scalar R , instead of R in Einstein-Hilbert action. One can achieve the dynamics of the universe without the need of dark energy in $f(R)$ gravity. The $f(R)$ theory of gravitation covers all domains from cosmological to the solar system scale. Moreover, no extra degree of freedom is required in $f(R)$ gravity. $f(R)$ gravity is equivalent with Brans-Dicke theory when coupling constant $\omega = 0$ [1]. Nojiri and Odintsov [2] obtained early time inflation and late time acceleration in the $f(R)$ theory of gravitation. Also, Nojiri et al. [3] presented a brief review on $f(R)$ theories of gravitation. However, the $f(R)$ theories have stability and viability issues [4-6].

Recently, Harko et al. [7] proposed a more general model of modified theory known as the $f(R, T)$ theory of gravity, in which $f(R, T)$ is an arbitrary function of the Ricci scalar R and the trace T of the energy momentum tensor. The coupling between matter and geometry leads to extra acceleration in $f(R, T)$ theory. There is a choice of the functions $f(R, T)$ depending on the nature of the matter field [8]. Harko et al. [7] also pointed out that the $f(R, T)$ gravity model depends on the source term, which represents the variation of the matter stress energy tensor with respect to the metric. The simplest model, $f(R, T) = R + \lambda T$, is equivalent

to a cosmological model with an effective cosmological constant. The term $f(T)$ in gravitational action modifies the gravitational interaction between matter and curvature. In $f(R, T)$ theory, cosmic acceleration comes not only from geometrical contribution but also from matter content [9]. The $f(R, T) = \mu R + \gamma T$ model shows quintessence-like behaviour of the equation of state (EoS) parameter [10]. Reddy et al. [11] investigated the Bianchi type-III space time in the $f(R, T)$ gravity and showed that the model does not have an initial singularity. Rao and Neelima [12] investigated Bianchi type-VI₀ cosmological models and discovered that $f(R, T)$ affects the matter distribution but has no effect on space-time geometry. Singh and Singh [13] discussed the behaviour of a scalar field in $f(R, T)$ gravity theory within the context of a flat FRW cosmological model that describes the early and late time evolution of the universe. Mishra et al. [14] investigated Bianchi type VIIh space-time in the presence of perfect fluid and discussed the dynamical feature of the models in $f(R, T)$ gravity. Recently, Chaubey and Shukla [15] have obtained the different cosmological solutions of Bianchi models in the framework of $f(R, T)$ gravity.

The first few minutes after the Big Bang are important in the history of the universe. When the temperature drops to some critical point, topological defects occur due to the phase transition of the universe. The idea of spontaneous symmetry breaking prompts that the topology of the vacuum manifold M is identified as domain walls. Topological defects are remnants of the phase transitions that may have occurred in the early universe [16-18]. Hill et al. [19] have proposed that domain walls are important in the formation of galaxies. Recently, Reddy and Naidu [20] have analyzed thick domain walls in scale-covariant theory of gravitation. Lazanu et al. [21] have studied the contribution of domain walls to the cosmic microwave background power spectrum. Adhav et al. [22] have investigated thick domain walls in the Brans-Dicke theory of gravitation. Further, Sahoo and Mishra [23] have studied string and domain walls for plane symmetric space times. Mahanta and Biswal [24] have studied string clouds and domain walls with quark matter in Lyra geometry. Maurya et al. [25] investigated domain walls and quark matter in Bianchi type-V universe with observational constraints in $f(R, T)$ gravity. The tension of domain walls plays an important role in the study of cosmological models. The effect of tension on domain walls in higher-dimensional space-times is also investigated by Rahaman and Kalam [26]. Biswal et al. [27] have studied Kaluza-Klein domain walls in $f(R, T)$ gravity. Negative tension of domain walls shows that they are invisible, which means that there is energy transfer from domain walls to other forms of matter [28]. Some authors [29,30] have investigated the domain walls cosmological models in different theories.

With the motivation of the above discussion, we aim to study Bianchi type-VI₀

domain wall cosmological models in the framework of the $f(R, T)$ theory of gravity. Section 2 is devoted to metric and $f(R, T)$ gravity. The field equations for the Bianchi type-VI₀ space-time in the presence of a domain wall are presented in section 3. The solutions of the field equations determined in section 4 for the case $C_1 \neq 0$. In section 5, the solution of the field equation is obtained for the case $C_1 = 0$. The conclusion is presented in section 6.

2. *Metric and $f(R, T)$ gravity.* The modified theories of gravitation are aimed to address the problem of gravitational interaction. Renormalizable theories of gravitation are based on the inclusion of higher order terms of curvature invariant in the Lagrangian. The action for $f(R, T)$ gravity theory is given as

$$S = \int \sqrt{-g} (f(R, T) + L_m) d^4x, \quad (1)$$

where L_m is the Lagrangian density of the matter. The field equations of the $f(R, T)$ theory of gravity are obtained by varying action S with respect to g_{ij} , which is given by

$$\begin{aligned} f_R(R, T)R_{ij} - \frac{1}{2}f(R, T)g_{ij} + (g_{ij}\square - \nabla_i\nabla_j)f(R, T) \\ = 8\pi T_{ij} - f_T(R, T)T_{ij} - f_T(R, T)\Theta_{ij}, \end{aligned} \quad (2)$$

where

$$\begin{aligned} T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta\sqrt{-g}}{\delta g_{ij}} L_m, \quad \Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{\alpha\beta} \frac{\partial^2 L_m}{\delta g^{ij} \partial g^{\alpha\beta}}, \\ f_R = \frac{\partial f(R, T)}{\partial R}, \quad f_T = \frac{\partial f(R, T)}{\partial T}. \end{aligned}$$

