

PROBING PSEUDOSCALARS WITH PULSAR POLARISATION DATA SETS

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Recently a data set containing linear and circular polarisation information of a collection of six hundred pulsars has been released. The operative radio wavelength for the same was 21 cm. Pulsars radio emission process is modelled either with synchrotron/superconducting self-Compton route or with curvature radiation route. These theories fall short of accounting for the circular polarisation observed, as they are predisposed towards producing, solely, linear polarisation. Here we invoke (pseudo)scalars and their interaction with photons mediated by colossal magnetic fields of pulsars, to account for the circular part of polarisation data. This enables us to estimate the pseudoscalar parameters such as its coupling to photons and its mass in conjunction as product. To obtain these values separately, we turn our attention to recent observation on 47 pulsars, whose absolute polarisation position angles have been made available. Except, a third of the latter set, the rest of it overlaps with the expansive former data set on polarisation type and degree. This helps us figure out, both the pseudoscalar parameters individually, that we report here.

Keywords: *ALP- γ (14.80.Va) mixing:pulsar (97.60.Gb):polarisation (42.25.Ja)*

1. Introduction. In the last two decades, scenarios in which pseudoscalar [1-6] particles and photons couple and subsequently mix (Fig.1) in the presence of magnetic fields. There are many ways in which photon can interact with axion like particles (ALPs). And these interaction can change the polarisation properties of electromagnetic radiations through different phenomena [7,8]. First may be, since the axions alter the photon dispersion relation, EM wave propagation in the presence of an axion backdrop can modify the polarisation of the wave. Different photon polarisation modes propagate through an axion cloud at different phase velocities. [9,10] provides the modified dispersion relation of photon polarisation modes. Second may be, the axion photon scattering process at the second level of coupling $O(g_{a\gamma}^2)$ can also produce circular polarisation by converting linear polarisation through Faraday conversion. The development of the Stokes parameters at the second order of $g_{a\gamma}$ is then computed using the quantum Boltzmann method. In third we study the scattering process of photons from a magnetic field by exchanging virtual axions in the intermediate states as a second order phenomena in terms of $O(g_{a\gamma}^2)$ using the quantum Boltzmann approach, i.e. when

plane polarized light coming from a source its parallel component which is parallel to magnetic field may interact with magnetic field and convert into axion like particles (ALPs) and these ALPs may again interact with magnetic field and reconvert back to the photons, while perpendicular component of plan polarized light will go as it is. Because ALPs have very little non zero mass so its velocity is smaller than photons. So, due to this conversion parallel component of plan polarized light will moves slowly and a phase difference will come in the parallel and perpendicular components of plane polarized light which will induce circular component in the incident light. This mixing have received a lot of attention [11-17], both phenomenologically [18-25] and observationally [26-33]. This is of particular interest in astrophysics, where this mixing of photons with pseudoscalars could make the universe transparent [34], change the polarisation properties of light [35] and is be potentially responsible for effects such as "Supernovae dimming" [34] or "Large-scale coherent orientation" [35] of the universe, also known as "Hutsemekers" effect. The best-known light pseudoscalar particle, the axion, was introduced long ago [36] to explain the absence of CP violation in Quantum Chromodynamics (QCD) [37]. One postulated the existence of a new spontaneously broken continuous Peccei-Quinn symmetry, so that the axion was a pseudo Goldstone boson. It was soon realised that one needed to introduce a very large scale in the theory in order to suppress the interactions of the axion, while preserving the Peccei-Quinn mechanism. The invisible axion [38] emerges at a unification scale, and the effective coupling is suppressed by this scale. The invisible axion, being closely related to QCD, has definite and interrelated expressions for its mass [39] and coupling strength [40] to other particles, given

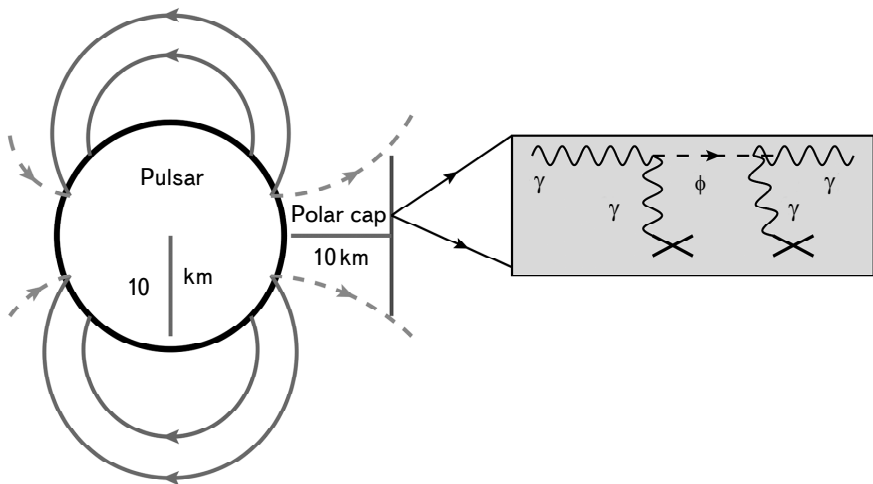


Fig.1. Axion photon mixing in the polar cap region.

a specific model [41,42]. Various cosmological and astrophysical bounds can be used to further constrain the parameters [40], and the allowed parameters do not lead to observable effects over cosmological scales. The mass of the pseudoscalar particle needs to be very close to the photon effective mass in order to mix in the rather weak magnetic fields of the extra galactic space. However, generic pseudoscalars or axion-like particles (ALP's) have been hypothesized by many extensions of the standard model of particle physics. Theories such as supergravity [43] and superstring theory [44] contain many broken U(1) symmetries, that can lead to very light scalar, or pseudoscalar, particles.

