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PLANE SYMMETRIC GRAVITATIONAL FIELDS IN (D+1)-DIMENSIONAL GENERAL RELATIVITY

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We consider plane symmetric gravitational fields within the framework of General Relativity in (D+1)-dimensional spacetime. Two classes of vacuum solutions correspond to higher-dimensional generalizations of the Rindler and Taub spacetimes. The general solutions are presented for a positive and negative cosmological constant as the only source of the gravity. Matching conditions on a planar boundary between two regions with distinct plane symmetric metric tensors are discussed. An example is considered with Rindler and Taub geometries in neighboring half-spaces. As another example, we discuss a finite thickness cosmological constant slab embedded into the Minkowski, Rindler and Taub spacetimes. The corresponding surface energy-momentum tensor is found required for matching the exterior and interior geometries.

Keywords: *plane symmetric gravitational fields: Taub's solution: Rindler's spacetime: cosmological constant*

1. *Introduction*. Exact solutions of Einstein's equations for the gravitational field are available only in geometries with relatively high symmetry (for reviews see [1,2]). In particular, they include spherical, axial and planar symmetric configurations. Another classes of solutions with maximally symmetric subspaces are used in cosmology. Despite their apparent simplicity, plane symmetric solutions remain an active subject of research. The investigations are motivated by interesting geometrical properties of those solutions and by their applications in different areas of gravitational physics. The latter include the domain wall type topological defects in field theories [3] and branes in string theory and in braneworld models with extra dimensions [4-6].

The static plane-symmetric vacuum solutions of Einstein's equations were already known in the early days of the development of the General Relativity [7] and were rediscovered later in [8]. Two classes of solutions are present. The first one corresponds to the Rindler spacetime and describes a flat geometry. It approximates the gravitational field near the black hole horizon and is among the most popular geometries in quantum field theory on backgrounds with horizons (see, for example, [9]). The second class of single parameter solutions corresponds to the Taub geometry. The characteristic feature of the latter is the presence of a curvature singularity on a plane with a fixed value of the coordinate along which the geometry is inhomogeneous. The test particle is repelled by the singularity. The nature of the singularity, the other properties of the Taub solution and its generalizations in the presence of the matter sources have been widely discussed in the literature (see, e.g., [10-34] and references therein). In the present paper we discuss several aspects of plane-symmetric static solutions in (D+1)-dimensional General Relativity. Higher dimensional gravitational configurations with planar symmetry appear in a number of models including braneworld scenarios, Anti-de Sitter/Conformal field theory (AdS/CFT) correspondence and fundamental branes in string theories and supergravity.

The organization of the paper is as follows. In the next section, the background geometry, gravitational field equations and the matching conditions in problems with different metric tensors in separate regions are presented. In Section 3 two classes of vacuum solutions corresponding to the Rindler and Taub spacetimes are considered. The solutions with a cosmological constant (CC) as the only source in the gravitational field equations are discussed in Section 4. In Section 5 we consider a slab with CC interior and with different exterior geometries. The corresponding surface energy-momentum tensors required by the matching conditions on the slab boundaries are given.

2. *Background geometry, the field equations and matching conditions*. We consider a plane symmetric (D+1)-dimensional spacetime with the line element

$$ds^2 = e^{2u_0} dt^2 - e^{2u_1} dx^2 - e^{2u_2} \sum_{i=2}^{D} \left(dx^i\right)^2 , \tag{1}$$

where $x^1 = x$, $u_l = u_l(x)$, $l = 0, 1, 2$. The nonzero components of the Ricci tensor are given by the expressions

$$\begin{aligned} R_0^0 &= e^{-2u_1}\left[u_0'' + u_0'^2 - u_0' u_1' + (D-1) u_0' u_2'\right], \\ R_1^2 &= e^{-2u_1}\left[u_0'' + u_0'^2 - u_1' u_0' - (D-1)\left(u_2'' + u_2'^2 - u_0' u_2'\right)\right], \\ R_2^2 &= e^{-2u_1}\left[u_2'' + u_0' u_2' - u_1' u_2' + (D-1) u_2'^2\right], \end{aligned} \tag{2}$$

and (no summation over i) $R_i^i = R_2^2$ for $i = 3, \ldots, D$. Here, the prime stands for the derivative with respect to the coordinate x. The Ricci scalar is expressed as

$$R = 2 e^{-2u_1}\left[u_0'' + u_0'^2 - u_0' u_1' + (D-1)\left(u_2'' + u_0' u_2' - u_1' u_2' + \frac{D}{2} u_2'^2\right)\right]. \tag{3}$$

