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This issue of Trans. Armenian NAS is dedicated to the 90-th anniversary of Academician V. A. AMBARTSUMIAN

ՀՀ ԳԱԱ «ԳԻՏՈՒԹՅՈՒՆ» ՀՐԱՏԱՐԱԿՉՈՒԹՅՈՒՆ

ԵՐԵՎԱՆ ЕРЕВАН

ԲՈՎԱՆԴԱԿՈՒԹՅՈՒՆ

СОДЕРЖАНИЕ

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This year 1998 is the year of 90-th anniversary of great Armenian scientist Academician V.A Ambartsumian, who more ten 50 years was leading the Armenian National Academy of Sciences and Byurakan Astrophysical Observatory.

This issue of Transactions of NAS RA allows readers to acquaint with modern situation in some directions of Science, connected with ideas of V.Ambartsumian.

The readers can be sure in profundity and diversity of ideas of world-famed scientist, which are accurately stated in his works.

Academician F.T.SARKISYAN
President of NAS RA

V.A.AMBARTSUMIAN (1908-1996)

Development of the Science in Armenia during the last 55 years inseparably linked with the name, scientific and organizational activities of Academician V.A.Ambartsumian.

Victor Amazasp Ambartsumian was born in 18 September 1908 in Tbilisi, in the family of Armenian intellectuals. In the early ages his unique mathematical abilities were displayed.

As a student of Leningrad State University he carried out a number of scientific works. Then he posed and solved the Sturm-Liuville Inverse problem, which became the beginning of new direction in Mathematics - the Theory of Inverse spectral problems for Differential Equations.

In 1928, after graduation from University, V.Ambartsumian entered the post-graduate course of Pulkovo Observatory. Under guidance of Academician A.A.Belopolsky he carried out important works in Physics of Stars and Nebules. During the Leningrad period of his sc. activity V.Ambartsumian suggested a quantitative theory of luminescence of Gas Nebulae. He laid the founds of Statistical Mechanics of stellar systems. Most important sc. discovery of V.Ambartsumian in this period is Principle of Invariance and its application in Astrophysical problems of Radiative Transfer. This fundamental physical principle and corresponding Ambartsumian Equation has large applications in various directions of Mathematics, Physics, Astrophysics, Geophysics etc.

V.Ambartsumian combined sc. activity in Leningrad University and Pulkovo Observatory with extensive pedagogical and organizational work. He became the founder of Soviet School of Theoretical Astrophysics. Many generations of astrophysicists studied the course of Theoretical Astrophysics by the textbooks of V.Ambartsumian. Many years he was the Pro-rector on Science of Leningrad University.

In 1939 V.Ambartsumian was elected the corresponding member of USSR Academy of Sciences, in 1953 he became the Academician. Many years he was the member of the Presidium of AS USSR and Russian AS.

After establishment of Armenian Academy of Sciences in 1943, V.Ambartsumian became its Vice-president. In 1947 he was elected the

President of Armenian AS and permanently was leading the Academy till 1993. Then he became the Honored President of NAS RA.

The beginning of Byurakan period of Ambartsumian's sc. activity was marked with the discovery of stellar systems of a new type - the Stellar Associations. These systems consist of the stars of early spectral classes, are unstable and quickly disintegrated. The discovery of Stellar Associations laid the basis for principal new Cosmogonical conception of Ambartsumian. The next discovery of V.Ambartsumian is his idea on activity of nuclei of Galaxies and their important role in evolution of Galaxies. At present the study of such activity represents one of central problems of Extragalactic Astronomy.

Subsequently V.Ambartsumian carried out fundamental investigations on Linear and Non-Linear problems of Radiative Transfer, on statistics of flare stars, on Theory of Baryon superdence stars.

During the activity of V.Ambartsumian as the President of Armenian AS our Republic was advanced in the world in some directions of science, largely due to the efforts of V.Ambartsumian.

Starting from 1965 Armenian AS is publishing All-Union sc. Journal "Astrophysics", which founder and General Editor was V.Ambartsumian. It's hard to exaggerate the role of the Journal in development of Astrophysics, in reassignment of the scientific prestige of Armenia.

V.A.Ambartsumian twice was awarded to the title Hero of Socialist Labor, became a National Hero of Armenia.

V.A.Ambartsumian was founded and lead a big scientific school. His followers actively continue the development of Ambartsumian's. sc. ideas and methods.

About 50 years he was leading The Dept of Astrophysics in Yerevan State University.

V.A.Ambartsumian was the President of International Astronomical Union, twice was elected as a President of International Council of Scientific Unions, was a member of many foreign Academies and Sc. Societies.

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ДОКЛАДЫ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

Том 98 1998 №3

MATHEMATICS

УДК 519.212

Academician of NAS RA **R. V. Ambartzumian**

Invariant Imbedding in Stochastic Geometry

(Submitted 19/ VI 1998)

INTRODUCTION

V. A. Ambartsumian has often pointed out ((1), Epilogue) that versions of the "invariance principle" he has used in the study of scattering of light may be effective in other mathematical problems as well.

R. Bellman and his followers (2) have developed and systematically applied similar ideas. Completely recognizing the priority of V. A. Ambartsumian, they coined the term of "Invariant Imbedding" to designate the corresponding mathematical approach, presumably to become useful in mathematical physics at large. Outside mathematical physics, an analytical procedure which can be attributed to invariant imbedding, has been applied in integral geometry in (3), where it helped to discover basic combinatorics governing the relation between measures in the space of lines and metrics.

The present article applies invariant imbedding in the related field of stochastic geometry. We derive differential equations for the probability distribution of the number of hits of a test segment by the lines of a random line process, valid under certain factorization assumptions. The imbedding parameters are the direction and the length of the test segment. The results are valid for random line processes that are translation invariant in distribution and possess first and second moment measures. There are also some smoothness assumptions.

Section 1 contains necessary prerequisites from the theory of translation invariant random line processes. The proofs of the properties listed in this section can be easily worked out within the standard framework of the "method of fixed realizations" as presented in (4).

In §2, invariant imbedding is applied towards derivation of differential relations involving Palm type probability distributions of the line process. In §3 we consider the marked point processes of hits induced on test lines by the lines of the line process. The marks are the angles at which the hits occur. We show that certain degree of independence between the point process of hits and the sequence of hit angles transforms these relations to differential equations of rather conventional nature. As stated by the concluding Theorem, under another additional assumption of "sufficient mixing" and absence of correlation between the cotangents of the angles, these equations can be resolved yielding Poisson distribution for random numbers of hits on test segments. Among earlier attempts to consider similar questions we mention (5) (this paper was the first to study the general random line processes), Chapter 10 in (4) and (6). They all used approaches different from the present one.

§1. TRANSLATION INVARIANT LINE PROCESSES

We consider random line processes in the Euclidean plane $\mathbb{R}^2$. A line process is defined to be (4) a random point process in the space of lines. Our notation will be g for a line in $\mathbb{R}^2$ and $\{g_i\}$ for a random line process. The latter notation stresses the fact that a line process is a countable random set of lines. Occasionally we use the letter $\mathbf{M}$ to denote the space of *realizations* of line processes, $\mathbf{m} \in \mathbf{M}$. By P we denote the probability distribution of $\{g_i\}$ (a probability measure on $\mathbf{M}$). We say that a line g "hits" a segment γ if $\gamma \cap g$ reduces to a point in the *relative interior* of γ.

Given a "test segment" γ, we will consider the event

$$\begin{pmatrix} \gamma \\ k \end{pmatrix} = \{\gamma \text{ is hit by exactly } k \text{ lines from } \{g_i\}\}.$$

Given two test segments γ_1 and γ_2 and two nonnegative integers k_1, k_2, we write $\begin{pmatrix} \gamma_1 & \gamma_2 \\ k_1 & k_2 \end{pmatrix}$ for the intersection of $\begin{pmatrix} \gamma_1 \\ k_1 \end{pmatrix}$ and $\begin{pmatrix} \gamma_2 \\ k_2 \end{pmatrix}$. This notation extends to any number of test segments. For the probabilities of the events we use notation like $P\begin{pmatrix} \gamma \\ k \end{pmatrix}$. In the definition that follows and elsewhere we write dg for the unique (up to constant factor) measure in the space of lines which is invariant with respect to Euclidean motions of $\mathbb{R}^2$.

Definition 1. A line process $\{g_i\}$ belongs to the class TICD2 if its probability distribution P is invariant with respect to the group of translations of the plane (Translation Invariant) and the first and second moment measures of $\{g_i\}$ are of the form $f_1 dg$ and (outside $g_1 = g_2$) $f_2 dg_1 dg_2$ respectively, with Continuous Densities f_1 and f_2.

This section contains a list of some properties of the processes from the class TICD2 that we will need for Invariant Imbedding in §2. Their proofs can be easily obtained by the method of "fixed realizations" presented in (4), and therefore are omitted.