Pulsars, discovered fifty years back [45], are a fusion fuel less state of a two to three solar mass $M_{\odot}$ star [46], wherein surmounting inward gravitational pull [47], in absence of a commensurate radiation pressure from fusion, makes it collapse [48], into a tiny object [49]. Two effects follow: the protons and neutrons coalesce together making the pulsars synonymous with neutron stars [50]; and, also during this compression phase the the magnetic flux is conserved, thereby promoting the magnetic induction field inside it to a colossal [50] value. Other effects such as the "pulsating" nature of the "star" in its last phase of stellar evolution, leading to the nomenclature and discovery of the same [50], won't be pursued here.

Pulsars have been harnessed to estimate the coloumn density [51], by observing pulsar dispersion measure. Also, the magnetic field of the interstellar medium (henceforth ISM) along the line of sight can be estimated by observing its rotation measure [51]. Pulsars were traditionally, observed on earth inside the radio frequency window specifically, from 100 MHz - 100 GHz [50]. However, over time, pulsars became known for emission in other wavebands like X-ray, γ -ray, etc. [52]. Despite, half a century of efforts, the mechanisms for such types of radiation and properties thereof, such as polarisation, are not very well understood [53]. This in turn banks heavily on the fact that pulsar atmosphere or its magnetosphere models are still in its infancy [54]. There are competing contenders as preferred models for pulsed emission and continuum radiation. Curvature radiation, synchrotron radiation, inverse Compton radiation, superconducting self Compton radiation, etc. are at the forefront, but none fits all observational features of pulsars [53]. We shall, however, restrict ourselves, polarisation properties of radiation inside the pulsar atmosphere without looking into the radiation origin. Here we shall harness the two pulsar properties the size, and magnetic field which in turn is deduced from period and associated derivative, to estimate the pseudoscalar parameters like the mass and its coupling to photons, with the help of 21 cm observations.

In section 2 we describe the polarimetric data set on six hundred pulsars [55], along with the quantities that can be derived from these observed parameters

Table 1

PULSAR POLARISATION PROPERTIES AT 1.4 GHz (sample)

Jname	P (ms)	$\dot{P}$ unitless	$\log(\dot{E})$ (ergs ⁻¹)	L/I %	V/I %	$ V /I$ %	err %	B Gev ²	χ rad
J0034-0721	943	4.24E-16	31.3	10.7	7.7	7.5	3	4.44E-08	0.303493832023204
J0051+0423	354.7	7.14E-18	30.8	13.1	-2.3	11.2	3.3	3.53E-09	0.287272232961884
J0108-1431	807.6	8.43E-17	30.8	76.7	15.5	13.1	3.1	1.83E-08	0.085016451375219
J0134-2937	137	8.21E-17	33.1	45.3	-17.2	16.9	3	7.44E-09	0.178538083464964
J0151-0635	1464.7	3.99E-16	30.7	29.1	-1.7	4.2	3.3	5.36E-08	0.070948527302082
J0152-1637	832.7	1.16E-15	31.9	15.1	1.1	6	3	6.90E-08	0.190253188556182
J0206-4028	630.6	1.27E-15	32.3	10.6	9.3	9.9	3.1	6.27E-08	0.366407550893253
J0211-8159	1077.3	3.17E-16	31	17	11.7	15.4	5.5	4.10E-08	0.502033554635695
J0255-5304	447.7	2.86E-17	31.1	7.3	-4.1	5.5	3	7.94E-09	0.329545023167104
J0304+1932	1387.6	1.35E-15	31.3	33.4	15.1	14.8	3	9.60E-08	0.209112164789615
J0343-3000	2597	5.59E-17	29.1	14.3	3.1	3.9	3.2	2.67E-08	0.107846382092558
J0401-7608	545.3	1.64E-15	32.6	28.6	-0.1	4.7	3	6.63E-08	0.08060916072996
J0448-2749	450.4	1.46E-16	31.8	23.9	-13.3	11.8	3	1.80E-08	0.225223527837328
J0450-1248	438	1.07E-16	31.7	25.3	2.5	6	3.4	1.52E-08	0.126047990334037
J0452-1759	548.9	5.28E-15	33.1	18.9	3.6	4.2	3	1.19E-07	0.109334472936971
J0459-0210	1133.1	1.47E-15	31.6	10.4	-12.9	9.6	3.8	9.05E-08	0.337370471111776
J0520-2553	241.6	2.84E-17	31.9	18.2	-4.3	5	3.6	5.81E-09	0.129970070094264
J0525+1115	354.4	7.12E-17	31.8	10.6	12.5	15.5	3	1.11E-08	0.478954335502063
J0528+2200	3745.5	4.21E-14	31.5	36.9	-4.9	4.6	3	8.81E-07	0.062683228726001
J0533+0402	963	1.8E-16	30.9	13.3	4	5.5	3.2	2.92E-08	0.211767678516289
J0536-7543	1245.9	6.17E-16	31.1	48.8	-11.1	11	3	6.15E-08	0.110852247980192
J0540-7125	1286	8.55E-16	31.2	14.2	3.1	15.8	4.8	7.35E-08	0.391121799864486
J0543+2329	246	1.5E-14	34.6	45.2	-8.2	7.9	3	1.35E-07	0.086515496781741
J0601-0527	396	1.25E-15	32.9	30.9	4.4	11.3	3	4.94E-08	0.175294355633881
J0614+2229	335	6.01E-14	34.8	72	20.3	20.1	3	3.15E-07	0.135938275210493

assuming a basic pulsar model [50]. The observation of circular polarisation is hitherto unexplained by radiation models, theoretical [56-58] and statistical [59] alike, so far. Thereafter, in the section 3, we invoke the light quanta to pseudoscalar interaction to step wise calculate the correlators, ab initio, between the three degrees of freedom. Thereby, in section 4 we digress to Stokes parameters; the experimental interface with theoretical quantities like ellipticity parameter and polarisation position angle, using the definition of correlators. In the next section 5, we discuss the extraction process of pseudoscalar parameters, for mixing case only, discarding another two cases and leaving the general case open that might arise, naturally. Also in this segment we estimate the values of regression parameters derived from statistical analysis of the data set tables. In the next section we present our results. Thereafter, we conclude by projecting the feasibility of our result and scope in future directions.