For discussion of geodesic motion one needs also to have the Christoffel symbols. The expressions for the corresponding nonzero components read (no summation over $i = 2, 3, \ldots, D$)

$$\Gamma^0_{01} = \Gamma^0_{10} = u_0' , \quad \Gamma^1_{00} = e^{2(u_0 - u_1)} u_0' , \quad \Gamma^1_{11} = u_1' ,$$
$$\Gamma^1_{ii} = -e^{2(u_2 - u_1)} u_2' , \quad \Gamma^i_{1i} = \Gamma^i_{i1} = u_2' . \tag{4}$$

The i th component of the acceleration for a test particle is given by $a^i = -\Gamma^i_{kl} w^k w^l$ with $w^i = dx^i / ds$ being the (D+1)-velocity. For a test particle at rest one has $w^i = \delta^i_0 e^{-u_0}$ and the acceleration is directed along the x-axis with $a^i = d^2 x^i / ds^2 = -\delta^i_1 e^{-2u_1} u_0'$.

For the gravitational field equations $R^k_i - \delta^k_i R/2 = 8\pi G T^k_i$, with T^k_i being the metric energy-momentum tensor, we get

$$-\frac{8\pi G T^0_0 e^{2u_1}}{D-1} = u_2'' + \frac{D}{2} u_2'^2 - u_1' u_2' , \quad -\frac{8\pi G T^1_1 e^{2u_1}}{D-1} = u_2' \left(u_0' + \frac{D-2}{2} u_2' \right),$$
$$-8\pi G T^2_2 e^{2u_1} = u_0'' + u_0'^2 - u_0' u_1' + (D-2)\left(u_2'' + u_0' u_2' - u_1' u_2' + \frac{D-1}{2} u_2'^2 \right). \tag{5}$$

In accordance with the problem symmetry one has (no summation over i) $T^i_i = T^2_2$ for $i = 3, \ldots, D$. Note that the quantity $-T^i_i$, $i = 1, 2, \ldots, D$, presents the effective pressure along the i th spatial dimension. From the covariant conservation equation $\nabla_k T^k_i = 0$ one finds

$$T_1^{1\prime} + \left(T^1_1 - T^0_0\right) u_0' + (D-1)\left(T^1_1 - T^2_2\right) u_2' = 0 . \tag{6}$$

This equation does not contain the function $u_1(x)$. For a source with barotropic equation of state, $T^i_i = -w_i T^0_0$ with constants w_i, we get

$$T^1_1 = \mathrm{const} \cdot \exp\left[-\left(\frac{1}{w_1} + 1 \right) u_0 + (D-1)\left(\frac{w_2}{w_1} - 1 \right) u_2 \right]. \tag{7}$$

In this case the components of the energy-momentum tensor, as functions of the coordinate x, do not change the sign.

The function $u_1(x)$ in (1) can be fixed by the choice of the coordinate x. The field equations are essentially simplified taking

$$u_1(x) = 0 . \tag{8}$$

This gives

$$-\frac{8\pi G T^0_0}{D-1} = u_2'' + \frac{D}{2} u_2'^2 , \quad -\frac{8\pi G T^1_1}{D-1} = u_2' \left(u_0' + \frac{D-2}{2} u_2' \right),$$
$$-8\pi G T^2_2 = u_0'' + u_0'^2 + (D-2)\left(u_2'' + u_0' u_2' + \frac{D-1}{2} u_2'^2 \right). \tag{9}$$

From these equations the following relations can be obtained:

$$\left[u_0' e^{u_0+(D-1)u_2}\right]' = \frac{8\pi G}{D-1}\left[(D-2)T_0^0 - T_1^1 - (D-1)T_2^2\right] e^{u_0+(D-1)u_2} ,$$
$$\left[u_0' e^{u_0+(D-1)u_2}\right]' = -\frac{8\pi G}{D-1}\left(T_0^0 + T_1^1\right) e^{u_0+(D-1)u_2} . \tag{10}$$