Let γ be an arbitrary segment in the plane. In this and the next section γ will remain fixed. We construct the rectangle R shown on Fig. 1. by attaching the lateral sides v_1 and v_2 to γ, otherwise called ***vertical windows***. Two segments h_1 and h_2 attached to γ to make continuations of γ we call ***horizontal windows***. The length of the windows, vertical or horizontal, let be l, and we will assume that l *tends to zero*.

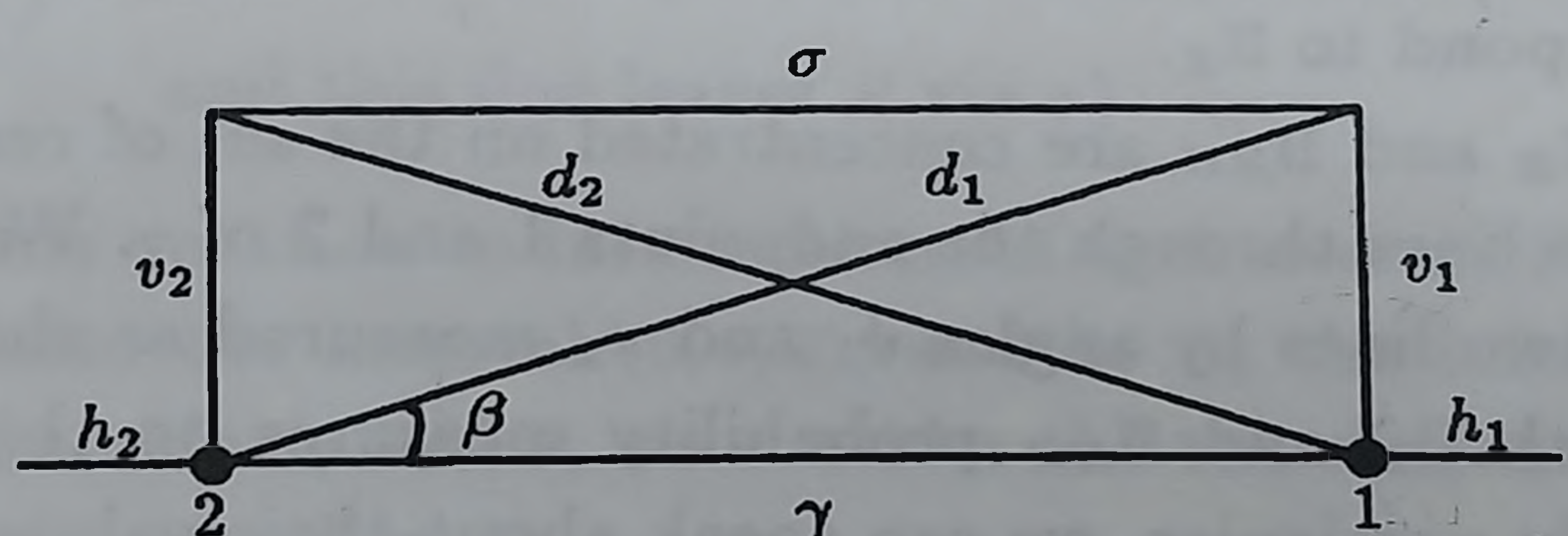

Fig. 1

Property P1 : for any window w, vertical or horizontal,

$$P\binom{w}{1} = O(l), \quad P\binom{w}{2} = O(l^2) \text{ and } P\binom{w}{k} = o(l^2) \text{ for } k > 2.$$

We will use special short notation

$$H = \binom{h_1}{1}, \quad \text{and } V = \binom{v_1}{1}.$$

The *intensities* of the process of intersection points induced by $\{g_i\}$ on a horizontal (λ_H) or vertical (λ_V) test lines are well defined :

$$\lambda_H = \lim_{l \to 0} l^{-1} P(H) \text{ and } \lambda_V = \lim_{l \to 0} l^{-1} P(V).$$

Property P2 : the following limits exist

$$\lim_{l\to 0} l^{-2}P(HH) = c_{HH}, \quad \lim_{l\to 0} l^{-2}P(A) = c_A \text{ and } \lim_{l\to 0} l^{-2}P(B) = c_B,$$

where $HH = \begin{pmatrix} h_1 & h_2 \\ 1 & 1 \end{pmatrix}$, while $A \subset \begin{pmatrix} v_1 & v_2 \\ 1 & 1 \end{pmatrix}$ is the subevent that occurs whenever v_1 and v_2 are intersected by the *same line* from $\{g_i\}$, while B is the relative complement of A within $\begin{pmatrix} v_1 & v_2 \\ 1 & 1 \end{pmatrix}$ i.e. B stands for the case where the intersections are caused by *two different lines* from $\{g_i\}$.

Property P3 :

$$\sum_{j_1+j_2\geq 3} P\begin{pmatrix} v_1 & v_2 \\ j_1 & j_2 \end{pmatrix} = o(l^2).$$

We will need the concepts of Palm type distributions Π_V, Π_H, Π_B and Π_{HH} of a process $\{g_i\}$ (for a rigorous geometrical theory of Palm distributions see ([4])).

Roughly, each of these Palm type distributions Π_Z is the limiting, as l tends to 0, conditional distributions of $\{g_i\}$, *conditional upon the event Z*. For each $Z \in \{H, V, B, HH\}$ we can speak about the line process that correspond to Π_Z.

Both Π_B and Π_{HH} are concentrated on the set of realizations that possess two lines through the endpoints 1 and 2 of γ. We parameterize the latter two lines by angles ψ_1 and ψ_2 measured as shown on Fig. 2, making both Π_B and Π_{HH} probability measures on the space $(0,\pi) \times (0,\pi) \times \mathrm{M}$. In particular, we can speak about their values on the events of the type $\begin{pmatrix} \gamma \\ k \end{pmatrix} \cap \{\Theta_1\} \cap \{\Theta_2\}$ where Θ_1, Θ_2 are two subintervals of $(0,\pi)$, $\{\Theta_i\}$ stands for the event $\psi_i \in \Theta_i$ *that occurs at the endpoint i*. Both probability distributions Π_H and Π_V are concentrated on the set of realizations that possess a line through the endpoint 1 of γ, i.e. both live on the $(0,\pi) \times \mathrm{M}$. In particular, the values of Π_H and Π_V on the events of the type $\begin{pmatrix} \gamma \\ k \end{pmatrix} \cap \{\Theta_1\}$ are well defined.

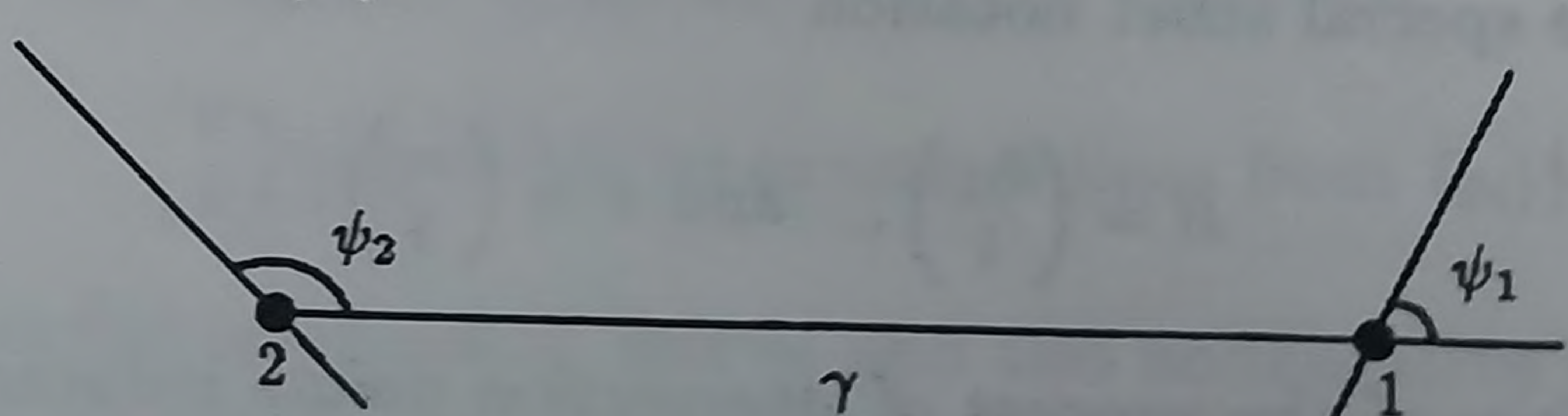

Fig. 2

We write E_Z for the *expectation* with respect to the probability measure Π_Z.

Property P4. Let $F(\Psi_1, \mathrm{m})$ *be a bounded function defined on* $(0,\pi) \times \mathrm{M}$. *If* $\{g_i\} \in TICD2$, *then for every choice of* γ

$$\lambda_H E_H [F(\psi_1, \mathrm{m}) \mid \cot \psi|] = \lambda_V E_V F(\psi_1, \mathrm{m}).$$

Property P5. Let $F(\Psi_1, \Psi_2, \mathrm{m})$ *be a bounded function defined on* $(0,\pi) \times (0,\pi) \times \mathrm{M}$. *If* $\{g_i\} \in TICD2$, *then for every choice of* γ

$$c_{HH} E_{HH} [F(\psi_1, \psi_2, \mathrm{m}) | \cot \psi_1 \cot \psi_2|] = c_B E_B F(\psi_1, \psi_2, \mathrm{m}).$$

Palm type probability of arbitrary event in M can not in general be calculated as a limit of the corresponding conditional probabilities of the same event. The following Proposition 4 illustrates this instability.