2. *Observation.* The following data shown in Table 1 is a small part of the data obtained from [55]. It contains the spin down luminosity $\dot{E}$ [48] and pulsar spin period P [49] all six hundred of them. Following the basic pulsar model [50,60] we may derive pulsar parameters such as spin period time derivative $\dot{P}$ and the magnetic field B_s .

$$B_s = 3.2 \cdot 10^{19} \sqrt{P\dot{P}}, \quad \dot{P} = \frac{\dot{E}P^3}{4\pi^2 I}. \quad (1)$$

Also, from the ratio between the percentage of circular to linear polarisation provides us with the ellipticity parameter χ .

$$\tan(2\chi) = \frac{V}{P_{lin}}. \quad (2)$$

We have extracted and separately tabulated these derived values for further use in section 5.

3. *Pseudoscalar photon mixing.* We begin our discussion with a derivation of the equations of motion for the axion-photon system [61] where the term "axion" stands generically for any light pseudoscalar particle. A suitable Lagrangian density is given by

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_a^2 a^2 + \frac{1}{4M}F_{\mu\nu}\tilde{F}^{\mu\nu}a \\ & + \frac{a^2}{90m_e^4} \left[(F_{\mu\nu}F^{\mu\nu})^2 + \frac{7}{4}(F_{\mu\nu}\tilde{F}^{\mu\nu})^2 \right], \end{aligned} \quad (3)$$

where a is the axion field, m_a its mass, $F_{\mu\nu}$ the electromagnetic field tensor, and $\tilde{F}^{\mu\nu} = \varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}/2$ its dual. The third term describes the CP-conserving interaction between the pseudoscalar and the electromagnetic field where the energy scale M is a phenomenological parameter to characterize the interaction strength i.e. $g_{a\gamma} = 1/M$, $g_{a\gamma}$ is coupling constant. The last term in Eq. (3) is the Euler-Heisenberg effective Lagrangian' arising from the vacuum polarizability. It describes photon-photon interactions in the limit where the photon frequencies are small in comparison with the electron mass m_e , and all field strengths are weak in comparison with the critical field strengths. In our case last term is negligible. We will solve equation of motion for above Lagrangian.

The following mixing matrix (4) provides for the necessary ingredient of photon pseudoscalar mixing mediated by a magnetic field [61]. Also this reference assumes a free space, for calculation, hence there are no Faraday effect (M_{21} , M_{12} entries) terms coupling the two photon polarisations. Inside pulsar magnetosphere this could hardly be the case. However, we may still ignore the Faraday terms. The reason being the smallness of it inside spaces with large magnetic fields; as shown in by one of the coauthors [62], by deriving the limiting propagation

frequency, below which Faraday effect holds significance.

$$\omega_L = \frac{\omega_p^2 \omega_B (\omega_p^2 - m_a^2) \cos \Theta}{|\mathcal{B}|^2 \sin^2 \Theta},$$

for the values derived from the pulsar database, such as the magnetic field, and the plasma frequency from literature [63], which is much smaller than the pseudoscalar mass, we see that Faraday effect can safely be neglected at the operating frequency of 1.4 GHz ($\gg \omega_L$), with which the observations were made.

$$M = \begin{pmatrix} A_1 & 0 & 0 \\ 0 & A_2 & T \\ 0 & T & -B \end{pmatrix}, \quad (4)$$

The symbols in the matrix (4) stands for

$$A_1 = 4\omega^2 \xi \sin^2 \Theta + \omega_p^2, \quad A_2 = 7\omega^2 \xi \sin^2 \Theta + \omega_p^2, \quad B = m_a^2, \quad T = g \mathcal{B} \omega. \quad (5)$$

where, $\mathcal{B}$ is the magnetic field, Θ is the angle between the $\vec{k}$ and the magnetic field $\mathcal{B}$, m_a the axion mass, and $\xi = (\alpha/45\pi)(e\mathcal{B}/m_f^2)^2$, with m_f [61] the lightest Fermion mass.

The non-diagonal 2×2 matrix, in Eq. (4) is given by,

$$M_2 = \begin{pmatrix} A_2 & T \\ T & -m_a^2 \end{pmatrix}. \quad (6)$$

One can solve for the eigen values of the Eq. (7), from the determinant equation,

$$\begin{vmatrix} A_2 - \lambda & T \\ T & -m_a^2 - \lambda \end{vmatrix} = 0, \quad (7)$$

and the roots are,

$$M_{\pm} = \frac{A_2 - m_a^2}{2} \pm \frac{1}{2} \sqrt{\left[(A_2 + m_a^2)^2 + 4T^2 \right]}. \quad (8)$$

3.1. Equation of motion. The equation of motion for the axion photon mixing, in the non-diagonal basis gets decoupled and can be written in the matrix form as:

$$\left[(\omega^2 + \partial_z^2) \mathbf{I} + \mathbf{M} \right] \begin{pmatrix} A_{\perp} \\ A_{\parallel} \\ a \end{pmatrix} = 0. \quad (9)$$

where $\mathbf{I}$ is a 3×3 identity matrix and $\mathbf{M}$ is the mixing matrix.