Note that $e^{u_0+(D-1)u_2} = \sqrt{|g|}$ with g being the determinant of the metric tensor g_{ik}. Additionally, by combining the equations (10) we get

$$\left[e^{u_0+(D-1)u_2}\right]'' = -\frac{8\pi G}{D-1}\left[T_0^0 + DT_1^1 + (D-1)T_2^2\right] e^{u_0+(D-1)u_2} . \tag{11}$$

The integration of relations (10) and (11) give conditions for the energy-momentum tensor to be compatible with given solutions for $u_0(x)$ and $u_2(x)$.

By using the set of equations (9) we can derive the matching conditions for the components of the metric tensor in problems where the geometry is described by two distinct metric tensors in regions separated by a planar boundary. As a separating boundary we take a hyperplane $x = L$. The energy-momentum tensor is decomposed into two contributions:

$$T_i^k = T_{(v)i}^k + T_{(s)i}^k , \quad T_{(s)i}^k = \tau_i^k \delta(x - L). \tag{12}$$

Here, $T_{(v)i}^k$ is the volume part and $T_{(s)i}^k$ corresponds to the surface energy-momentum tensor localized on the interface $x = L$. Generally, the volume part is different in the regions $x < L$ and $x > L$. Assuming that the metric tensor is continuous at $x = L$, the discontinuities in its first order derivatives are found by integrating the equations (9) in the region $[L-\varepsilon, L+\varepsilon]$, $\varepsilon > 0$, and then taking the limit $\varepsilon \to 0$. The continuity conditions for the metric tensor read

$$u_0\big|_{L_-}^{L_+} = \lim_{\varepsilon\to 0}\left[u_0(L+\varepsilon) - u_0(L-\varepsilon)\right] = 0 , \quad u_2\big|_{L_-}^{L_+} = 0 . \tag{13}$$

Under these conditions, by taking into account that $\lim_{\varepsilon\to 0}\int_{L-\varepsilon}^{L+\varepsilon} dx u_l'^2 = 0$ and $\lim_{\varepsilon\to 0}\int_{L-\varepsilon}^{L+\varepsilon} dx u_0' u_2' = 0$, for the first order derivatives we get

$$u_0'\big|_{L_-}^{L_+} = 8\pi G\left(\frac{D-2}{D-1}\tau_0^0 - \tau_2^2\right), \quad u_2'\big|_{L_-}^{L_+} = -\frac{8\pi G}{D-1}\tau_0^0 , \quad \tau_1^1 = 0 . \tag{14}$$

The discontinuities in the derivatives of the metric tensor are completely determined by the surface energy-momentum tensor. The corresponding conditions can also be obtained from the Israel matching conditions in terms of the extrinsic curvature tensor of the separating boundary.

3. *Vacuum solutions*. We start with the vacuum solutions of the set of equations (9). For them one has $T_i^k = 0$. By having the coordinate x fixed by the condition (8), we have two possibilities. For the first one $u_2' = 0$ and the first and second equations in (9) are satisfied identically. From the last equation we

get $u_0'' + u_0'^2 = 0$. The solution $u_0' = 0$ corresponds to a flat spacetime in the Minkowskian coordinates. The solution for $u_0' \neq 0$ is obtained after a simple integration: $e^{2u_0} = (x + C)^2$. Taking $C = 0$ we get the line element

$$ds_R^2 = x^2 dt_R^2 - dx^2 - \sum_{i=2}^{D} \left(dx^i\right)^2 , \tag{15}$$

which corresponds to the Rindler spacetime. Note that in the representation (15) the Rindler time coordinate t_R is dimensionless. Introducing new coordinates (T, X) in accordance with $T = x \sinh t_R$, $X = \mathrm{sgn}(X) x \cosh t_R$, the line element (15) takes the Minkowskian form. The coordinates ($t_R, x, x^2, ..., x^D$) cover the Rindler wedges $|X| > |T|$ of the Minkowski spacetime. The worldline with fixed ($x, x^2, ..., x^D$) describes a uniformly accelerated observer having the proper acceleration $1/x$. The hypersurface $x = 0$ corresponds to the Rindler horizon.