For a side u of the rectangle R, $u \neq v_1$, we define the event $\begin{pmatrix} v_1 \\ u \end{pmatrix} \subset \begin{pmatrix} v_1 \\ 1 \end{pmatrix}$ as

$$\begin{pmatrix} v_1 \\ u \end{pmatrix} = \{\text{there is exactly one line in } \{g_i\} \text{ that hits } v_1 \text{ and this line leaves } R \text{ via } u\}$$

and extend this notation to intersections of such events. From now on by S_1 we denote the interval $(0, \pi/2)$, and by S_2 the interval $(\pi/2, \pi)$. Along with the sides γ and σ, we consider the two diagonals d_1 and d_2 of the rectangle R on Fig. 1.

Property P6. If $\{g_i\} \in TICD2$, *then four limit relations of the form*

$$\lim_{l \to 0} l^{-2} P \begin{pmatrix} u & v_1 \\ k & u_1 \end{pmatrix} = c_V \Pi_V \left[\begin{pmatrix} \gamma \\ k - r \end{pmatrix} \cap \{S_i\} \right]$$

hold. The map

$$(u, u_1) \mapsto (r, i)$$

on which they depend is given by

$$(\gamma, \gamma) \mapsto (1,1) \qquad (d_1, \gamma) \mapsto (0,1) \qquad (\gamma, d_1) \mapsto (0,2) \qquad (d_1, d_1) \mapsto (1,2)$$

Property P7. If $\{g_i\} \in TICD2$, *then sixteen limit relations of the form*

$$\lim_{l \to 0} l^{-2} P \begin{pmatrix} u & v_1 & v_2 \\ k & u_1 & u_2 \end{pmatrix} = c_B \Pi_B \left[\begin{pmatrix} \gamma \\ k - r \end{pmatrix} \cap \{S_i\} \cap \{S_j\} \right]$$

hold. The map

$$(u, u_1, u_2) \mapsto (r, i, j)$$

on which they depend is given by the table

$(\gamma,\gamma,\gamma) \mapsto (2,1,2)$ $(\gamma,\gamma,\sigma) \mapsto (1,1,1)$ $(\gamma,\sigma,\gamma) \mapsto (1,2,2)$ $(\gamma,\sigma,\sigma) \mapsto (0,2,1)$

$(\sigma,\gamma,\gamma) \mapsto (0,1,2)$ $(\sigma,\gamma,\sigma) \mapsto (1,1,1)$ $(\sigma,\sigma,\gamma) \mapsto (1,2,2)$ $(\sigma,\sigma,\sigma) \mapsto (2,2,1)$

$(d_1,\gamma,\gamma) \mapsto (1,1,2)$ $(d_1,\gamma,\sigma) \mapsto (0,1,1)$ $(d_1,\sigma,\gamma) \mapsto (2,2,2)$ $(d_1,\sigma,\sigma) \mapsto (1,2,1)$

$(d_2,\gamma,\gamma) \mapsto (1,1,2)$ $(d_2,\gamma,\sigma) \mapsto (2,1,1)$ $(d_2,\sigma,\gamma) \mapsto (0,2,2)$ $(d_2,\sigma,\sigma) \mapsto (1,2,1)$

Limit conditioning by the event A has a special status. In fact, the method of fixed realizations yields ([4]) the existence of Palm distribution Π_A only for line processes that are invariant with respect to the group of Euclidean motions (translations *and* rotations). For $\{g_i\} \in TICD2$ a condition of existence of the limit

$$x_k = \lim_{l\to 0}[P(A)]^{-1}P(\binom{\chi}{k} \cap A) = c_A^{-1} \lim_{l\to 0} l^{-2}P(\binom{\chi}{k} \cap A). \tag{1}$$

is contained in Proposition 2 of the next section. In (1), the segment χ is defined as follows : whenever the event A occurs, $\{g_i\}$ contains a line which hits both v_1 and v_2, and we take χ to be the segment cut from that unique line by the vertical windows.

§2. INVARIANT IMBEDDING

We formulate the two propositions of the present section for the line processes from the class TICD2, although application of the invariant imbedding approach requires only the properties P 1-P7 listed in §1, rather than proper translation invariance of $\{g_i\}$. We use the notation

$\Delta x_k = x_k - x_{k-1}$ for the first and

$\Delta^2 y_k = y_k - 2y_{k-1} + y_{k-2}$ for the second difference with respect to k.

Proposition 1. If $\{g_i\} \in TICD2$, then the following limit exists

$$\begin{aligned}\lim_{l\to 0}(\lambda_V l)^{-1}\left[P\binom{d_1}{k} - P\binom{\gamma}{k}\right] &= \Pi_V\left(\binom{\gamma}{k-1} \cap \{S_2\}\right) + \\ + \Pi_V\left(\binom{\gamma}{k} \cap \{S_1\}\right) - \Pi_V\left(\binom{\gamma}{k} \cap \{S_2\}\right) &- \Pi_V\left(\binom{\gamma}{k-1} \cap \{S_1\}\right),\end{aligned} \tag{2}$$

where S_1 is the interval $(0, \pi/2)$, S_2 is the interval $(\pi/2, \pi)$.

The proof of this proposition, based on P1 and P4, we leave to the reader because it is a simplified ("first order") version of the proof of Proposition 2, which we give in complete detail.

Proposition 2. If $\{g_i\} \in TICD2$ and the limit

$$L_k(\gamma) = \lim_{l \to 0} l^{-2} \left[P\begin{pmatrix} d_1 \\ k \end{pmatrix} - P\begin{pmatrix} \gamma \\ k \end{pmatrix} - P\begin{pmatrix} \sigma \\ k \end{pmatrix} + P\begin{pmatrix} d_2 \\ k \end{pmatrix} \right]$$

exists, then the limits x_k in (1) exist and

$$L_k(\gamma) = -2c_A \, \Delta x_k + c_B \Delta^2 y_k, \tag{3}$$

where

$$\begin{aligned} y_k = \Pi_B \left(\begin{pmatrix} \gamma \\ k \end{pmatrix} \cap \{S_1\} \cap \{S_1\} \right) + \Pi_B \left(\begin{pmatrix} \gamma \\ k \end{pmatrix} \cap \{S_2\} \cap \{S_2\} \right) - \\ - \Pi_B \left(\begin{pmatrix} \gamma \\ k \end{pmatrix} \cap \{S_1\} \cap \{S_2\} \right) - \Pi_B \left(\begin{pmatrix} \gamma \\ k \end{pmatrix} \cap \{S_2\} \cap \{S_1\} \right) \end{aligned} \tag{4}$$

with S_1 and S_2 same as in Proposition 1.

Proof : For convenience in writing, we occasionally use the notation (see Fig. 1) $\gamma = \sigma_1$ and $\sigma = \sigma_2$. For each choice of τ from the collection $\{\sigma_1, \sigma_2, d_1, d_2\}$ we represent $\begin{pmatrix} \tau \\ k \end{pmatrix}$ as a union of mutually exclusive events

$$\begin{pmatrix} \tau \\ k \end{pmatrix} = \bigcup_{j_1, j_2 \geq 0} \begin{pmatrix} \tau & v_1 & v_2 \\ k & j_1 & j_2 \end{pmatrix}.$$

By P3, when $l \to 0$

$$\sum_{j_1 + j_2 \geq 3} P\begin{pmatrix} \tau & v_1 & v_2 \\ k & j_1 & j_2 \end{pmatrix} = o(l^2),$$

and therefore

$$P\begin{pmatrix} \tau \\ k \end{pmatrix} = \sum_{0 \leq j_1 + j_2 \leq 2} P\begin{pmatrix} \tau & v_1 & v_2 \\ k & j_1 & j_2 \end{pmatrix} + o(l^2). \tag{5}$$

A line which enters a triangle crossing one of its sides leaves the triangle crossing one of the remaining sides. Therefore we have the set identities

$$\begin{pmatrix} \sigma_1 & v_1 & v_2 \\ k & 0 & j_2 \end{pmatrix} = \begin{pmatrix} d_1 & v_1 & v_2 \\ k & 0 & j_2 \end{pmatrix}, \quad \begin{pmatrix} \sigma_1 & v_1 & v_2 \\ k & j_1 & 0 \end{pmatrix} = \begin{pmatrix} d_2 & v_1 & v_2 \\ k & j_1 & 0 \end{pmatrix}, \tag{6}$$

as well as similar identities for σ_2.