We assume the function $f(R, T)$ in the following form:

$$f(R, T) = R + \mu T. \quad (3)$$

The late-time dominance of the domain wall network over the energy density provides a very small contribution to anomalies in the cosmic microwave background (CMB). The frozen domain wall is a proposed candidate of dark energy [31]. The energy momentum tensor for a thick domain wall [32] has a form

$$T_{ij} = \rho(g_{ij} + u_i u_j) + p u_i u_j, \quad (4)$$

where ρ and p stand for energy density and pressure of the domain wall, respectively, and u_i is the unit space-like vector in the same direction with $u_i u^i = -1$. Here $\rho = \rho_b + \sigma_d$ and $p = p_b - \sigma_d$, in which the quantities ρ_b and p_b stand for energy density and pressure of the barotropic fluid, and σ_d is the tension of the domain wall. It is believed that there is an anisotropic phase of evolution in the universe, i.e., the universe may be anisotropic in the early phase of evolution. Bianchi type I - XI space times are important in the study of the

anisotropic universe. We consider the Bianchi type VI₀ space time as

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2\alpha x} dy^2 + C^2 e^{2\alpha x} dz^2, \tag{5}$$

where A, B, C are the functions of t and α is a non-zero constant.

Bianchi type VI₀ space-time is of particular interest since it is a simple generalization of Bianchi type space and, at the same time, is sufficiently complex. Primordial helium abundance and isotropization in a special sense are explained by Barrow [33] with the help of Bianchi type VI₀ space-time. Weaver [34] used magnetism to investigate the Bianchi type VI₀ metric. Recently, the Bianchi type VI₀ space-time has been studied by Pradhan and Bali [35] in general relativity, whereas Rao and Neelima [12] and Ram et al. [36] in $f(R, T)$ theory of gravity, by using different physical fluids.

3. *Field equations.* In the $f(R, T)$ theory, the coupling of matter and gravity contributes equally to cosmic acceleration. The field equations of $f(R, T)$ gravity are derived in metric formalism as

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} + \frac{\alpha^2}{A^2} = (8\pi + 5\mu)\rho + \mu p \tag{6}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{\alpha^2}{A^2} = (8\pi + 5\mu)\rho + \mu p \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{B}\dot{A}}{BA} - \frac{\alpha^2}{A^2} = (8\pi + 5\mu)\rho + \mu p \tag{8}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{B}\dot{C}}{BC} - \frac{\alpha^2}{A^2} = -(8\pi + \mu)\rho + 3\mu p \tag{9}$$

$$\frac{\dot{C}}{C} - \frac{\dot{B}}{B} = 0, \tag{10}$$

where the overhead dot represents the derivative with respect to cosmic time. From equation (10), we obtain

$$C = \beta B, \tag{11}$$

where β is an integrating constant. The expansion scalar θ and shear scalar σ are defined as

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \tag{12}$$

$$\sigma^2 = \frac{1}{2} \left[\sum_{i=1}^3 H_i^2 - \frac{1}{3} \theta^2 \right] = \frac{1}{3} \left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} + \frac{\dot{C}^2}{C^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{A}\dot{C}}{AC} - \frac{\dot{B}\dot{C}}{BC} \right), \tag{13}$$

where $H = \frac{1}{3} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right)$ is the Hubble parameter, and $H_i, i = 1, 2, 3$ are the

directional Hubble parameters in the directions of the x, y, z axes, respectively. As discussed above, the universe is expanding and accelerating. The expansion scalar is used to predict how fast the universe is expanding mathematically. The shear scalar is the rate of shear of the fluid congruence. According to the theorem, "For any space-time in which the matter content consists of a perfect fluid and whose flow vector field forms an expanding geodesic and hypersurface orthogonal congruence, if the cosmological constant is zero, $\rho \propto \theta^2$ implies $\sigma^2 \propto \theta^2$ and $R \propto \theta^2$ " [37].

Using equation (11), the system of field equations (6) to (10) reduces to three equations in four unknowns A, B, p and ρ . We require one more condition to solve the system. According to recent observations of the velocity-redshift relationship for extragalactic sources, the Hubble expansion of the universe is now isotropic to within ≈ 30 percent. The redshift studies suggest that $\sigma/H \leq 0.30$, where σ is shear scalar and H is Hubble parameter [38,39]. The importance of this physical condition is discussed by Collins and Ellis [40]. The CMB is the most accurate measure of anisotropy. The entire universe is almost uniform. Notwithstanding, perturbation in density causes small fluctuation at the level 10^5 in one part. The new limit of an isotropic expansion is $\sigma/H < 4.7 \cdot 10^{-11}$ (95% C. L.) [41]. It should be noted that we can assume a physical relationship or a mathematical condition. However, assuming a mathematical condition may lead to an unphysical solution, and assuming a physical condition may lead to an unsolvable equation. Therefore, even though recent observational data shows that the universe is isotropic, we follow the literature and assume that the expansion scalar θ is proportional to the shear scalar σ , i.e., $\theta \propto \sigma$, to avoid any unphysical situation. Using equations (11), (12) and (13), we get