For the second class of the vacuum solutions we have $u_2' \neq 0$ and from the first equation in (9) we find $e^{2u_2} = \mathrm{const} \cdot |x + C|^{4/D}$. With this function $u_2(x)$, the second equation gives $e^{2u_0} = \mathrm{const} \cdot |x + C|^{-2(D-2)/D}$. For these expressions of $u_0(x)$ and $u_2(x)$ the last equation in (9) is obeyed identically. Specifying the constants, the solution is presented in the Taub form:

$$ds_T^2 = |1 - \sigma x|^{-2(2-D)/D} dt^2 - dx^2 - |1 - \sigma x|^{4/D} \sum_{i=2}^{D} \left(dx^i\right)^2 , \tag{16}$$

where σ is another constant. This solution has a singularity at $x = 1/\sigma$. For $D = 3$ it is reduced to the Taub solution in General Relativity. The higher dimensional generalization of the Taub solution has also been considered in [27]. For a test particle at rest with the coordinate x, the acceleration in the geometry (16) is expressed as $a^i = \delta_1^i (1 - 2/D)/(x - 1/\sigma)$. This corresponds to the repulsion from the wall at $x = 1/\sigma$ in both regions $x < 1/\sigma$ and $x > 1/\sigma$. Introducing the notations

$$n_D = 2\frac{D-1}{D}, \quad \sigma' = n_D^{(D-2)/2(D-1)} \sigma , \tag{17}$$

and new coordinates x'^i in accordance with

$$t' = n_D^{(2-D)/2(D-1)} t , \quad 1 - \sigma' x' = \frac{(1 - \sigma x)^{n_D}}{n_D}, \quad x'^i = n_D^{1/(D-1)} x^i , \quad i = 2, ..., D , \tag{18}$$

the line element is written in the form

$$ds_T^2 = |1 - \sigma' x'|^{(2-D)/(D-1)} \left(dt'^2 - dx'^2\right) - |1 - \sigma' x'|^{2/(D-1)} \sum_{i=2} \left(dx'^i\right)^2 . \tag{19}$$

As a simple example with two different metric tensors in the regions $x > 0$ and $x < 0$, we take $ds^2 = ds_T^2$ in the region $x < 0$ (given by (16) with $\sigma > 0$) and

$$ds_R^2 = (1 + x/b)^2\, dt^2 - dx^2 - \sum_{i=2}^{D} \left(dx^i\right)^2 , \tag{20}$$

in the region $x > 0$. The latter corresponds to the Rindler spacetime and is obtained from (15) redefining $x \to x + b$ and passing to a new time coordinate $t = bt_R$. For both regions $T_{(v)i}^k = 0$ and the metric tensor is regular. From (14) one gets the surface energy-momentum tensor required by the matching conditions:

$$8\pi G \tau_0^0 = -2\sigma \frac{D-1}{D}, \quad \tau_1^1 = 0, \quad 8\pi G \tau_2^2 = -\frac{1}{b} - \sigma \frac{D-2}{D} . \tag{21}$$

Note that the corresponding energy density is negative. In the special case $\sigma = 1/b$ we obtain $\tau_2^2 = \tau_0^0$ and τ_i^k describes a CC-type source localized on the plane $x = 0$.

4. *Solutions with cosmological constant*. In this section we consider the solutions of the gravitational field equations (9) with the CC Λ as the only source. For the corresponding energy-momentum tensor one has

$$T_i^k = T_{(\Lambda)i}^k = \frac{\Lambda}{8\pi G} \delta_i^k . \tag{22}$$

4.1. *AdS spacetime*. For a negative CC from the first equation we have a special solution

$$u_2' = \pm \frac{1}{a}, \quad a = \sqrt{\frac{D(D-1)}{2|\Lambda|}} . \tag{23}$$

With this solution, the second equation in (9) gives $u_0' = \pm 1/a$. The third equation is automatically satisfied. Fixing the integration constants, the line element corresponding to this solution takes the form

$$ds^2 = e^{\pm 2x/a} \left[dt^2 - \sum_{i=2}^{D} \left(dx^i\right)^2 \right] - dx^2 . \tag{24}$$