In the expression

$$D = P\begin{pmatrix} d_1 \\ k \end{pmatrix} - P\begin{pmatrix} \gamma \\ k \end{pmatrix} - P\begin{pmatrix} \sigma \\ k \end{pmatrix} + P\begin{pmatrix} d_2 \\ k \end{pmatrix}.$$

we replace the individual probabilities by their decompositions (5). Further, the probability of each event $\begin{pmatrix} \sigma_i & v_1 & v_2 \\ k & j_1 & j_2 \end{pmatrix}$, where either j_1 or

j_2 or both equal zero we replace according to (6) (or the analog of (6) for σ_2). In the resulting sum the probabilities of each event $\begin{pmatrix} d_i & v_1 & v_2 \\ k & j_1 & j_2 \end{pmatrix}$ where at least one of the indices j_1 or j_2 equals zero, will enter twice with opposite signs. So these terms cancel out and we get

$$D = -\sum_{i=1,2} P\begin{pmatrix} \sigma_i & v_1 & v_2 \\ k & 1 & 1 \end{pmatrix} + \sum_{i=1,2} P\begin{pmatrix} d_i & v_1 & v_2 \\ k & 1 & 1 \end{pmatrix} + o(l^2). \tag{7}$$

We have

$$\begin{pmatrix} \tau & v_1 & v_2 \\ k & 1 & 1 \end{pmatrix} = \bigcup_{(u_1,u_2)\in U} \begin{pmatrix} \tau & v_1 & v_2 \\ k & u_1 & u_2 \end{pmatrix}, \tag{8}$$

where $U = \{v_2, \gamma, \sigma\} \times \{v_1, \gamma, \sigma\}$, and the events under the union are mutually exclusive.

Therefore the probabilities in (7) can be replaced by sums of probabilities of the events according to (8). By P3, for pairs $(u_1, u_2) = (v_2, \gamma)$, (v_2, σ), (γ, v_1) and (σ, v_1) we have

$$P\begin{pmatrix} \gamma & v_1 & v_2 \\ k & u_1 & u_2 \end{pmatrix} = o(l^2),$$

and (7) takes the form

$$D = -\sum_{i=1,2} P\begin{pmatrix} \sigma_i & v_1 & v_2 \\ k & v_2 & v_1 \end{pmatrix} + \sum_{i=1,2} P\begin{pmatrix} d_i & v_1 & v_2 \\ k & v_2 & v_1 \end{pmatrix} +$$

$$+\sum_{i=1,2}\left[\sum_{(u_1,u_2)\in U_1}\left[-P\begin{pmatrix} \sigma_i & v_1 & v_2 \\ k & u_1 & u_2 \end{pmatrix} + P\begin{pmatrix} d_i & v_1 & v_2 \\ k & u_1 & u_2 \end{pmatrix}\right]\right] + o(l^2), \tag{9}$$

where in the last sum $U_1 = \{\gamma, \sigma\} \times \{\gamma, \sigma\}$. Note that $\begin{pmatrix} v_1 & v_2 \\ v_2 & v_1 \end{pmatrix}$ coincides with A as defined in P2.

We divide (9) by l^2 and let $l \to 0$. Using P7, one can check that

$$\lim_{l\to 0} l^{-2} \sum_{i=1,2}\left[\sum_{(u_1,u_2)\in U_1}\left[-P\begin{pmatrix} \sigma_i & v_1 & v_2 \\ k & u_1 & u_2 \end{pmatrix} + P\begin{pmatrix} d_i & v_1 & v_2 \\ k & u_1 & u_2 \end{pmatrix}\right]\right] = c_B \Delta^2 y_k.$$

By the definition of the segment χ

$$\begin{pmatrix} \sigma_i \\ k \end{pmatrix} \cap A = \begin{pmatrix} \chi \\ k \end{pmatrix} \cap A \quad \text{and} \quad \begin{pmatrix} d_i \\ k \end{pmatrix} \cap A = \begin{pmatrix} \chi \\ k-1 \end{pmatrix} \cap A.$$

The existence of the limits, $i = 1, 2$

$$\lim_{l\to 0} l^{-2} P\begin{pmatrix} \sigma_i & v_1 & v_2 \\ k & v_2 & v_1 \end{pmatrix} = c_A x_k \quad \text{and} \quad \lim_{l\to 0} l^{-2} P\begin{pmatrix} d_i & v_1 & v_2 \\ k & v_2 & v_1 \end{pmatrix} = c_A x_{k-1}$$

follows from the existence of the limits $L_k(t, \alpha)$ (first for x_0 because $x_{-1} = 0$ and then in succession, for all x_k). Proposition 7 is completely proved.

§3. FACTORIZATION AND SUFFICIENT MIXING CONDITIONS

By translation invariance, the probabilities $P\binom{\gamma}{k}$ depend solely on the length t and direction α of the segment γ. So we can use "functional" notation

$$P\binom{\gamma}{k} = p_k(t,\alpha)$$

and the left-hand side of (2) reduces to

$$\lim_{l\to 0} \frac{p_k(\sqrt{t^2+l^2},\alpha+\beta) - p_k(t,\alpha)}{l} = t^{-1}\frac{\partial p_k(t,\alpha)}{\partial\alpha},$$

while for (3) applying Taylor expansion we find

$$L_k(\gamma) = \lim_{l\to 0} \frac{p_k(\sqrt{t^2+l^2},\alpha+\beta) - 2p_k(t,\alpha) + p_k(\sqrt{t^2+l^2},\alpha-\beta)}{l^2} =$$

$$= t^{-1}\cdot\frac{\partial p_k(t,\alpha)}{\partial t} + t^{-2}\cdot\frac{\partial^2 p_k(t,\alpha)}{\partial\alpha^2}. \tag{10}$$

Validity of (10) is essentially the *smoothness assumption* to which we refer in the Theorem below. Turning to the right-hand sides of (2) and (3), we first transform them using P4 and P5. By a remarkable interplay of signs

$$\lambda_V \Pi_V\left(\binom{\gamma}{k}\cap\{S_1\}\right) - \lambda_V \Pi_V\left(\binom{\gamma}{k}\cap\{S_2\}\right) = \lambda_H E_H I\binom{\gamma}{k}\cot\psi_1, \tag{11}$$

where I stands for the indicator function of the corresponding event (dependence on $\mathbf{m}\in\mathbf{M}$ suppressed). Similarly, for the quantities y_k in (8) we have

$$y_k = E_{HH} I\binom{\gamma}{k}\cot\psi_1\cot\psi_2.$$

We are ready to consider the consequences of certain *factorization* assumptions F1,F2 and F3, expressed in terms of the probability distribution of random *marked point process* $\{P_i,\psi_i\}_g$ of intersections induced by $\{g_i\}$ on a test line g. In this notation, $P_i = g\cap g_i$, while the mark ψ_i is the angle at which the intersection at P_i occurs. We note in advance, that jointly, the three assumptions F1,F2 and F3 are essentially less restrictive than the *Cox independence* well known in stochastic geometry ([5]). We say, that $\{P_i,\psi_i\}_g$ has Cox independence property, if for test line g of any direction α, the sequence of angles $\{\psi_i\}$ is independent of the point process $\{P_i\}$, and $\{\psi_i\}$ is a sequence of independent angles. Doubly stochastic Poisson line process $\{g_i\}$

governed by random measure of the form $\xi \cdot f_1(\phi)dg$, where the factor ξ is random while $f_1(\phi)$ is nonrandom (Cox line processes), all have this property.

Assumption F1 : for any direction α, any t and k the random variables $\cot\psi_1$ and $I\binom{\gamma}{k}$ are uncorrelated, i.e.

$$E_H I\binom{\gamma}{k}\cot\psi_1 = \Pi_H\binom{\gamma}{k} E_H \cot\psi_1.$$

We have

$$\lambda(\alpha) = \int \sin\psi f_1(\phi)d\psi,$$

where f_1 is the density of the first moment measure of $\{g_i\}$ (because of translation invariance, f_1 depends solely on the direction ϕ of the line g), ψ is the angle between the directions ϕ and α. Therefore the probability density of random angle ψ_1 is $(\lambda(\alpha))^{-1}\sin\psi f_1(\phi)d\psi$. Hence

$$E_H \cot\psi_1 = (\lambda(\alpha))^{-1}\int \cos\psi f_1(\phi)d\psi = -(\lambda(\alpha))^{-1}\lambda'(\alpha),$$

with $\lambda'(\alpha)$ denoting the first derivative in α. By so called Palm formulae for point processes in one dimention, see [4]

$$\lambda(\alpha)\left[\Pi_H\binom{\gamma}{k-1} - \Pi_H\binom{\gamma}{k}\right] = \frac{\partial p_k(t,\alpha)}{\partial t}. \tag{12}$$

We come to the conclusion that under the assumption F1, the relation (2) transforms to the differential equation

$$\frac{\partial p_k(t,\alpha)}{\partial \alpha} = t\cdot(\lambda(\alpha))^{-1}\lambda'(\alpha)\frac{\partial p_k(t,\alpha)}{\partial t}. \tag{13}$$

The equation (13) can be easily solved by standard method of characteristics. Its general solution has the form

$$p_k(t,\alpha) = q_k(\lambda(\alpha)t), \tag{14}$$

where $q_k(\cdot)$ is some function of one argument.