The uncoupled and the coupled equations can further be written as,

$$\left[(\omega^2 + \partial_z^2) + A_1 \right] (A_\perp) = 0 \quad (10)$$

and

$$\left[(\omega^2 + \partial_z^2) \mathbf{I} + M_2 \right] \begin{pmatrix} A_{||} \\ a \end{pmatrix} = 0. \quad (11)$$

It is possible to diagonalise Eq. (11) by a similarity transformation (we would denote the diagonalising matrix by O), leading to the form,

$$\left[(\omega^2 + \partial_z^2) \mathbf{I} + M_D \right] \begin{pmatrix} \bar{A}_{||} \\ \bar{a} \end{pmatrix} = 0. \quad (12)$$

when the diagonal matrix M_D is given by:

$$M_D = \begin{pmatrix} M_+ & 0 \\ 0 & M_- \end{pmatrix}. \quad (13)$$

3.2. Dispersion relations. Defining the wave vectors in terms of k_i 's, as:

$$k_\perp = \sqrt{\omega^2 + A_1}, \quad k_+ = \sqrt{\omega^2 + M_+}, \quad k_- = -\sqrt{\omega^2 + M_+} \quad (14)$$

and

$$k'_+ = \sqrt{\omega^2 + M_-}, \quad k'_- = -\sqrt{\omega^2 + M_-}. \quad (15)$$

3.3. Solutions. The solutions for the gauge field and the axion field, given by (12) as well as the solution for eqn. for $A_\perp$ in k space can be written as,

$$\bar{A}_{||}(z) = \bar{A}_{||+}(0)e^{ik_+z} + \bar{A}_{||-}(0)e^{-ik_-z}, \quad (16)$$

$$\bar{a}(z) = \bar{a}_+(0)e^{ik'_+z} + \bar{a}_-(0)e^{-ik'_-z}, \quad (17)$$

$$A_\perp(z) = A_{\perp+}(0)e^{ik_\perp z} + A_{\perp-}(0)e^{-ik_\perp z}. \quad (18)$$

The diagonal matrix can be written as

$$M_D = O^T M_2 O \quad (19)$$

when

$$O = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \equiv \begin{pmatrix} c & -s \\ s & c \end{pmatrix} \quad (20)$$

in short hand notation.

3.4. Similarity transformation. The diagonal matrix

$$M_D = \begin{pmatrix} c & -s \\ -s & c \end{pmatrix} \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} c & -s \\ s & c \end{pmatrix}, \quad (21)$$

With $M_{11} = A_2$, $M_{12} = T$, $M_{21} = T$ lastly $M_{22} = -B$.

The value of the parameter θ , is fixed from the equality,

$$M_D = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} c & -s \\ s & c \end{pmatrix} = \begin{pmatrix} M_+ & 0 \\ 0 & M_- \end{pmatrix}, \quad (22)$$

leading to,

$$\begin{pmatrix} c^2 M_{11} + s^2 M_{22} + 2csM_{12} & M_{12}(c^2 - s^2) + cs(M_{22} - M_{11}) \\ M_{12}(c^2 - s^2) + cs(M_{22} - M_{11}) & s^2 M_{11} + c^2 M_{22} - 2csM_{12} \end{pmatrix} = \begin{pmatrix} M_+ & 0 \\ 0 & M_- \end{pmatrix}. \quad (23)$$

Equating the components of the matrix equation (23), one arrives at:

$$\tan(2\theta) = \frac{2M_{12}}{M_{11} - M_{22}} = \frac{T}{A_2 - m_a^2}. \quad (24)$$

3.5. Correlation functions. The solutions for propagation along the +ve z axis, is given by,

$$\bar{A}_{||}(z) = \bar{A}_{||}(0)e^{ik_+z} \quad (25)$$

$$\bar{a}(z) = \bar{a}(0)e^{ik'_+z} \quad (26)$$

that can further be written in the following form,

$$\begin{pmatrix} \bar{A}_{||}(z) \\ \bar{a}(z) \end{pmatrix} = \begin{pmatrix} e^{ik_+z} & 0 \\ 0 & e^{ik'_+z} \end{pmatrix} \begin{pmatrix} \bar{A}_{||}(0) \\ \bar{a}(0) \end{pmatrix}. \quad (27)$$

Since,

$$\begin{pmatrix} \bar{A}_{||}(z/0) \\ \bar{a}(z/0) \end{pmatrix} = O^T \begin{pmatrix} A_{||}(z/0) \\ a(z/0) \end{pmatrix} \quad (28)$$

it follows from there that,

$$\begin{pmatrix} A_{||}(z) \\ a(z) \end{pmatrix} = O \begin{pmatrix} e^{ik_+z} & 0 \\ 0 & e^{ik'_+z} \end{pmatrix} O^T \begin{pmatrix} A_{||}(0) \\ a(0) \end{pmatrix}. \quad (29)$$

Using Eq. (29) we arrive at the relation,

$$A_{||}(z) = [e^{ik_+z} \cos^2 \theta + e^{ik'_+z} \sin^2 \theta] A_{||}(0) + [e^{ik_+z} - e^{ik'_+z}] \cos \theta \sin \theta a(0) \quad (30)$$

$$a(z) = [e^{ik_+z} - e^{ik'_+z}] \cos \theta \sin \theta A_{||}(0) + [e^{ik_+z} \sin^2 \theta + e^{ik'_+z} \cos^2 \theta] a(0). \quad (31)$$