$$\frac{1}{\sqrt{3}} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) = l \left(\frac{\dot{A}}{A} - \frac{2\dot{B}}{B} \right), \tag{14}$$

where l is a constant. Equation (14) further reduces to

$$\frac{\dot{A}}{A} = n \frac{\dot{B}}{B}, \tag{15}$$

where $n = (1 + 2l\sqrt{3}) / (1 - l\sqrt{3})$. After integrating, equation (15) leads to

$$A = kB^n, \tag{16}$$

where k is a constant of integration. From equations (6)-(8) and (16), we get

$$2\ddot{B} + 2(n+1)\frac{\dot{B}^2}{B} = \frac{4\alpha^2}{(n-1)k^2 B^{2n-1}}. \tag{17}$$

Let $\dot{B} = L(B)$, then $\ddot{B} = LL'$ with $L' = dL/dB$ and we get the reduced equation (17) in the following form

$$L^2 B^{2(n+1)} = \frac{\alpha^2}{(n-1)k^2} B^4 + C_1^2, \quad (18)$$

where C_1^2 is the integration constant. We have two possibilities, depending on whether the value of the integration constant is zero or not. Note that the $f(R, T)$ gravity models depend on a source term, so each choice of L_m would generate a specific set of field equations. Particular cases of $f(R, T)$ functions, including $f(R, T) = R + \mu T$, are discussed by Harko et al. [7]. The importance of this model is that the coupling between matter and geometry becomes effective and time-dependent. We would like to mention that one may get different solutions by taking different values of constants and parameters. Consideration of all possibilities of $f(R, T)$ functions and other parameters is left as work for the future. Therefore, we consider the following two cases:

4. *Case I.* In this case, we have assumed that the non-zero integration constant i.e. $C_1 \neq 0$. The equation (18) is reduced to following equation:

$$\frac{B^{n+1} dB}{\sqrt{\frac{\alpha^2}{(n-1)k^2} B^4 + C_1^2}} = dt. \quad (19)$$

For simplicity, we take a particular value of constant, $n=0$ and by solving equation (19), we arrive at,

$$\frac{du}{\sqrt{C_1^2 - \frac{\alpha^2}{k^2} u^2}} = 2 dt, \quad (20)$$

where $u = B^2$.

Simplifying and integrating, we get

$$B = \sqrt{\left(\frac{kC_1}{\alpha}\right) \sin\left(\frac{2kt}{\alpha}\right)} \quad (21)$$

$$C = \beta \sqrt{\left(\frac{kC_1}{\alpha}\right) \sin\left(\frac{2kt}{\alpha}\right)} \quad (22)$$

$$A = k. \quad (23)$$

The scale factors are oscillatory. The oscillatory models are physically important since they are alternatives to inflation. From equations (6)-(9), we obtain the expressions for energy density and pressure are as follows:

$$\rho = \frac{6\pi b^2 \cos^2(2kt/\alpha)}{l_1 \sin^2(2kt/\alpha)} - \frac{8\pi\alpha^2}{3l_1 k^2} \quad (24)$$

$$p = \frac{(-2\pi + \mu)b^2 \cos^2(2kt/\alpha)}{l_1 \sin^2(2kt/\alpha)} + \frac{(2\pi + \mu)\alpha^2}{3l_1 k^2}, \tag{25}$$

where $b = 2k/\alpha$ and $l_1 = 64\pi^2 + 48\pi\mu + 8\mu^2$. From Fig.1, it is clear that the energy density is negative for $\mu = -6.29$ and positive for $\mu = -6.28$, therefore we have a viable model for $\mu \geq -6.28$ with increasing time. The pressure of domain wall is negative throughout the evolution of the universe depicted in Fig.2. Thus, domain walls play a role in dark energy. Also, the energy density of barotropic fluid is found to be

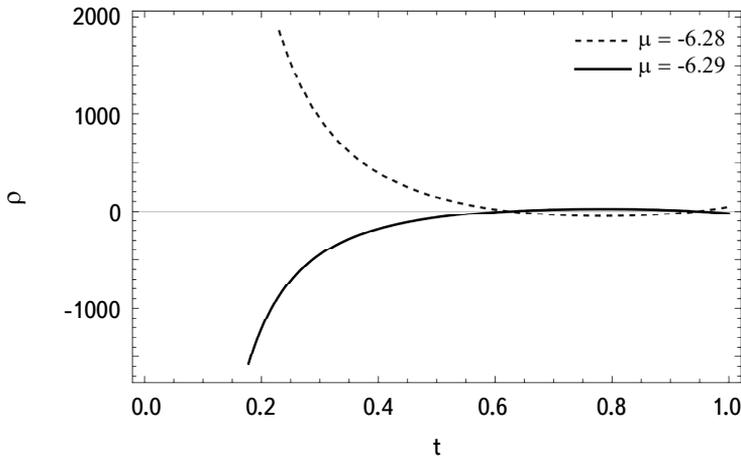


Fig.1. Plot of energy density ρ vs. cosmic time t for $\alpha = k = 0.1$.