Assumption F2 : for any direction α, any t and k the random variables $\cot\psi_1\ \cot\psi_2$ and $I\binom{\gamma}{k}$ are uncorrelated, i.e.

$$E_{HH} I\binom{\gamma}{k}\cot\psi_1\cot\psi_2 = \Pi_{HH}\binom{\gamma}{k} E_{HH}\cot\psi_1\cot\psi_2.$$

One can easily derive the second order analog of (12) :

$$c_{HH}\Delta^2\Pi_{HH}\binom{\gamma}{k} = \frac{\partial^2 p_k(t,\alpha)}{\partial t^2}.$$

We conclude that under assumption F2, (3) reduces to

$$t\cdot\frac{\partial p_k(t,\alpha)}{\partial t} + \frac{\partial^2 p_k(t,\alpha)}{\partial \alpha^2} = -2\,c_A t^2\,\Delta x_k + t^2 a(t,\alpha)\frac{\partial^2 p_k(t,\alpha)}{\partial t^2}, \tag{15}$$

where $a(t,\alpha) = E_{HH}\cot\psi_1\cot\psi_2$.

We observe ([4]) that $c_A(\gamma) = t^{-1} f_1(\alpha)$ and $2f_1(\alpha) = \lambda(\alpha) + \lambda''(\alpha)$. By a direct substitution of (14) into (15) we get the following corollary of F1 and F2 acting jointly :

$$(\lambda + \lambda'')\, q_k' + t[(\lambda')^2 - \lambda^2 \cdot a(t,\alpha)]\, q_k'' = -(\lambda + \lambda'')\, \Delta x_k. \tag{16}$$

Assumption F3 : for any direction α and any t

$$E_{HH} \cot\psi_1 \cot\psi_2 = E_H \cot\psi_1 E_H \cot\psi_2 = [\lambda'(\alpha)]^2[\lambda(\alpha)]^{-2}$$

Under F3 the equation (16) transforms to

$$q_k' = -\Delta x_k. \tag{17}$$

This infinite system of equations is easily solved if we make one more additional assumption that

$$x_k = P\binom{\gamma}{k} = p_k(t,\alpha) \tag{18}$$

meaning limiting independence of the events A and $\binom{\chi}{k}$. We call (18) the assumption of ***sufficient mixing***. Roughly, (18) means that the circumstance that χ is chosen to lie on one of the lines from the random collection $\{g_i\}$ can be ignored, as far as the distribution of the number of hits on that segment is considered. In the limit, as $l \to 0$, χ receives length t and direction α.

Under (18), the solution of (17) satisfying natural initial conditions $q_0(0) = 1$ and $q_k(0) = 0$ for $k > 0$ yields Poisson probabilities with unit parameter $q_k(t) = \frac{t^k}{k!} e^{-t}$. This result we formulate as a theorem.

THEOREM. Let $\{g_i\} \in TICD2$ possesses smooth hitting probabilities $p_k(t,\alpha)$. If for any direction α and length t, $\{g_i\}$ possesses the three factorization properties F1, F2 and F3, as well as the property of sufficient mixing, then $p_k(t,\alpha)$ are Poisson probabilities with parameter $\lambda(\alpha)t$, where $\lambda(\alpha)$ is the sin*-transform of the density of the first moment measure.*

We note in conclusion, that if the condition of sufficient mixing is removed, then the Theorem becomes invalid, as demonstrated by any Cox line processes $\{g_i\} \in TICD2$ for which the factor ξ is essentially random. For them the probabilities $p_k(t,\alpha)$ become mixtures of Poisson probabilities. The latter reduce to Poisson probabilities whenever ξ is nonrandom. But in that case the line process $\{g_i\}$ becomes Poisson. Clearly, for Poisson $\{g_i\}$ the sufficient mixing condition is satisfied.

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Հայաստանի ԳԱԱ ակադեմիկոս Ռ. Վ. ՀԱՄԲԱՐՁՈՒՄՅԱՆ

Հաստատագիր ներդրման եղանակը ստոխաստիկ երկրաչափության մեջ

Հոդվածում հաստատագրի ներդրման եղանակը կիրառվում է ստոխաստիկ երկրաչափության մի խնդրում: Հարթության վրա դիտարկվում են տեղաշարժների նկատմամբ հաստատագրի ուղիղների երկրորդ կարգի պատահական պրոցեսներ:

Ցույց է տրված, որ եթե բավարարված են ֆակտորիզացիայի և խառնելիության որոշ պայմաններ, ապա փորձնական հատվածը հարող պրոցեսին պատկանող ուղիղների քանակի բաշխումը միշտ Պուասոնյան է:

Академик НАН Армении **Р. В. АМБАРЦУМЯН**

Инвариантное вложение в стохастической геометрии

В статье принцип инвариантного вложения применяется в одной задаче стохастической геометрии. При некоторых предположениях факторизации выводятся дифференциальные уравнения для вероятностей, описывающих распределение числа пересечений тестового сегмента прямыми, принадлежащими трансляционно-инвариантному случайному процессу прямых на плоскости.

Показано, что при дополнительном условии т.н. "достаточного перемешивания" полученные уравнения допускают лишь пуассоновские решения.

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MATHEMATICAL PHYSICS

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On Ambartsumian Equation and its Applications

Submitted by Academician of NAS RA R.V.Ambartzumian 22 June 1998

V. Ambartsumian's Principle of Invariance (PI) ([1-7]) honoured of wide application in linear problems of Mathematical Physics and Mathematics of Non-Linear functional equations- Ambartsumian Eq. (AE) and its generalizations. This paper is a breaf review of some results in this direction, obtained by representatives of Byurakan school of Mathematical Physics.

OPERATOR FORM OF AE IN RADIATIVE TRANSFER ([7]).

Let in space is given the cartesian system of cordinates (x,y,z) and $\mathbf{n}$ is the ort of axies OZ. Let $\Omega=\{\omega\}$ be an unite sphere of directions with measure $d\omega$ (body angle). Lets $\omega\omega'$ be the scalar product of $\omega,\omega'\in\Omega$. Let $\Omega^+\subset\Omega$ is the half-space of positive directions of ω for which $\omega\cdot\mathbf{n}>0$ and $\prod(a,b)$ is the plane layer, bounded by planes z=a and z=b, where $-\infty\le a\le b\le+\infty$.

Consider the linear stationary Radiative Transfer (RT) problem in the homogeneous medium $\prod_r=\prod(0,r), r\le+\infty$. We 'll assume that the intensity of radiation J into a medium depends on z, direction ω and on set w of other characteristics of radiation field (energy or frequency, polarization degree and etc.): $J=J(z,\omega,w)$. Let W={w} be a phase space with measure dw.

We denote $J^{\pm}(z,\omega,w)=J(z,\pm\omega,w)$, $\omega\in\Omega^+$. Let the vector – functions $J^{\pm}(z)$ are follows:

$$J^{\pm}(z)(\omega,w)=J\pm(z,\omega,w).$$

The Integral – Differential RT equation for wide class of Linear RT problems in $\prod_r$, in vector – operator notations, permits the following form:

$$\pm\frac{dJ^{\pm}(z)}{dz}=-A(z)J^{\pm}(z)+L^{+}(z)J^{\pm}(z)+L^{-}(z)J^{\pm}(z) \tag{1}$$

Here A(z) is the operator of multiplication on function $a(z,w)(\omega\mathbf{n})^{-1}$, where $a(z,w)>0$ is the volume coefficients of absorption, $L^{\pm}$ are the integral operators, describing the redistribution of radiation by direction, energy and etc. at elementary act of scattering:

$$L^{\pm}f(\omega,w)=\int\limits_{\Omega xw}L^{\pm}(\omega,w,\omega',w')f(\omega',w')d\omega'dw'$$

The condition of absence of radiation fission is the following:

$$\iint\left[L^{+}(\omega,w,\omega',w')+L^{-}(\omega,w,\omega',w')\right]'dwd\omega=\lambda a(w)(\omega n)^{-1},\ \ \lambda\le 1 \quad (2)$$

The functions $L^{\pm}$ possesse the condition of symmetry:

$$(\omega\mathbf{n})L^{\pm}(\omega,w,\omega',w')=(\omega'\mathbf{n})L^{\pm}(\omega',w'\omega,w) \quad (3)$$

The following boundary conditions are joined to Eq.(1)

$$J^{+}(0)=J_0,\ \ J^{-}(r)=J_0 \quad (4)$$

At $r=+\infty$ (the case of homogeneous half-space) the boundary conditions for (1) are

$$J^{+}(0)=J_0,\ \ J^{-}(z)=o\left(e^{Az}\right),\ z\to+\infty \quad (5)$$

In particualar case of coherent isotropic scattering the operators A and $L^{\pm}$ are (where $\eta=\omega n$)

$$(Af)(\eta)=\frac{1}{\eta}f(\eta),\qquad \left(L^{\pm}f\right)(\eta)=\frac{\lambda}{2\eta}\int\limits_0^1 f(\zeta)d\zeta,\quad 0\le\lambda\le 1 \quad (6)$$

PI (in operator form) for the problem (1),(5) means that exists integral operator $\rho=\text{const}$, such that

$$J^{-}(z)=\rho J^{+}(z),\ \ z\ge 0. \quad (7)$$

From (1) and (7) AE for ρ (in general operator form) is obtained

$$A\rho+\rho A=L^{-}+\rho L^{+}+L^{+}\rho+\rho L^{-}\rho \quad (8)$$

If ρ is known, then J^{+} is determined by Cauchy problem

$$\frac{dJ^{+}(z)}{dz}=-GJ^{+},\ \ J^{+}(0)=J_0,\qquad G=A-L^{+}-L^{-}\rho \quad (9)$$

J^{-} is determined by (7).