If the axion field is zero to begin with, i.e

$$a(0) = 0. \quad (32)$$

Then the solution for the gauge fields take the following form,

$$A_{||}(z) = [e^{ik_+z} \cos^2 \theta + e^{ik'_+z} \sin^2 \theta] A_{||}(0) \quad (33)$$

$$A_{\perp}(z) = e^{ik_{\perp}z} A_{\perp}(0). \quad (34)$$

The correlations of different components take the following form:

$$\langle A_{\parallel}^*(z) A_{\parallel}(z) \rangle = [\cos^4 \theta + \sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta \cos[(k_{\perp} - k'_{\perp})z]] \langle A_{\parallel}^*(0) A_{\parallel}(0) \rangle \quad (35)$$

$$\langle A_{\parallel}^*(z) A_{\perp}(z) \rangle = [\cos^2 \theta e^{i(k_{\perp} - k'_{\perp})z} + \sin^2 \theta e^{i(k_{\perp} - k'_{\perp})z}] \langle A_{\parallel}^*(0) A_{\perp}(0) \rangle \quad (36)$$

$$\langle A_{\perp}^*(z) A_{\perp}(z) \rangle = \langle A_{\perp}^*(0) A_{\perp}(0) \rangle. \quad (37)$$

4. *Stokes parameters.* Using the definitions of the Stokes parameters, in terms of the correlators:

$$I = \langle A_{\parallel}^*(z) A_{\parallel}(z) \rangle + \langle A_{\perp}^*(z) A_{\perp}(z) \rangle, \quad (38)$$

$$Q = \langle A_{\parallel}^*(z) A_{\parallel}(z) \rangle - \langle A_{\perp}^*(z) A_{\perp}(z) \rangle, \quad (39)$$

$$U = 2 \operatorname{Re} \langle A_{\parallel}^*(z) A_{\perp}(z) \rangle, \quad (40)$$

$$V = 2 \operatorname{Im} \langle A_{\parallel}^*(z) A_{\perp}(z) \rangle. \quad (41)$$

Using the relations for the corresponding correlators, the Stokes parameters turn out to be

$$\begin{aligned} I &= [\cos^4 \theta + \sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta \cos[(k_{\perp} - k'_{\perp})z]] \langle A_{\parallel}^*(0) A_{\parallel}(0) \rangle + \langle A_{\perp}^*(0) A_{\perp}(0) \rangle \\ Q &= [\cos^4 \theta + \sin^4 \theta + 2 \sin^2 \theta \cos^2 \theta \cos[(k_{\perp} - k'_{\perp})z]] \langle A_{\parallel}^*(0) A_{\parallel}(0) \rangle - \langle A_{\perp}^*(0) A_{\perp}(0) \rangle \\ U &= 2 \left([\cos^2 \theta \cos[(k_{\perp} - k'_{\perp})z]] + \sin^2 \theta \cos[(k_{\perp} - k'_{\perp})z] \right) \langle A_{\parallel}^*(0) A_{\perp}(0) \rangle \\ V &= 2 \left([\cos^2 \theta \sin[(k_{\perp} - k'_{\perp})z]] + \sin^2 \theta \sin[(k_{\perp} - k'_{\perp})z] \right) \langle A_{\parallel}^*(0) A_{\perp}(0) \rangle. \end{aligned} \quad (42)$$

The Stokes parameters are also expressed as such

$$I = I_p, \quad (43)$$

$$Q = I_p \cos 2\psi \cos 2\chi, \quad (44)$$

$$U = I_p \sin 2\psi \cos 2\chi, \quad (45)$$

$$V = I_p \sin 2\chi. \quad (46)$$

where χ , ψ are usual ellipticity parameter and the polarisation position angle. The degree of (linear /) polarisation is given by,

$$p = \frac{\sqrt{Q^2 + U^2 + V^2}}{I_p}, \quad p_{lin} = \frac{\sqrt{Q^2 + U^2}}{I_p} \quad (47)$$

and the linear polarisation angle is given by

$$\tan 2\psi = \frac{U}{Q}, \quad \tan 2\chi = \frac{V}{p_{lin}}. \quad (48)$$

It has been noted in [64], that in case, we make any coordinate transformation around the axis of photon propagation the two linear polarisation become mixed. Hence, we need to be careful, as our solution process entails a similarity transformation. To see this we define the density matrix

$$\rho(z) = \begin{pmatrix} \langle A_{\parallel}^*(z) A_{\parallel}(z) \rangle & \langle A_{\parallel}(z) A_{\perp}^*(z) \rangle \\ \langle A_{\parallel}^*(z) A_{\perp}(z) \rangle & \langle A_{\perp}^*(z) A_{\perp}(z) \rangle \end{pmatrix} = \frac{1}{2} \begin{pmatrix} I(z) + Q(z) & U(z) - iV(z) \\ U(z) + iV(z) & I(z) - Q(z) \end{pmatrix}, \quad (49)$$

if we rotate the density matrix by an amount α about an axis perpendicular the plane containing $A_{\parallel}(z)$, $A_{\perp}(z)$, the density matrix transforms as $\rho(z) \rightarrow \rho'(z)$ given such to be

$$\rho'(z) = \frac{1}{2} R(\alpha) \begin{pmatrix} I(z) + Q(z) & U(z) - iV(z) \\ U(z) + iV(z) & I(z) - Q(z) \end{pmatrix} R^{-1}(\alpha), \quad (50)$$

where,

$$R(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}. \quad (51)$$

Under such transformation the $I(z)$ and $V(z)$ remains unaltered. However, the $Q(z)$ and $U(z)$ starts mixing with each other by the following

$$\begin{pmatrix} Q'(z) \\ U'(z) \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix} \begin{pmatrix} Q(z) \\ U(z) \end{pmatrix}. \quad (52)$$

We conclude this section by mentioning that in such a case the ellipticity parameter remains unaltered but the polarisation position angle changes by 2α as given below

$$\tan(2\chi') = \tan(2\chi), \quad \tan(2\psi') = \tan(2\alpha + 2\psi). \quad (53)$$

5. Ellipticity parameter and polarization position angle. As a follow-up to the analytical expressions given in the previous section/s, we consider two special case of the Stokes parameter where either one of the two effects, namely, the mixing effect or, the vacuum birefringence effect would be absent. Thereafter we shall consider the general formula. In each case, we would like to obtain the value of the ellipticity angle χ after propagation a fixed distance z of light and determine it's frequency dependence. For all the three cases we shall assume the light to be completely plane polarised in the transverse direction, or U polarised. This is common observance in pulsar polarisation cases.