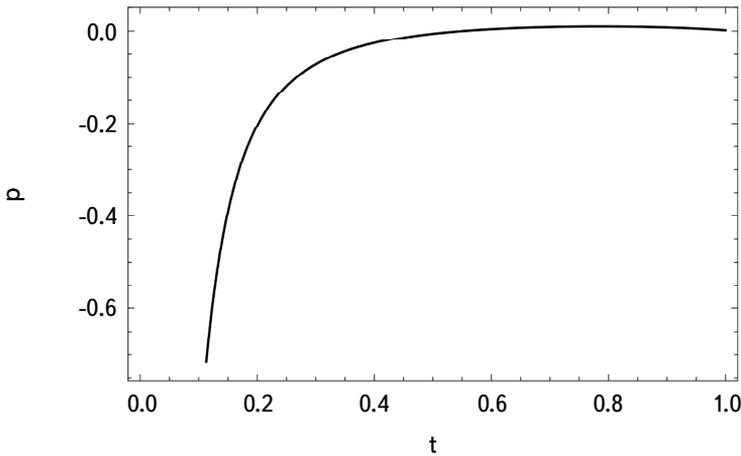


Fig.2. Plot of pressure p vs. cosmic time t for $\alpha = \mu = k = 0.1$.

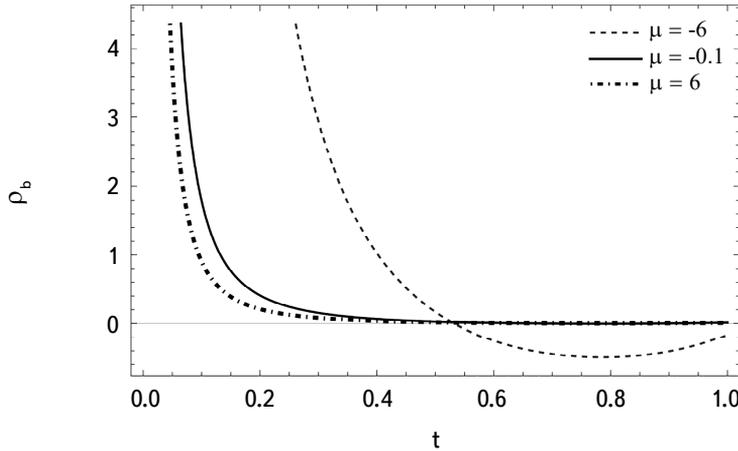


Fig.3. Plot of energy density of barotropic fluid ρ_b vs. cosmic time t for $\alpha = k = 0.1$.

$$\rho_b = \frac{(4\pi + \mu)b^2 \cos^2(2kt/\alpha)}{l_1(w+1)\sin^2(2kt/\alpha)} + \frac{(-2\pi + 3\mu)\alpha^2}{3l_1(w+1)k^2}. \tag{26}$$

Fig.3 depicts the behaviour of barotropic fluid energy density ρ_b for different values of parameter μ . It is clear that, for small values of μ , ρ_b is decreasing function of t . It is positive and large near $t=0$ and tends to zero with increasing time t . Furthermore, the tension of domain walls σ_d is obtained as

$$\sigma_d = \frac{(6\pi w + 2\pi - \mu)b^2 \cos^2(2kt/\alpha)}{l_1(w+1)\sin^2(2kt/\alpha)} - \frac{(8\pi w + 6\pi + 3\mu)\alpha^2}{3l_1(w+1)k^2}. \tag{27}$$

The Fig.4 shows the graphical behaviour of the tension of domain walls σ_d for $\mu = -6$, $\mu = -0.1$ and $\mu = 6$. It is observed that the tension of domain walls is positive decreasing function of time. Initially, it is very large and decays to zero with increasing time. The surface tension of domain walls control its motion. Zeldovich et al. [42] have discussed the properties of domain interfaces, cosmological expansion, and homogeneity of the universe. Domain walls must be disappear at a quite early stage of the evolution of the universe for isotropy and homogeneity. When domain walls disappear, the energy density transformed into energy of massive quanta or into equilibrium radiation. Here, $\sigma_d > 0$ near $t=0$, which means that domain walls exists in the early stage of the universe. They survive for a very small period of time and vanish, which is as per requirement stated by Zeldovich et al. [42]. This confirms our earlier work in $f(R, T)$ theory for FRW and axially symmetric space time [43]. The physical parameters such as expansion scalar θ , deceleration parameter q and shear scalar σ are obtained as