Solution of problem (9) has the form

$$J^{+}(z)=X(z)J_0 \quad (10)$$

where $X(z)=e^{-Gz}$ is the semigroup of operators determining by Cauchy problem

$$\frac{dX}{dz}=-GX,\ \ X(0)=I \quad (11)$$

I- is the identity operator.
From (8) one can pass to the functional form of AE in regard to kernel ρ of integral operator ρ. For problem (6), (8) we obtain

$$\left(\frac{1}{\eta}+\frac{1}{\zeta}\right)\rho(\eta,\zeta)=\frac{\lambda}{2\eta}\left[1+\int_0^1\rho(\eta',\zeta)d\eta'\right]\left[1+\eta\int_0^1\rho(\eta,\eta')\frac{d\eta'}{\eta'}\right] \tag{12}$$

Denote

$$\varphi(\zeta)=1+\int_0^1\rho(\eta',\zeta)d\eta' \tag{13}$$

From (12) one can obtain the Ambartsumian φ - Equation

$$\varphi(\eta)=1+\frac{\lambda}{2}\eta\varphi(\eta)\int_0^1\frac{\varphi(\zeta)d\zeta}{\eta+\zeta} \tag{14}$$

The Eq. (12) and (14) were obtained in ([1,2]) by means of application of PI to the problem of diffuse reflection from homogeneous half space. The functional form of AE for case of coherent anisotropic scattering was obtained in ([2,3]).

The physical solution of AE

The solution of AE (14) is nonunique. In ([8]) it was shown, that it is possible to construct "physical solution" (PS) of (14). Let consider the following iterations for (8):

$$A\rho_{m+1}+\rho_{m+1}A=L^-+\rho_m L^+ + L^+\rho_m+\rho_m L^-\rho_m, \qquad \rho_0=0, \quad m=0,1,2,..... \tag{15}$$

Since A is an operator of multiplication on positive function, then from (15) are subsequently determined the kernels ρ_m of integral operators ρ_m. The sequence ρ_m strongly converges in corresponding functional spaces, to the integral operator ρ. This limit ρ is called the Basic solution (BS) of AE (8). Its kernel ρ possesses the properties

i. ρ is the minimal positive solution of AE .

ii. ρ satisfies to inequality

$$\int_{\Omega x W}\rho(\omega,w,\omega',w')d\omega dw\le\lambda\le1 \tag{16}$$

which is in accordance with physical meaning of ρ.

iii. ρ satisfies to the principle of mutuality

$$(\omega n)\rho(\omega,w,\omega',w')=(\omega' n)\rho(\omega',w',\omega,w) \tag{17}$$

iv. ρ - is the strong limit at $r\to\infty$ of reflection operator $R(r)$ from homogeneous layer Π_r.

v. ρ - is the limit $(\text{at } t \to +\infty)$ of the operator of reflection for corresponding nonstationary problem

vi. By means of ρ the solution of problem (1), (4), or (1), (5) is constructed.

ρ is the unique solution of AE (8), possessing all above enumerated properties and therefore is the physical solution of (8).

Miln's problem of anisotropic scattering

One of classical problems of RT is the conservative Miln's problem in anisotropic coherent scattering.

This problem is the following: to construct a positive solution of conservative RT equation in homogeneous half-space at absence initial sources of energy in finite part of medium.

The Integral-Differential RT Equation has a form.

$$\frac{\partial J(\tau,\omega)}{\partial \tau} + J(\tau,\omega) = \int_{\Omega} g(\omega,\omega')J(\tau,\omega')d\omega' \tag{18}$$

with boundary conditions

$$J^{+}(0,\omega) = 0; J^{-}(\tau,\omega) = o\left(\exp\left\{\tau(\omega n)^{-1}\right\}\right) \text{ at } \tau \to +\infty \tag{19}$$

Here g - is the indicatrix of scattering:

$$g \geq 0; \quad \int_{-1}^{1} g(\mu)d\mu = 1$$

In work ([9]) the theory of Eq. (18) and Miln's problem is developed, assuming that the function g is integrable with quadrat $\left(g \in L_2(-1;1)\right)$. This assumption allows to apply the methods of Hilbert spaces. In ([10,11]) for the first time Miln's problem in natarally assumption ($g \in L_1(-1,1)$ was studied and solved.

The solution of problem it became possible in virtue of application of AE and a number of mathematical constructions.It was shown, that together with properties (16), (17), for reflection function ρ the equality takes place

$$\int_{\Omega} \rho(\omega,\omega')d\omega = 1$$

It is essential for construction of solution of Miln's problem.

The formal procedure of construction of solution of Miln's problem it was early partially known, and consist from the following stages.

i. The function ρ is constructed.

ii. The function f is determined by formulae $f(\omega) = C\left[\omega n + \int_{\Omega} \rho(\omega,\omega')(\omega' n)d\omega'\right]; \; C = \text{const}$ (20)

iii. J^{+} - is determined from Cauchy problem.

$$\frac{dJ^{+}}{d\tau} = L^{-} f + \left[-A + L^{+} + L^{-}\rho\right]J^{+}; J^{+}(0) = 0 \tag{21}$$

iv. J^- - is determined by formulae

$$J^- = f + \rho J^+$$

Solvability, asymptotic behaviour and other properties of solution are established in [10].

On method of Spheric Harmonics

One of general methods of solution anisotropic scattering is the method of spheric harmonic, ([3,5,6]), which is based on replacement of indicatrix g by finite linear combinatoon of Legendre polinomials P_k

$$g(\mu) \approx g_m(\mu) = \sum_{k=0}^{n} C_k P_k(\mu)$$

Then Eq.(18) (and more general equations of anisotropic scattering) on separate equaitions in regard to asimuthal harmonic are reduced.

In the capacity of g_m it is usualy taken the partial sum of expansion of g in series of Fourier by Legendre polinomials. It provides best root square proximity g_m to g for fixed m. It turn out to be, hewever, that proximity between solutions of initial and reduced equations is provided by proximity g_m to g in metric L_1, i.e. in sense

$$\delta_n = \int_{-1}^{1} \left| g(\mu) - g_m(\mu) \right| d\mu \quad \text{(for fixed n)}$$

It is also desirable provision of inequality $0 \le g_m \le g$.
Such approximation problem it is still possible to solve by numerical methods.

The Layer of finite thickness

It's well known that RT problems in layer of finite thickness are essentially complicated comparing with RT in homogeneous half-space. In ([4,5]) basing on PI was suggested method of singular Eq. for layer of finite thickness (φ and ψ functions of Ambartsumian or X,Y functions of Chandreseckhar). However this method was not obtain wide applications.

Riccati equation method, reducing the boundary-value Transfer problem to Cauchy problems doesn't take into account specification of homogeneity of medium.

In ([7]) the new variant of PI was developed. By this way for the first time the solution method for RT problem in homogeneous layer Π_r, $r < +\infty$ by means of Ambartsumian function ρ for Π_∞ was found.

Let $R = R(r)$ and $T = T(r)$ are operators of reflection and transmission for Π_r. Let $U = \rho \mathbf{X}(r)$, where the semigroup $\mathbf{X}(r)$ is determined according to (11). The following relations take place.:

$$\rho = R(r) + TU(r), \qquad \mathbf{X}(r) = \mathbf{T}(r) + R(r)U(r) \tag{22}$$

The linear system (22) in regard to R and T is easy solved. Then the estimations $\|\rho\| \le \lambda$ and $\|X(r)\| \le 1$ following from (16), the important role are played.

The mathematical substantiation and development of this method are contained in ([11,12,13]).

The Inverse problems

The operator approach and other analytical constructions, describing on previous sections, were allowed to solve a number of Inverse problems on determination of local act of interaction of radiation with medium by known reflection and transmission properties of all layer ([15,14,13]). Such problems are of interest in Atmosphere optics, Neutron physics, Messbauer spectroscopy, and etc. Below we'll consider only two Inverse problems.

Inverse problem 1. (Division of layer into half).