5.1. Case - I: Mixing only. Here we assume that the vacuum birefringence terms (i.e. ξ term inside the diagonal ones A_1 , A_2) are absent. We also assume a pseudoscalar mass which is much less than the plasma frequency here. This greatly simplifies calculation without being much deviant from the reality, if we

consider the parameters of the pulsar environment. Next we consider how the circular polarisation varies in this case. Assuming $\theta \ll 1$ one have

$$V = \left[\sin[(k_{\perp} - k_{+})z] + \left[\frac{g \mathfrak{B} \omega}{\omega_p^2 + m_a^2} \right]^2 \sin[(k_{\perp} - k'_{+})z] \right] \langle A_{\parallel}^*(0) A_{\perp}(0) \rangle. \quad (54)$$

Following the set of Eqs. (14)-(15) we can simplify the arguments of the remaining sinusoids of Eq. (54) as given below:

$$k_{\perp} - k_{+} = - \left\{ \frac{(g \mathfrak{B} \omega)^2}{2(\omega_p^2 + m_a^2)\omega} \right\}, \quad k_{\perp} - k'_{+} = + \left\{ \frac{(g \mathfrak{B} \omega)^2}{2(\omega_p^2 + m_a^2)\omega} + \frac{m_a^2}{2\omega} \right\}. \quad (55)$$

So, if $\xi = 0$, then the ellipticity parameter to its lowest order ($\propto \theta^2$) is found to be as follows, which matches well with [64,65], though the later most probably has a typo⁽¹⁾.

$$\chi \approx \frac{1}{96\omega} (g \mathfrak{B} m_a)^2 z^3. \quad (56)$$

Similarly, we may now turn our attention to two linear polarisation degrees of freedom, where the mixing angle $\theta \ll 1$, is small, to figure out the polarisation position angle.

$$\tan(2\psi) = \frac{U}{Q}. \quad (57)$$

However, in the beginning of this section we have already mentioned that $U \approx 1$. This is true for the parameters of interest used here and the observational cases to be discussed later. This makes the polarisation position angle inversely proportional to Q . But before we evaluate the expression for Q , we note that in the case of mixing the beam is assumed to propagate at an angle $\pi/4$ as compared to the magnetic field of the pulsar. Hence we need to change our expression for polarisation position angle accordingly. As discussed during derivation of Eq. (53), we have;

$$\tan\left(2\psi + \frac{\pi}{2}\right) = \frac{1}{Q}. \quad (58)$$

Next, we evaluate Q keeping in mind the approximations made before. Keeping terms up to order θ^2 in the expression for Q , we have,

$$Q = -2\theta^2 \left[\sin^2 \left(\frac{\langle k_{\perp} - k'_{+} \rangle z}{2} \right) \right]. \quad (59)$$

Again, following the set of Eqs. (14)-(15), we have

⁽¹⁾ It claimed concurrence with the former but is actually at variance, with it

$$\langle k_+ - k'_+ \rangle \simeq \frac{m_a^2}{2\omega}. \quad (60)$$

Substituting, one gets, in conjunction with [65]

$$\psi = \frac{1}{16} (g \mathfrak{B} z)^2. \quad (61)$$

However, unlike the circular polarisation, which was attributed to its entirety, to the mixing effect, one can not ascribe the entire pulsar linear polarisation [66] to this tiny mixing effect, where the mixing angle $\theta \ll 1$. So, we note that the pulsar radio emission is inherently linearly polarised to a large degree, due to curvature, synchrotron and superconducting self Compton effects thereof. We use $U \simeq 1$ and only the Q part is modelled via pseudoscalar photon mixing; where

$$Q = \frac{1}{8} (g \mathfrak{B} z)^2 \quad (62)$$

along with the definitive couple of Eq. (48) to note that the linear polarisation observed is equal to

$$p_{lin} = Q \sec\left(2\psi + \frac{\pi}{2}\right). \quad (63)$$

We note that the determination process of absolute pulsar polarisation [67] position

Table 2

REGRESSION RESULT FOR THE COUPLING OF THE PSEUDOSCALAR

Coefficients	Mean	Std.-Error	<i>F</i> -statistics	<i>t</i> -value	Pr(> <i>t</i>)
Slope	2.404e-25	4.567e-26	27.7196	5.265	1.21e-05

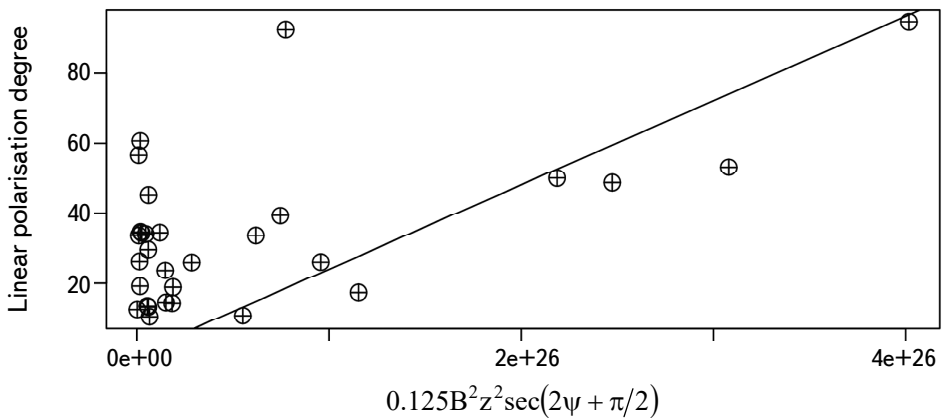


Fig.2. Linear regression between PPAs and Lin. Pol. Abscissa is in GeV^2 units and the ordinate is dimensionless.