$$\theta = \frac{k \cos(2kt/\alpha)}{\alpha \sin(2kt/\alpha)} \tag{28}$$

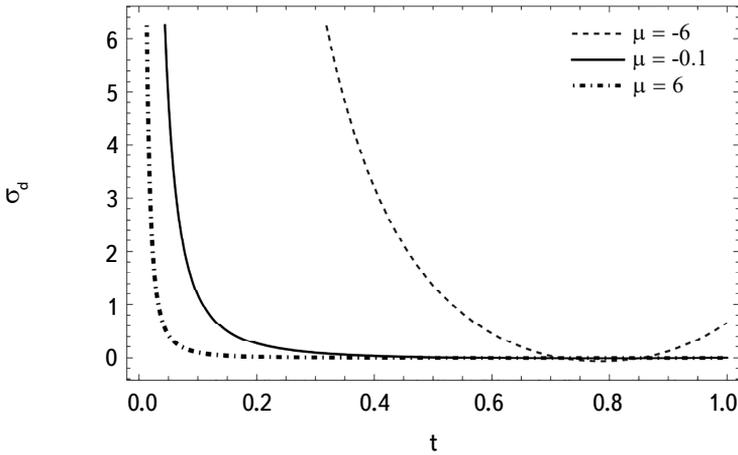


Fig.4. Plot of the tension of domain walls σ_d vs. cosmic time t for $\alpha = k = 0.1$.

$$q = -1 + 3\sec^2(2kt/\alpha) \tag{29}$$

$$\sigma = -\frac{k \cos(2kt/\alpha)}{\sqrt{3}\alpha \sin(2kt/\alpha)}. \tag{30}$$

The sign of deceleration parameter indicates whether the universe is accelerating or decelerating. The positive sign of q corresponds to decelerating universe, whereas the negative sign corresponds to an accelerating universe. Here, the deceleration parameter is positive i.e. the model indicates a decelerating universe. The expansion scalar θ and shear scalar σ are decreasing function of time; at $t=0$, θ and σ are infinite i.e. the rate of expansion of the universe was very high at the Big Bang. Also they are cyclic. It is noted that $\sigma/\theta \neq 0$ and σ/θ is constant, not depends on t , therefore the universe is anisotropic throughout the evolution. The volume of universe V , is found to be

$$V = \frac{\beta k^2 c_1}{\alpha} \sin(2kt/\alpha). \tag{31}$$

From equation (31), it is clear that the universe is cyclic which undergoes an endless sequence of cosmic epochs that begin with a bang and end in a crunch [44]. Basu [45] stated that when the galaxies are found to lie along the curve $q=1$ or above, the universe will expand to a maximum extension and then retrace back to its path in the contraction. The universe contracts to a state of high temperature and density, and at the end of contraction, it will vanish into singularity. The present observational status would suggest that the distance galaxies rather lie along the curve for $q=1$ i.e., the universe is expanding and decelerating.

Linder [46] has suggested novel models for parametrizing the equations of state

in the study of dark energy to extend the parametrization of dark energy to redshifts $z > 1$. Liu et al. [47] have studied the parametrized equation of the state of dark energy for a variety of models with identical parameters using SNIa data to determine the best way to parametrize it. The parametrized equation of state ω in term of average scale factor a is defined as

$$\omega = \omega_0 + \omega_1 \left(1 - \frac{a}{a_0} \right)^r, \tag{32}$$

where ω_0, ω_1 are two undefined parameter, a is average scale factor, a_0 is the present value of scale factor and $r = 1, 2, \dots$. Therefore, the new parametrized ω is obtained as

$$\omega = \omega_0 + \omega_1 \left[1 - \frac{\beta k^{2/3} c_1^{1/3}}{\alpha^{1/3}} (\sin(2kt/\alpha))^{1/3} \right]^r. \tag{33}$$

The graphical representation of parametrized w is shown in Fig.5. It is observed that the range of w is $-0.90 \leq w \leq -0.65$ for the particular values of the parameters $w_0 = 0.1$ and $w_1 = -1$. The observational data indicates the range of the EoS parameter w is $-1.013^{+0.068}_{-0.073}$ [48], whereas Liu et al. [47] obtained the

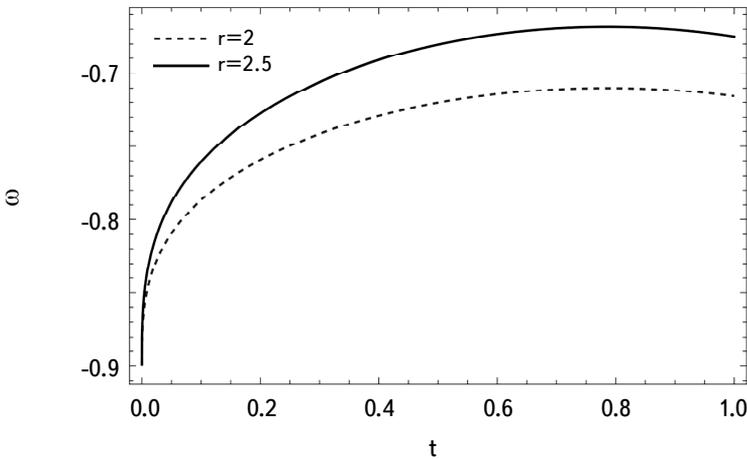


Fig.5. Plot of parametrized ω vs. cosmic time t for $\alpha = \beta = c_1 = k = 0.1$.

range of the EoS parameter as $-1.5 \leq w \leq 0.5$. Also, Scolnic et al. [49] obtained the range of the EoS parameter as $w = -1.026 \pm 0.041$. It is clearly found that the range of EoS parameters obtained in our model is consistent with values obtained by Suzuki et al. [48] and Scolnic et al. [49].