Let's the operators R(r) and T(r) for homogeneous layer Π_r are given. It is required to construct $R\left(\frac{r}{2}\right)$ and $T\left(\frac{r}{2}\right)$. From (22) we have

$$U(r) = \left[R(r) + T(r)U(r)\right]\left[T(r) + R(r)\right]U(r) \tag{23}$$

The Eq. (23) is an nonlinear Eq. in regard to W(r). It's physical solution is the limit of simple iterations W_n, $W_0 = 0$. It follows from $\|R + T\| \le 1$ (no Fission). We have $\|U\| \le 1$.

Solution of problem 1 consists of following stages.

i. The physical solution of (8) is constructed.

ii. By means of formulae (22) the operators ρ and $X(r)$ are determined

iii. The positive quadric root from $X(r)$ is extracted $X\left(\frac{r}{2}\right) = \left[X(r)\right]^{1/2}$. Then $X\left(\frac{r}{2}\right) = \rho X\left(\frac{r}{2}\right)$ is determined.

iv. The linear system in ragerd to $R\left(\frac{r}{2}\right)$ and $T\left(\frac{r}{2}\right)$ is solved. This system is obtained from (22) by means of replacement r by $\frac{r}{2}$.

Inverse problem 2. By means of known operators $R(r)$ and $T(r)$ how to determine the operators A and $L^{\pm}$.

The solution of this problem is based on relations

$$A - L^{+} = \lim_{\tau\to 0}\frac{1}{\tau}\left[I - T(\tau)\right],\ L^{-} = \lim_{\tau\to 0}\frac{1}{\tau}R(\tau). \tag{24}$$

By using the approach, described above, one can construct the operators $X\left(2^{-k}r\right)$ $k \ge 1$, $R\left(2^{-k}r\right)$ and $T\left(2^{-k}r\right)$. If we take k enough greater then from (24) may be approximately determined $A - L^{+}$ and L^{-}.

The operators A and $L^{\pm}$ may be immideately determined in the following two cases.

i. The symmetric case $L^{+} = L^{-} = L$.

ii. The case, where the operator A is known. (for example $a(\omega, w) = 1$).

The case i. includes in particularly the problems of isotropic scattering and anisotropic coherent scattering where the indicatrix of scattering is a even function.

In general case the full solution of problem is based on the relation (2).

Wiener - Hopf Eguation

PI and Ambartsumian Eq. have important applications in theory and effective solution of various classes of Integral equations. They are in organic connection with volterrian factorization of Integral operators. More complete results in this direction are obtained in the case of Wiener-Hop Eq. (WHE (16-22)) and some more general convolution Eq.(22-24). Let consider WHE

$$f(x) = g(x) + \int_0^\infty K(x-t) f(t) dt \tag{25}$$

or in operator form

$$(\mathrm{I} - \mathbf{K})\mathrm{f} = \mathrm{g}, \quad (\mathbf{K}f)(x) = \int_0^\infty K(x-t) f(t) dt$$

We assume that the kernel K is an even function and represents in the form

$$K(x) = \int_a^b e^{-|x|s} G(s) ds \equiv \int_a^b e^{-|x|s} d\sigma(s), 0 \le a < b \le +\infty \tag{26}$$

where

$$G(s) \ge 0; \ \sigma(s) = \int_0^s G(p) dp; \ \lambda = 2\int_a^b \frac{G(s)ds}{s} \le 1$$

The considered Equation has an important applications in RT, Kinetic theory of Gases and etc.

Consider the factorization

$$\mathrm{I} - \mathbf{K} = (\mathrm{I} - \mathbf{V}_-)(\mathrm{I} - \mathbf{V}_+) \tag{27}$$

where $\mathbf{V}_\pm$ are volterrian operators to be found

$$(\mathbf{V}_+ f)(x) = \int_0^x V(x-t) f(t) dt \qquad (\mathbf{V}_- f)(x) = \int_x^\infty V(t-x) f(t) dt \tag{28}$$

Let φ -is a solution of AE

$$\varphi(s) = 1 + \varphi(s) \int_a^b \frac{\varphi(p) G(p) dp}{s+p} \tag{29}$$

satisfying the condition

$$\mu \equiv \int_a^b \frac{1}{s} |\varphi(s)| G(s) ds \le +\infty$$

Then the factorization (27) holds, where

$$V(x)=\int_a^b e^{-xs}\varphi(s)G(s)ds.$$

The factorization interpretation of AE is very simple and fruitfull.
AE (29) possesses Basic solution (BS) φ, which is the limit of simple iterations. It possesses the following properties.

$$\varphi(s)\downarrow s;\ \varphi\in C[0,+\infty);\ \varphi(+\infty)=1;\ \varphi(0)=(1-\lambda)^{-1/2}.$$

In conservative case $\lambda=1$, we have $\varphi(0)=+\infty$. Let α_k be moments of function φ

$$\alpha_m=m!\int_a^b\frac{\varphi(s)G(s)ds}{s^{m+1}}ds\ (\le+\infty)$$

Then $\alpha_o=1;\ \alpha_1=\sqrt{\int_a^b\frac{G(s)ds}{s^3}}\le+\infty$

It $\alpha_1<+\infty$, then

$$\varphi(s)=\frac{1}{\alpha_1 s}+\frac{\alpha_2}{2\alpha_1^2}+\varphi_1(s),$$

where $\varphi_1(0^+)=0$.
The factorization (27) reduced WHE (25) to the solution of two volterrian-type Eq.

$$F(x)=g(x)+\int_x^\infty V(t-x)F(t)dt,\ f(x)=F(x)+\int_0^x V(x-t)f(t)dt$$

The resolvent function Φ is determined by Renewal Eq.

$$\Phi(x)=V(x)+\int_0^x V(x-t)\Phi(t)dt$$

and allows the representation ([25])

$$\Phi(x)=\int_\alpha^\beta e^{-xp}d\rho(p);\ \ \rho(p)\uparrow p$$

The method of approximate solution of Eq. (25), (26) based on constructions, described above, will be discussed bellow.

The AE for non-symantic kernel Kand system of WHE were studied in ([16-21]).

Using Ambartsumyan function in works ([26-28]) the effective solution method on convolution type integral equation of finite interval with kernel (26) is suggested.

Convolution-type equation with direct and inverse shifts.

Consider the following Equation

$$f(x)=g(x)+\int_0^\infty K(x-t)f(t)dt+\int_0^\infty K_o(x+t)f(t)dt \tag{30}$$

where k is given by (26) and

$$K_o(x)=\int_{a_0}^{b_0} e^{-xs}G_o(s)ds,\ g(x)=\int_{a_1}^{b_1} e^{-xs}G_1(s)ds.$$

Such equation arises in RT and in Kinetic Theory of Gases, (KTG), when we take into account reflection of rediation (or gas) from boundary medium. In virtue of factorization method of ([23]) and factorization approach of AE, it became possible to receive exact analytical results on Eq. (30).

Rewrite (30) in operator form $(\mathrm{I}-\mathbf{K}-\mathbf{K}_o)\mathrm{f}=\mathrm{g}$.

Let φ -be the Basic Solution of (29). Then the following factorization holds.

$$\mathrm{I}-\mathbf{K}-\mathbf{K}_o=(\mathrm{I}-\mathbf{V}_-)(\mathrm{I}-\mathbf{T})(\mathrm{I}-\mathbf{V}_+) \tag{31}$$

Here $\mathbf{V}_\pm$ are given by (28) and **T** is the following operator

$$(\mathbf{T}\mathrm{f})(\mathrm{x})=\int_{a_0}^{b_0}\mathrm{T}(\mathrm{x}+\mathrm{t})\mathrm{f}(\mathrm{t})\mathrm{dt},\ \ \mathrm{T}(\mathrm{x})=\int_{a_0}^{b_0}e^{-xs}\varphi^2(s)G_0(s)ds$$

As application of factorization (31) let's consider an important problem on construction of possitive solution S of conservative $(\lambda=1)$ homogeneous equation (30). In particulary, when $\mathrm{K}_o=\varepsilon\mathrm{K}$, at $0\le\varepsilon\le1$, the factorization (31) allows to reduce this problem to the solution of three coupled equations:

$$(\mathrm{I}-\mathbf{V}_-)\mathrm{F}=0,\ \ (\mathrm{I}-\mathbf{T})\mathrm{H}=\mathrm{F},\ \ (\mathrm{I}-\mathbf{V}_+)\mathrm{S}=\mathrm{H} \tag{32}$$

The first equation possesses non-trivial solution $\mathrm{F}(\mathrm{x})=\mathrm{C}=\mathrm{const}$. The second Eq. (where $\mathrm{F}(\mathrm{x})=\mathrm{C}$) possesses unique bounded solution $\mathrm{H}(\mathrm{x})=\mathrm{ch}(\mathrm{x})$. The solution S of last Eq. (32) possesses asymptotic $\mathrm{S}(\mathrm{x})\sim a\mathrm{x}$ at $\mathrm{x}\to+\infty$.
The following formulae is of interest in KTG

$$\mathrm{S}(\mathrm{x})-a\mathrm{x}\sim\frac{a\alpha_2}{\alpha_1}+\frac{1}{\alpha_1}\int_0^\infty \mathrm{H}(\mathrm{x})\mathrm{dx}$$

On an approximate solution of the problems (25) and (30).