This line element describes AdS spacetime in Poincaré coordinates. Introducing a new coordinate $z = \mp a e^{\mp x/a}$, $-\infty < \pm z < 0$, the line element is written in a conformally flat form

$$ds^2 = \frac{a^2}{z^2} \left[dt^2 - \sum_{i=2}^{D} \left(dx^i\right)^2 - dz^2 \right] . \tag{25}$$

Here, the hypersurfaces $z = \mp\infty$ and $z = 0$ correspond to the AdS horizon and boundary, respectively. The acceleration of a test particle in the geometry (24) is given by $a^i = \mp \delta_1^i / a$ and it does not depend on the location of the particle. The latter property is a consequence of the maximal symmetry of the AdS spacetime. The acceleration is directed towards of the AdS horizon.

In the *D*-dimensional generalization of the Randall-Sundrum 1-brane model

[35] the background line element reads

$$ds^2 = e^{-2|x|/a}\left[dt^2 - \sum_{i=2}^{D}\left(dx^i\right)^2\right] - dx^2 , \tag{26}$$

and the brane is located at $x=0$. By taking into account that the volume energy-momentum tensor is given by (22) in both regions $x<0$ and $x>0$, from the matching conditions (14) we get

$$\tau_0^0 = \tau_2^2 = \frac{D-1}{4\pi Ga}, \quad \tau_1^1 = 0. \tag{27}$$

This correspond to a positive CC localized on the brane.

4.2. *General solution for negative CC.* For a negative cosmological constant the first integral of the first equation in (9) is given by

$$u_2' = \frac{1}{a}\tanh w, \quad w \equiv \frac{D(x-x_0)}{2a}, \tag{28}$$

with x_0 being an integration constant. Substituting this in the second equation we get

$$u_0' = \frac{1}{2a}\left[D\coth w - (D-2)\tanh w\right]. \tag{29}$$

Now it can be checked that with these solutions for u_0' and u_2' the third equation in (9) is obeyed identically. The simple integration of (28) and (29) gives the functions $u_0(x)$ and $u_2(x)$. The corresponding line element reads

$$ds^2 = \frac{\sinh^2 w}{(\cosh w)^{2(D-2)/D}}dt^2 - dx^2 - (\cosh w)^{4/D}\sum_{i=2}^{D}\left(dx^i\right)^2 . \tag{30}$$

Let us consider the asymptotic of the line element (30) for small and large values of $|w|$. For $|w| << 1$, keeping the leading terms we get

$$ds^2 \approx x'^2 dt'^2 - dx'^2 - \sum_{i=2}^{D}\left(dx^i\right)^2 , \tag{31}$$

where $x' = x - x_0$ and $t' = Dt/2a$. The right-hand side of (31) is the line element for the Rindler spacetime (compare with (15)). For large values of $|w|$, $|w| >> 1$, keeping the leading terms we get

$$ds^2 \approx e^{\pm 2x/a}\left[dt'^2 - \sum_{i=2}^{D}\left(dx'^i\right)^2\right] - dx^2 , \tag{32}$$

with $t' = 2^{-2/D} e^{\mp x_0/a} t$ and $x'^i = 2^{-2/D} e^{\mp x_0/a} x^i$. Here, the upper and lower signs correspond to the cases $w>0$ and $w<0$, respectively. Hence, in this limit the asymptotic geometry corresponds to the AdS spacetime.

For the acceleration of a test particle at rest one has $a^i = -\delta_1^i u_0'$ with u_0' given by (29). It is positive in the region $w<0$ and negative in the region $w>0$ and, hence, the acceleration is directed towards the hyperplane $w=0$ which corresponds to the Rindler horizon. At large distances from the horizon, corresponding to $|w|>>1$, we get $a^i \approx -\delta_1^i \,\mathrm{sgn}(w)/a$. Near the horizon the leading term in the asymptotic expansion is given by $a^i \approx \delta_1^i/(x_0-x)$. This term does not depend on the value of CC.