5. *Case II.* In this case, we consider that the constant of integration is zero i.e. $C_1 = 0$, then the equation (18) reduced to the following form

$$L^2 B^{2(n+1)} = \frac{\alpha^2}{(n-1)k^2} B^4. \tag{34}$$

Simplifying the above equation and integrating, we arrive at

$$B = E^{1/n} t^{1/n} \tag{35}$$

$$C = \beta E^{1/n} t^{1/n} \tag{36}$$

$$A = kEt, \tag{37}$$

where $E = n\alpha/\sqrt{n-1}k$. It is clear that metric potentials are real for $n > 1$ and imaginary for $n < 1$.

The expression of energy density and pressure are calculated as follows

$$\rho = \frac{n+2}{3\mu n^2 t^2} + \frac{(8\pi + \mu)[(2-n) - l_2(n+2)]}{l_1 n^2 t^2}, \tag{38}$$

where $l_2 = (8\pi + 5\mu)/3\mu$. From Fig.6, it is clear that the energy density is a decreasing function of time. It is large near $t=0$. A dark energy model for domain walls is also obtained by Caglar and Aygun [50] in case of a higher dimensional FRW universe.

$$p = \frac{3\mu[(n-2) - l_2(n+2)]}{l_1 n^2 t^2}. \tag{39}$$

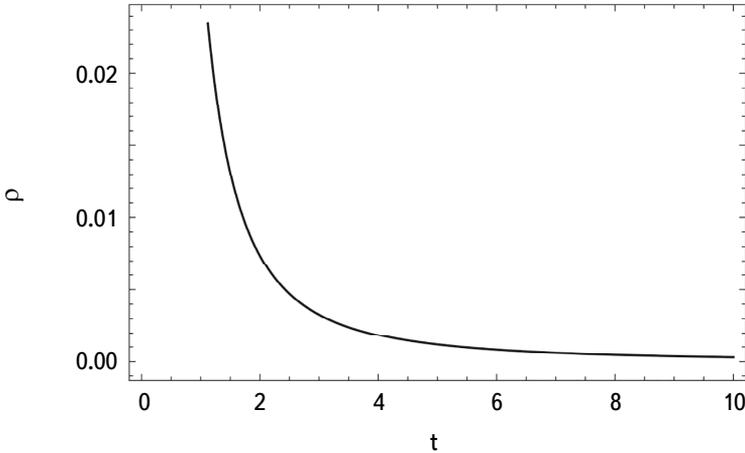


Fig.6. Plot of energy density ρ vs. cosmic time t for $\alpha = \mu = 0.1$ and $n = 1.1$.

Also, Fig.7 shows that the pressure is negative. Therefore, it is important to note that, we may say, the domain walls acts as dark energy candidates. The energy density of barotropic fluid is found to be

$$\rho_b = \frac{(2-n) - l_2(n+2)}{(8\pi + 2\mu)(w+1)n^2 t^2} + \frac{n+2}{3\mu(w+1)n^2 t^2}. \tag{40}$$

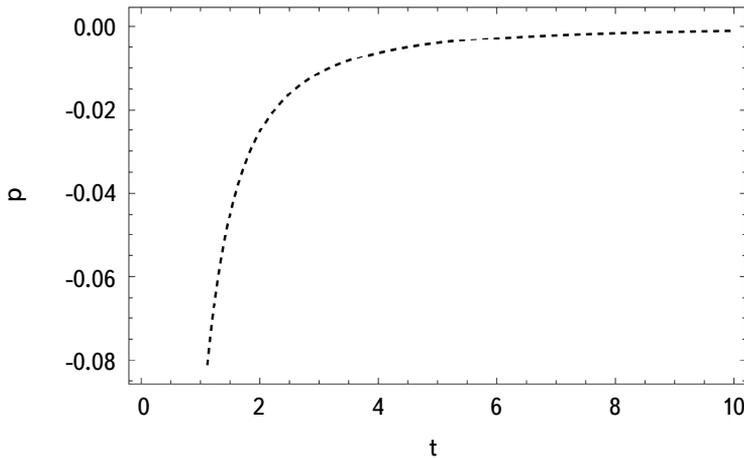


Fig.7. Plot of pressure p vs. cosmic time t for $\alpha = \mu = 0.1$ and $n = 1.1$.

Fig.8 shows that the plot of energy density of a barotropic fluid ρ_b is positive for $n=1.1$, $\mu = -25.133$ and it is negative for $n=1.1$, $\mu = -25.132$. We have a viable model for $\mu \leq -25.133$ and for other values of μ , the energy density is negative.