Table 3

SMALL SAMPLE OF ABSOLUTE PULSAR POLARISATION
POSITION ANGLES, FROM [68]

Pulsar	PA_ν , deg	PA_0	Ψ , deg
B0011+47	+136(3)	43(7)	-87(8)
B0136+57	-131(0)	43(3)	6(3)
B0329+54	119(1)	20(4)	99(4)
B0355+54	48(1)	-41(4)	89(5)
B0450+55	108(0)	-23(16)	-94(16)
B0450-18	40(5)	47(3)	-7(6)
B0540+23	58(19)	-85(3)	-37(19)
B0628-28	294(2)	26(2)	88(3)
B0736-40	227(5)	-44(5)	91(7)

angles, is now experimentally feasible and the same values have already been scraped out for 30 odd pulsars. The literature contains a little less than fifty absolute PPAs from [68], out of which only 30 cross matched with that of our old set of 537 data, used to calculate the ellipticity parameter.

The expression for Q has only one unknown, the coupling of pseudoscalar with photons. Hence, we may do a regression analysis here, too, to estimate the same. The summary table is given in Table 2.

For the sake of brevity, we post a small segment of total 47 pulsar given in [68] in Table 3. The pulsar names here are catalogued in B1950 almanac standard, which were then converted to J2000 almanac standard and cross matched with the original and usable 537 strong population data on pulsar polarisation. 30 odd

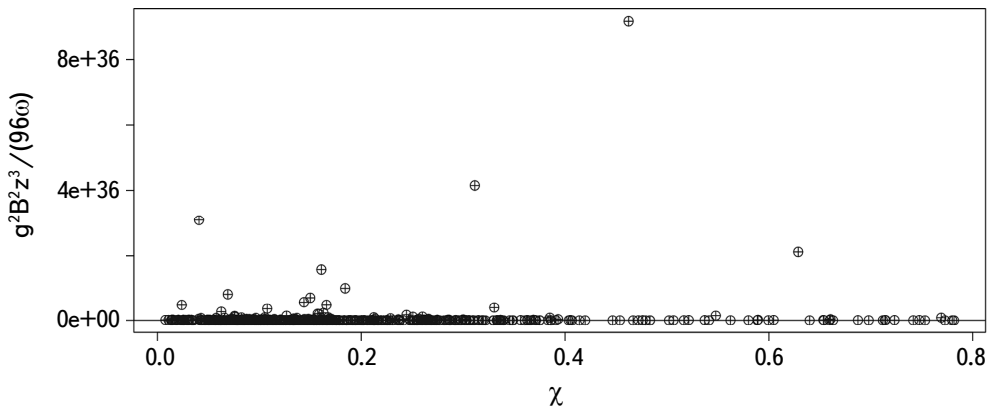


Fig.3. Linear regression between observed and (scaled) theoretical ellipticity parameter. Abscissa is dimensionless and the ordinate is in GeV^{-2} units.

Table 4

REGRESSION RESULT FOR THE MASS OF THE PSEUDOSCALAR

Coefficients	Mean	Std. error	F-statistics	t-value	Pr(> t)
Slope	7.471e-38	2.251e-38	11.0164	3.319	0.000964

samples of them were found to be common in both.

Now, we turn our attention back to the ellipticity parameter given in Eq. (56). Being a small angle, we have relegated the tangent as equivalent to its angular argument. The regression analysis thus to be undertaken is between circular and linear random variables, lying on LHS (ellipticity parameter) and RHS (magnetic field) respectively. Suffice it to say that the LHS is readily read off from the Table 1. The result of correlation study is given in the Table 4.

5.2. *Case - II: Vacuum birefringence only.* Here if we assume the mixing to be absent then we get $\theta = 0$ and hence we get the circular polarisation as

$$V = 2(\sin[(k_{\perp} - k_{+})z]) \langle A_{\parallel}^{*}(0) A_{\perp}(0) \rangle. \quad (64)$$

Here, we need to evaluate only one argument and it is the same as given above:

$$k_{\perp} - k_{+} = \frac{1}{2\omega} \{-3\xi\}. \quad (65)$$

We note, that, the mass of the pseudoscalar cancels and the mixing term is assumed zero. We see, that, the circular polarisation has now become inversely proportional to frequency assuming the argument to be considerably small as in the other case. Given that no circular polarisation would be produced in this case, it would be uninteresting to ponder over here.

5.3. *Case III: Limiting case.* We note the essential non-linearity resulting from the two effect taken in conjunction. Since we can not just add the two effects separately, even if they both are small and perturbative, to obtain the final result. We also note that the two effects shall be competing with each other when the following condition is met

$$\omega\xi \approx g\mathcal{B}.$$

Leaving aside the numerical prefactors - tentatively we see that, unless the value of magnetic field is much larger than normal pulsars (as in magnetars) and the beam frequency used in experiment is quite high (unlike the present case), even the modest values coming from astronomical bounds on pseudoscalar may not be comparable with the vacuum birefringence effect and the former is in fact larger in effect.