4.3. *General solution for positive CC*. We turn to the case of $\Lambda > 0$. By steps similar to those described in the previous subsection we can show that

$$u_0' = \frac{1}{2a}\left[D\cot w + (D-2)\tan w\right], \quad u_2' = -\frac{1}{a}\tan w. \tag{33}$$

The further integrations of these relations lead to the line element

$$ds^2 = \frac{\sin^2 w}{|\cos w|^{2(D-2)/D}}dt^2 - dx^2 - |\cos w|^{4/D}\sum_{i=2}\left(dx^i\right)^2. \tag{34}$$

In this case the metric is a periodic function of w with the period equal to π. This corresponds to the periodicity with respect to the coordinate x with the period equal to $2\pi a/D$. The asymptotic of the line element near the point $w=0$ is described by the Rindler line element (31). Near the point $w=\pi/2$ the line element is approximated by the Taub solution:

$$ds^2 \approx \frac{dt^2}{|w-\pi/2|^{2(D-2)/D}} - dx^2 - |w-\pi/2|^{4/D}\sum_{i=2}\left(dx^i\right)^2. \tag{35}$$

Note that in this point we have a singularity.

By taking into account that the metric tensor is periodic, let us consider the acceleration of the test particle, given as $a^i = -\delta_1^i u_0'$, in the region $-\pi/2 < w < \pi/2$. It is positive for $-\pi/2 < w < 0$ and negative for $0 < w < \pi/2$. This means that, similar to the case of negative CC, the acceleration is directed towards the Rindler horizon $w=0$ with the near horizon asymptotic $a^i \approx \delta_1^i/(x_0-x)$. The singular walls $w=\pm\pi/2$ are repulsive and near of them the asymptoic of the acceleration is given by $a^i \approx \delta_1^i(2-D)\,\mathrm{sgn}(w)/\left[a(\pi-2|w|)\right]$.

5. *CC slab with finite thickness*. As an application of the matching procedure and of the solutions described above, here we consider a finite thickness slab with the CC energy-momentum tensor (22) in the region $-L \le x \le L$. Different geometries in the exterior regions $x<-L$ and $x>L$ will be discussed. Assuming a symmetric configuration with respect to the plane $x=0$, firstly we consider the interior line element (30) with $x_0=0$, corresponding to a negative cosmological constant Λ. We have $ds^2 = g_{ik}^{(\Lambda)}dx^i dx^k$ with the metric tensor

$$g_{ik}^{(\Lambda)}(x) = \mathrm{diag}\left(\frac{\sinh^2 w}{(\cosh w)^{2(D-2)/D}}, -1, -\cosh^{4/D} w, \ldots, -\cosh^{4/D} w\right), \tag{36}$$

in the region $|x| \le L$ and $w = Dx/2a$. Rescaling the time and spatial coordinates x^i, $i = 2, \ldots, D$, the interior line element can be written in the form

$$ds^2 = g_{ik}(x) dx^i dx^k , \quad g_{ik}(x) = \frac{g_{ik}^{(\Lambda)}(x)}{\left|g_{ik}^{(\Lambda)}(L)\right|}, \quad |x| \le L. \tag{37}$$

With this normalization $g_{ik}(L) = \mathrm{diag}(1, -1, \ldots, -1)$. This normalization is convenient in the consideration of the matching conditions discussed below.

5.1. *Minkowski exterior*. We start the discussion by the Minkowskian geometry in the exterior regions:

$$ds_M^2 = dt^2 - dx^2 - \sum_{i=2}^{D} \left(dx^i\right)^2 , \quad |x| > L. \tag{38}$$

With the choice (37) for the interior line element, the metric tensor is continuous on the boundaries $x = \pm L$. The surface energy-momentum tensor is determined by the matching conditions (14). By taking into account that

$$u_0' = \frac{1}{2a}\left[D \coth w - (D-2) \tanh w\right], \quad u_2' = \frac{1}{a} \tanh w , \tag{39}$$

in the region $|x| < L$ and $u_0' = u_2' = 0$ for $|x| > L$, from (14) we get

$$\tau_0^0 = \frac{D-1}{8\pi G a} \tanh w_L , \quad \tau_1^1 = 0, \quad \tau_2^2 = \frac{D-2}{D-1}\frac{\tau_0^0}{2} + \frac{D \coth w_L}{16\pi G a}, \tag{40}$$

where $w_L = DL/(2a)$. The surface energy density and the stresses (no summation over i) $\tau_i^i = \tau_2^2$, $i = 2, \ldots, D$, are positive. Note that the effective pressure along the i th spatial direction is given by $-\tau_i^i$ and in the example under consideration it is negative.