The tension of the domain walls is obtained as

$$\sigma_d = \frac{l_3[(2-n)-l_2(n+2)]}{l_1(w+1)n^2t^2} + \frac{(n+2)w}{3\mu(w+1)n^2t^2}, \tag{41}$$

where $l_3 = [(8\pi + \mu)w - 3\mu]$. Fig.9 shows that the graph of tension of domain walls σ_d is positive for $n=1.1$ and $\mu = -12.566$, but it is negative for $n=1.1$ and $\mu = -12.567$. The tension of domain walls is negative for a viable model, that is, for $\mu = -12.567$. It indicates that the domain wall transforms into another form

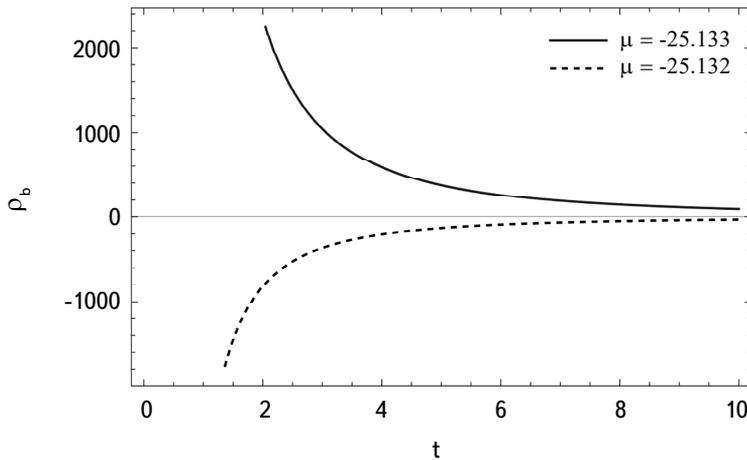


Fig.8. Plot of energy density of barotropic fluid ρ_b vs. cosmic time t for $\alpha = 0.1$ and $n = 1.1$.

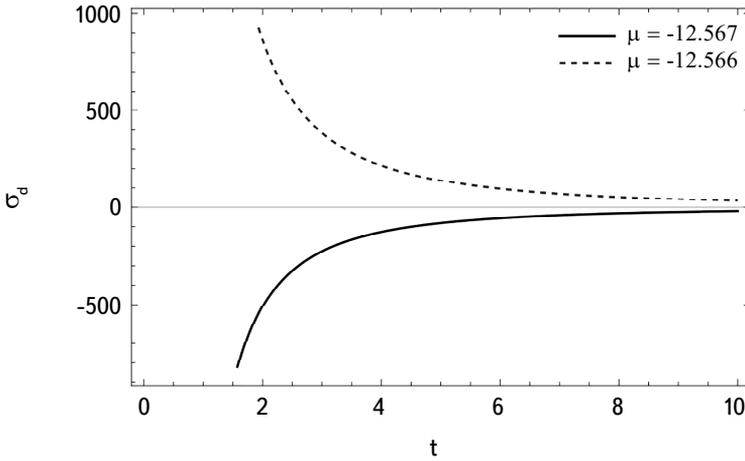


Fig.9. Plot of tension of the domain walls σ_d vs. cosmic time t for $\alpha = 0.1$ and $n = 1.1$.

of matter. The expansion scalar θ , deceleration parameter q and shear scalar σ is obtained as

$$\theta = \frac{n+2}{nt} \tag{42}$$

$$q = -1 + \frac{3n}{n+2} \tag{43}$$

$$\sigma = \frac{n-1}{\sqrt{3}nt}. \tag{44}$$

From expression (44), it is clear that the expansion rate of universe is decreasing with increasing time. It is highest near $t=0$. The shear scalar is also a decreasing function of time. At $n=1$, it is zero. Therefore, the model has isotropy at $n=1$ throughout the evolution. Also, $\sigma/\theta \neq 0$ for other values of n , $\sigma/\theta \neq 0$ i.e. the universe does not approach to isotropy at late time. Recently, Aluri et al. [51] studied consistency of isotropy with observational data and found that there is a deviation from isotropy in topological anomalies in CMB. In the present study, our model does not approach isotropy, and for consistency with the conclusion drawn by Aluri et al. [51], we will study this in the future work. The deceleration parameter is negative for $n < 1$ and positive for $n > 1$. As mentioned above, metric potentials A, B, C are real for $n > 1$. Therefore, q is positive in that case i.e. the universe is decelerating. The volume of universe V is found to be

$$V = \beta k \left[\frac{n\alpha t}{k\sqrt{n-1}} \right]^{(n+2)/n}. \tag{45}$$

From Fig.10, it is clear that the volume of universe is expanding with increasing time. The expansion of the universe is faster for $n = 1.1$ than $n = 2$. Also, the

parametrize equation of state ω is obtained as

$$\omega = \omega_0 + \omega_1 \left[1 - \beta^{1/3} k^{(n-6)/3n} \left(\frac{n\alpha t}{\sqrt{n-1}} \right)^{(n+2)/n} \right]^r. \tag{46}$$

Fig.11 shows that the graphical representation of parametrized w is an increasing function of cosmic time. For the particular values of the parameters $w_0 = 0.01$ and $w_1 = -1$, the range of the EoS parameter is $-1.0 \leq w \leq -0.1$, i.e., initially the model behaves like Λ CDM, and as time increases, it behaves like a quintessence dark energy model. In this case, the range of w is smaller than that obtained by Liu et al. [47], Suzuki et al. [48], and Scolnic et al. [49].