5.4. *Case IV: General case [69]*. Here in this subsection we calculate the amount of circular polarisation, vide the Stokes parameter V , without resorting to any of the approximations made in the preceding two subsections, for completeness. Here the expression for the V becomes

$$V = \left\{ \cos^2 \left[\frac{g \mathfrak{B} \omega}{7 \xi \omega^2 \sin^2 \alpha + \omega_p^2 + m_a^2} \right] \sin [(k_{\perp} - k_+)z] \right. \\ \left. + \sin^2 \left[\frac{g \mathfrak{B} \omega}{7 \xi \omega^2 \sin^2 \alpha + \omega_p^2 + m_a^2} \right] \sin [(k_{\perp} - k'_+)z] \right\} \langle A_{\parallel}^*(0) A_{\perp}(0) \rangle. \quad (66)$$

Hence at lowest order the first term, in $(\theta \ll 1)$ limit, would not change anything from what the first term, for the case of the pure mixing effect, did. But the second term, even at the lowest order shall render the expression for V qualitatively different from what it was then at pure mixing effect. Needless to say that pure vacuum birefringence effect does not match, even qualitatively, with any of them, either. For completeness, we write down the values of the wave vectors again.

$$k_{\perp} - k_+ = \frac{1}{2\omega} \left\{ \xi (4 \cos^2 \alpha - 7 \sin^2 \alpha) + \left[\frac{(g \mathfrak{B} \omega)^2}{7 \xi \omega^2 \sin^2 \alpha + m_a^2} \right] \right\} \\ k_{\perp} - k'_+ = \frac{1}{2\omega} \left\{ 4 \xi \cos^2 \alpha + m_a^2 - \left[\frac{(g \mathfrak{B} \omega)^2}{7 \xi \omega^2 \sin^2 \alpha + m_a^2} \right] \right\}. \quad (67)$$

Thus far we have only shown the difference of results of all three separate cases in terms of the V parameters depicting circular polarisation. This can be done with other two linear polarisation degrees of freedom, too. We leave this for a future endeavour.

6. *Result*. By the careful analysis of [55], containing 600 Pulsar polarisation data, of which 537 are used here and that of [68], containing 47 absolute PPAs for pulsars, 30 of which are common to the above, we came to the following result given in Table 5. We however note that more data samples on absolute PPAs are required to obtain a more statistically significant result on the coupling, which is deduced, from this parameter. Currently a little over fifty pulsars are

Table 5

THE RESULT OF THIS ANALYSIS

Results Obtained		
Parameters	Values	Significance level
$g_{\phi\gamma}$	$4.903 \cdot 10^{-13} \text{ GeV}^{-1}$	$\approx 0.001\%$
m_a	$2.733 \cdot 10^{-10} \text{ eV}$	$< 0.1\%$

amenable to this type of absolute PPA studies. The second quantity, namely the mass, has the numbers (>500) on its side. Nonetheless, its extraction from the ellipticity parameter, in turn, hinges on the coupling value, indirectly affecting the confidence interval found from the population. Also, for the sake of thoroughness we mention that the degree of linear polarisation is claimed to be dependent on frequencies in which they are observed [70]. The PPAs that are quoted in [68], are for various radio frequencies, e.g. 327 MHz, 691 MHz, 3.1 GHz etc., including that of 1.4 GHz, which corresponds to 21 cm. Since, there has been no connection to PPAs are made with frequency, to our knowledge to this date, we did not investigate this further.

7. Discussion and outlook. Taking advantage of new age of data explosion arising out from newer observational techniques and that of machine tools, we tried to estimate pseudoscalar particle mass and its coupling to photons. The results thus obtained do not match any standard axion models such as DFSZ or KSVZ etc. Hence these finding must be accommodated in the fold of axion like particles (ALPs) outside of the QCD realm. Surprisingly, our bottom up study, has automatically, led us to values, that are comparable and between the contemporary theories on cosmic axion background radiation (CAB), leading to soft X-ray excesses observed from Coma cluster [71] and that of the extra-galactic background light (EBL) to ALPs conversion and oscillation, leading to an observed anomalous γ ray transparency of the universe [72]. Fortunately, the previous constraints set on the mass and coupling of pseudoscalars, either by the changes in of quasar polarisation, hypothetically by ALPs [73], or by the γ ray burst SN1987A [74], occurring through a so called ALPs burst, are not in conflict with our results, either.

As mentioned in section 5 a future incorporation of vacuum birefringence effect into this study, may be performed, so as to see how the result on these estimates may change, for better or worse. These parameters may also be harnessed for devising CDM/WDM models and to obtain their relic densities.

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ИССЛЕДОВАНИЕ ПСЕВДОСКАЛЯРОВ С ПОМОЩЬЮ НАБОРОВ ДАННЫХ О ПОЛЯРИЗАЦИИ ПУЛЬСАРОВ

К.ЧАНДА, С.МАНДАЛ

Недавно был опубликован новый набор данных, содержащий информацию о линейной и круговой поляризации 600 пульсаров на радиоволне 21 см. Существующие модели радиоизлучения пульсаров, такие как синхротонное/автокомптоновое или искривленное излучение, не могут объяснить наблюдаемую круговую поляризацию, поскольку они предсказывают только линейную поляризацию. Чтобы решить эту проблему, мы предлагаем использовать (псевдо)скалярные частицы, взаимодействующие с фотонами в присутствии интенсивных магнитных полей пульсаров, для объяснения круговой поляризации. Это позволяет нам оценить псевдоскалярные параметры, такие как связь с фотонами и массу в виде их произведения. Чтобы получить эти значения по отдельности, мы воспользовались недавними наблюдениями 47 пульсаров, для которых определены абсолютные позиционные углы поляризации. Две трети этого нового набора наблюдений пересекается с обширным предыдущим набором данных о типе и степени поляризации. Это позволило определить оба псевдоскалярных параметра индивидуально.

Ключевые слова: *ALP- γ (14.80.Va)смещение:пульсар (97.60.Gb):поляризация (42.25.Ja)*

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