5.2. *Rindler exterior*. For the exterior Rindler geometry the line element is given by (15) in the regions $|x| > L$ and for the interior geometry we have (37). Introducing a new Rindler time coordinate t in accordance with $t = Lt_R$, we see that the metric tensor is continuous on the boundaries $x = \pm L$. The derivatives in the matching conditions are given by (39) in the region $|x| < L$ and by $u_0' = 1/x$, $u_2' = 0$ in the region $|x| > L$. From (14) one finds

$$\tau_0^0 = \frac{D-1}{8\pi G a} \tanh w_L , \quad \tau_1^1 = 0, \quad \tau_2^2 = \frac{D-2}{D-1}\frac{\tau_0^0}{2} + D\frac{\coth w_L - 1/w_L}{16\pi G a}. \tag{41}$$

The surface energy density is the same as that for the Minkowski exterior, whereas the stresses are different. Note that, depending on the value of the parameter w_L, the effective pressure $-\tau_2^2$ can be either negative or positive.

5.3. *Taub exterior*. The exterior geometry is described by the line element (16). Redefining the coordinates and the constant σ, we rewrite it in the form

$$ds_T^2 = \left(\frac{1+\sigma|x|}{1+\sigma L}\right)^{2(D-2)/D} dt^2 - dx^2 - \left(\frac{1+\sigma|x|}{1+\sigma L}\right)^{4/D} \sum_{i=2}^{D} \left(dx^i\right)^2 . \tag{42}$$

For $\sigma > 0$ the metric tensor is regular. With this line element in the region $|x| > L$ and with the line element (37) for $|x| < L$, the metric tensor is continuous at $x = \pm L$. By taking into account that

$$u_0' = \frac{2-D}{D}\frac{\sigma \operatorname{sgn}(x)}{1+\sigma|x|}, \quad u_2' = \frac{2}{D}\frac{\sigma \operatorname{sgn}(x)}{1+\sigma|x|}, \tag{43}$$

for $|x| > L$, the matching conditions at $x = L$ give

$$\tau_0^0 = \frac{D-1}{8\pi G a}\left(\tanh w_L - \frac{2}{D}\frac{\sigma a}{1+\sigma L}\right), \quad \tau_2^2 = \frac{D-2}{D-1}\frac{\tau_0^0}{2} + D\frac{\coth w_L}{16\pi G a}, \tag{44}$$

and $\tau_1^1 = 0$. Note that for the Taub exterior, depending on the relative values of a and L, the surface energy density can be either positive or negative.

5.4. *Slab with positive CC.* Now we turn to the slab with positive CC. The interior line element is given by (34) with the metric tensor

$$g_{ik}^{(\Lambda)}(x) = \operatorname{diag}\left(\frac{\sin^2 w}{|\cos w|^{2(D-2)/D}}, -1, -|\cos w|^{4/D}, \ldots, -|\cos w|^{4/D}\right), \tag{45}$$

where $w = Dx/(2a)$. Again, rescaling the coordinates the line element is presented in the form (37) with $g_{ik}^{(\Lambda)}(L) = \operatorname{diag}(1, -1, \ldots, -1)$. We will assume that $L < \pi a/D$. In this case the metric tensor is regular inside the slab. The derivatives of the functions $u_0(x)$ and $u_2(x)$ in the region $|x| < L$ are given by (33). For the case of $\Lambda > 0$, the components of the surface energy-momentum tensor for the exterior Minkowski, Rindler and Taub geometries are obtained from the formulas given above for $\Lambda < 0$ by the replacements

$$\tanh w_L \to -\tan w_L , \quad \coth w_L \to \cot w_L . \tag{46}$$

The surface energy density is negative for all those geometries.

6. *Conclusion.* We have considered plane symmetric solutions of General Relativity for general number of spatial dimensions. For the metric tensor given by (1), the field equations are presented in the form (5) and the covariant continuity equation for the energy-momentum tensor is reduced to (6). The set of gravitational equations is simplified by the choice of the coordinate x in accordance with (8). By using those equations one can derive the matching conditions for the metric tensor in the problems where the geometry is described by two distinct line elements in neighboring half-spaces. The metric tensor is continuous on the separating boundary and the discontinuity of its first order

derivative is given by (14), where τ_i^k is the surface energy-momentum tensor.