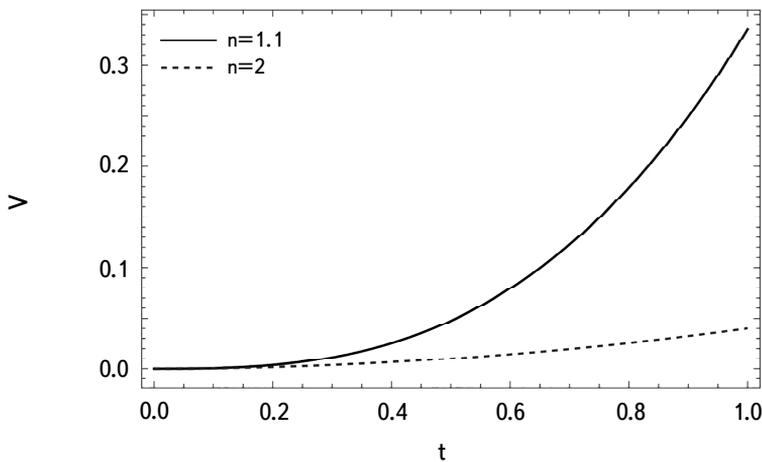


Fig.10. Plot of volume of universe V vs. cosmic time t for $\alpha = \beta = k = 0.1$.

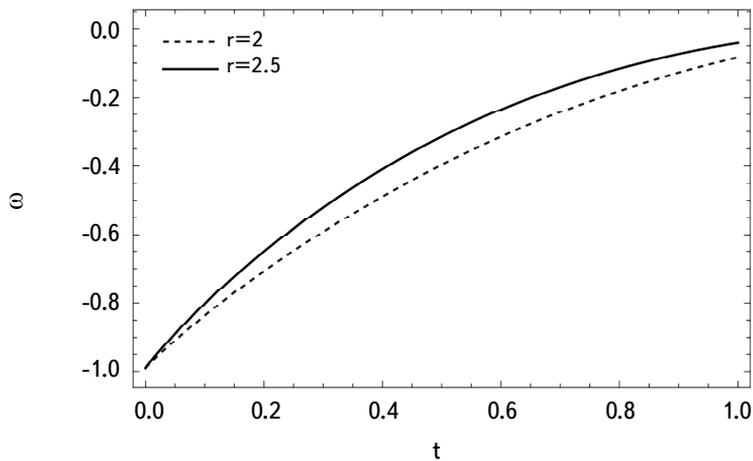


Fig.11. Plot of parametrize ω vs. cosmic time t for $\alpha = \beta = k = 0.1$ and $n = 1.1$.

6. *Conclusion.* In this paper, we have studied Bianchi type-VI₀ cosmological models in the presence of domain walls in $f(R, T)$ theory of gravity using the function $f(R, T) = R + 2f(T)$. The solutions of field equations are determined for the two cases such as $C_1 \neq 0, n = 0$ and $C_1 = 0$.

- In case I, the total energy density is positive for $\mu \geq -6.28$ with increasing time. The energy density of barotropic fluid is decreasing as a function of time. It is very large near $t=0$ and diminishes as t tends to infinity. As the tension of domain walls indicates the existence of domain wall, $\sigma_d > 0$, therefore domain walls exist in the early era of the universe and vanish with increasing time by transforming their energy into other forms of matter. In our case, q is positive, which indicates that the decelerating nature of the universe. Moreover, the expansion scalar and shear scalar are periodic functions and their ratio $\sigma/\theta \neq 0$, indicating that the universe is anisotropic. Similar results are also obtained by Adhav et al. [52] for domain walls. The novelty of work is that domain walls may behave like dark energy due to the negative pressure. The nature of the universe is cyclic, which begins with Big Bang and ends in a Big Crunch.

- In case II, we observe that the sign of deceleration parameter is negative for $n < 1$, thus the universe is accelerating for $n < 1$. Also, q is positive for $n > 1$ and $q = 0$ for $n = 1$. As the metric potentials A, B, C are imaginary for $n < 1$ and infinite for $n = 1$, we discard these values of n and we consider $n > 1$, for which we get q positive. Therefore, the universe is decelerating. The total energy density is a decreasing function of time. It is large near $t=0$. We observe that the energy density of barotropic pressure ρ_b is negative, whereas tension of the domain walls σ_d is positive. The ratio $\sigma/\theta \neq 0$ i.e. the study indicates an anisotropic universe. The universe is expanding.

- The range of EoS parameters in cases I and II are $-0.90 \leq w \leq -0.65$ and $-1.0 \leq w \leq -0.1$, respectively, which resemble the findings of Suzuki et al. [48] and Scolnic et al. [49].

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ВСЕЛЕННАЯ ТИПА БЪЯНКИ VI_0 С ДОМЕННЫМИ СТЕНКАМИ В РАМКАХ ТЕОРИИ ГРАВИТАЦИИ $f(R, T)$

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Рассмотрено пространство-время типа Бьянки VI_0 с доменными стенками в рамках модифицированной теории гравитации $f(R, T)$. Для решения уравнений поля предполагается, что скаляр сдвига σ пропорционален скаляру расширения θ . Рассматривается также параметризация уравнения состояния баротропной жидкости и обсуждается влияние на доменные стенки. Было замечено, что доменные стенки могут вести себя как темная энергия. Также подробно обсуждаются некоторые физические параметры.

Ключевые слова: *доменные стенки: пространство-время типа Бьянки VI_0 ; гравитация $f(R, T)$*

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