Two classes of the vacuum solutions of the gravitational field equation are presented. The first one corresponds to the Rindler spacetime and the second one is a higher-dimensional generalization of the well known Taub solution. By an appropriate choice of the integration constants the latter is given by (16) (see also [27]). It has a singularity at $x = 1/\sigma$ that presents a repulsive wall for test particles. As a simple example of geometry with two distinct metric tensors in two different regions we have considered the combination of the Rindler and Taub geometries separated by a planar boundary. The components of the corresponding surface energy-momentum tensor are expressed by (21).

As an example of the source in gravitational field equations we have considered the CC Λ. For negative CC there is a special solution that corresponds to $(D+1)$-dimensional AdS spacetime. In Poincaré coordinates the line element has the form (24). In the Randall-Sundrum 1-brane model two copies of the AdS half-space are combined in the form of Eq. (26). The surface energy-momentum tensor on the separating brane is given by (27). The general solutions of the field equations for negative and positive CC are given by (30) and (34), respectively. In the case of a negative CC the geometry is non-singular. For small and large values of the variable $|w|$ it is approximated by the Rindler and AdS spacetimes, respectively. For a positive CC the metric tensor is a periodic function of x with the period $2\pi a/D$. In this case one has singularities at the points corresponding to $w = (n+1/2)\pi$. Near these points the geometry is approximated by the Taub solution. For both solutions with negative and positive CC the hyperplane $w = 0$ $(x = x_0)$ corresponds to a horizon that is the analog of the Rindler horizon. The acceleration of a test particle at rest is directed towards the horizon.

By using the solutions with a CC we have constructed a simple model of a finite thickness slab symmetric with respect to the central plane. The volume energy-momentum tensor inside the slab is given by (22) and in the exterior regions we have used the vacuum solutions of the field equations. Three different cases have been considered with the Minkowski, Rindler and Taub geometries. For the latter geometry the singularity-free Taub solution is employed. The corresponding surface energy-momentum tensors are expressed by (40), (41) and (44), respectively. For a slab with positive CC the interior geometry is non-singular for the half-thickness obeying the condition $L < \pi a/D$.

The setup considered in the present paper can be used for the investigation of the backreaction effects of the vacuum polarization of quantum fields induced by boundaries with $x = \text{const}$. The boundary conditions imposed on quantum fields lead to the modification of the spectrum for vacuum fluctuations and, as a consequence, the vacuum expectation values of physical observables are changed.

In particular, the vacuum energy-momentum tensor for planar boundaries has been widely considered in the literature. The simplest example is the Casimir effect (see, for example, [36]) for perfectly conducting parallel plates in the Minkowski spacetime. Already in that simple example the vacuum stresses are anisotropic. The planar boundaries in the Rindler spacetime, corresponding to uniformly accelerated plates in the Fulling-Rinlder vacuum, have been considered in [37-40]. The references for the corresponding investigations in the AdS bulk can be found in [41].

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ПЛОСКО-СИММЕТРИЧНЫЕ ГРАВИТАЦИОННЫЕ ПОЛЯ В (D+1)-МЕРНОЙ ОБЩЕЙ ТЕОРИИ ОТНОСИТЕЛЬНОСТИ

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Рассмотрены плоско-симметричные гравитационные поля в рамках общей теории относительности в (D+1)-мерном пространстве-времени. Два класса вакуумных решений соответствуют многомерным обобщениям пространства-времени Риндлера и Тауба. Представлены общие решения для положительной и отрицательной космологической постоянной в качестве единственного источника гравитации. Обсуждаются условия сшивки на плоской границе двух областей с различными плоско-симметричными метрическими тензорами. Рассмотрен пример с геометриями Риндлера и Тауба в соседних полупространствах. В качестве другого примера обсуждается плоско-параллельная пластина конечной толщины с космологической постоянной, погруженная в пространство-время Минковского, Риндлера и Тауба. Найден соответствующий поверхностный тензор энергии-импульса, необходимый для согласования внешней и внутренней геометрии.

Ключевые слова: *плоско-симметричные гравитационные поля: решение Тауба: пространство-время Риндлера: космологическая постоянная*

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