The described approach and some other numerical method are suggested in works ([29-31]). The principal methods of approximate analytical and numerical solution of Eq. (25) and (30) are based on the modification of method of discrete ordinates (MDO) of Chandrasekhar ([5]). The application of MDO is equivalent to the replacement of kernel K in (26) by finite linear combination of exponentials.

$$K(x)\approx T(x)=\sum c_m e^{-|x|s_m}=\int_a^b e^{-|x|s}\,d\tilde{\sigma}(s)$$

where $a < s_1, < s_2 < \cdots < s_n < b \le +\infty$, and $\tilde{\sigma}$ is the step by step constant function. The proximity of solutions of Eq. (25) and reducing Equation (with kernel T) are estimated by means of quantity (see ([29]))

$$\delta=\|K-T\|_{L_1(-\infty,+\infty)}$$

In conservative case it is essentially to provide carring out of condition $T \le K$.

The problem of optimal selection of nodes $\{s_k\}$ and weight factors $\{c_m\}$, which are minimize the quantity δ for fixed n, is suggested in ([29]).

In virtue of MDO, AE is reduced to the finite nonlinear algebraic systems. It may be solved by iteration.

$$\varphi_i=1+\varphi_i\sum_{k=1}^{n}\frac{\varphi_k c_k}{s_i+s_k};i=1,2,\cdots n,\varphi_i=\varphi(s_i)$$

The function Φ has a form

$$\Phi(x)\approx\tilde{\Phi}(x)=\sum q_m e^{-p_m x}$$

where p_m - are determined from the characteristic equation $\sum_{k=1}^{n}\frac{c_k\varphi_k}{s_k-p}=1$. They are arrange according to $0 < p_1 < s_1 < p_2 < s_2 < \cdots < p_n < s_n$. The numbers $q_m > 0$ are determined from linear algebraic system with Cauchy matrix:

$$\sum_{k=1}^{n}\frac{q_k}{s_j-p_k}=1,\quad j=1,2,\cdots n$$

Let's consider the equation (25), in case, when $g(x)=\int_{a_0}^{b_0} e^{-xs}G_o(s)\approx\sum_{m=1}^{n} b_m e^{-xs_m}$.

Then the approximate solution of initial equation (25) is expressed by numbers (φ_k), (p_k), (q_k), (b_m)

$$f(x)=\sum_{m=1}^{n} b_m\varphi_m e^{-xs_m}+\sum_{k=1}^{n}\sum_{m=1}^{n} b_m q_k\varphi_m\frac{e^{-xp_k}e^{-xs_m}}{s_m-p_k}.$$

Byurakan Astrophysical Observatory NAS RA

Ն.Բ. Ենգիբարյան, Ա.Խ. Խաչատրյան, Մ.Հ. Մուրադյան

Համբարձումյանի հավասարումը և նրա կիրառությունները

Աշխատանքը հանդիսանում է Վ.Համբարձումյանի հավասարման (ՀՀ), նրա ընդհանրացումների և կիրառությունների ուղղությամբ Մաթեմատիկական Ֆիզիկայի Բյուրականյան դպրոցի ներկայացուցիչների կատարած որոշ աշխատանքների ակնարկ: Բերված են ՀՀ-ի օպերատորային և սկալյար տեսքերը, նրա ֆիզիկական լուծման կառուցման անալիտիկ և թվային մեթոդներ: Շարադրված են համասեռ կիսատարածությունում և վերջավոր հաստության շերտում ճառագայթման տեղափոխման և Գազերի կինետիկ տեսության որոշ ուղիղ և հակադարձ խնդիրների լուծման մեթոդներ, որոնք հիմնված են Համբարձումյանի հավասարման կիրառման վրա:

Н.Б. Енгибарян, А.Х. Хачатрян, М.Г. Мурадян

Уравнение Амбарцумяна и его применения

Работа является обзором некоторых работ представителей Бюраканской школы Математической физики по уравнвнию В. Амбарцумяна (УА), его обобщениям и приложениям. Приведены операторная и скалярная форма УА, аналитические и численные методы построения Физичекого решения УА. Изложены методы решения ряда прямых и обратных задач Теории переноса излучения и Кинетической теории газов в однородном полупространстве и в плоском слое конечной толщины, с применением УА.

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ДОКЛАДЫ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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PHYSICS

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Physics of Neutron Stars. (Short review)

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1.In the thirties,after the discovery of neutron by Chadwick(1932), in the works of Baade and Zwicki (1934) Oppenheimer and Volkoff(1939) the conception about neutron stars was formed: about the superdense celestial bodies, composed of degenerate ideal neutron gas. However, it was soon established that neutron is a nonstable particle, and it meant that the existence of celestial bodies, composed of neutrons only, is not possible. Thus, certainly, more valid models of superdense stars could be constructed, composed of mixtures of neutrons, protons and electrons degenerate gases; but this was not done. Later, for two decades the conception of neutron stars was ignored

The idea of superdense stars, composed of degenerate matter was again revived in the beginning of the sixties, in a completely new understanding and on a scientifically based level in the joint works of Hambartsumian and Sahakian. Those works were based on the achievements of physics of elementary particles made in the previous years.

In the white dwarfs especially in the stars of large density neutron stars) the temperature is visibly lower than the degeneration temperature of the matter components, that's why in those celestial bodies the matter is in condition of strong degeneration. When we approach to the center of star, the chemical composition of the plasma undergoes essential changes. Depending on the density, visible changes appear even at the densities lower than nuclear in so-called "Ae"-envelope of the neutron star, but in over nuclear region changes in the plasma are striking. In neutron stars at usual nuclear density ($\rho_0 = 2.85.10^{14}$ g/cm^3) the matter consists of neutrons, protons and negative pions with about the same concentrations, approximately and negative pions with about the same concentrations, approximately with 1% of electrons, which are necessary for guarantee of this state stability

With the increase of the plasma density, hyperons appear gradually as well as some other particles as stable components. In the degenerated hadron matter the stability of these particles, which are not stable in the usual conditions, is provided by the Pauli's principle. For this reason the decays of π^- and τ^- mesons will be forbidden, if the boundary energy of electrons will be higher than definite values, therefore in the corresponding medium they become stable particles. So in a more common case, when there is a sufficiently high density in the degenerate matter as stable particles there will be represent all kinds of baryons, leptons and mesons with negative charge. Such plasma will be called hadronic.

The chemical composition and particles concentration in the equilibrium hadronic plasma, as well the threshold of stabilization of particles in determined by following system of equations:

$$\mu^0_i=\mu_V,\ \mu^-_i=\mu_V+\mu_e,\ \mu^+_i+\mu_e=\mu_V,\ \mu_\pi=\mu_e,$$

$$\sum n^+_i+\sum n^-_i+\sum n^0_i=n,\ \sum n^+_i-\sum n^-_i=n_e+n_\pi \tag{1}$$

Here μ_i,n_i are is chemical potential and number density of i-th particle, signs -,0 and + indicate the electric charge of baryons, and n is the density of the total number of baryons. The first four equations refer to the conditions of thermodynamic equilibrium between the components of the plasma, the fifth and sixth express the law of baryons number conservation and the condition of local electroneutrality of the plasma. Though in all physical processes the total number of leptons is kept, in superdense celestial bodies it isn't kept, as neutrino is not held in them. If neutrino had a rest mass we would have to take into account the three laws of conservation of the number of leptons of three different kinds as well. In all the equations with the participation of the baryons, the electrical and baryonic charges from the left and the right are similar, which is a direct consequence of conservation these charges in elementary actions. These relations between the chemical potentials express the conditions of equilibrium between the components of the plasma that is why they correspond to the minimum of the system energy. In conditions of degenerated hadron matter, after a new particle appeared, a degenerated gas from these partiales is formed right away.

The chemical potential of the baryon is equal to:

$$\mu_\kappa=\varepsilon_\kappa+V_\kappa(p_\kappa), \tag{2}$$

where ε_κ is the boundary Fermi energy, V_κ is the energy of interaction of this baryon with the media, it depends on the boundary impulse. Here we speak about the potential energy of nuclear interactions.

In a hadron plasma no number of neutrons prevails the number of other baryons. On the whole the concentrations all kinds of baryons the same order, but the concentration of electrons is less than 1%. So, the name "neutron star" is not proper for celestial bodies consisting of hadron matter, nevertheless, we consider that this historically significant title should be preserved in our later works.

In the fundamental work ([4]) of Hambartzumian and Sahakian a condition of hadronic plasma in approximation of a ideal gas was investigated, i.e. in (1) it was assumed $|V_k<<1|$. Actually this approximation is not so bad. Thresholds of stabilization and concentration of particles in a plasma were calculated, the basic possibility of formation if negative pions condensate was justified. In a result the energy density and the pressure of hadronic matter was calculated. In the most common case the state equation of hadronic matter in approximation of ideal gas may be written in the following form

$$\rho c^2=K_n\sum\delta_k\left(sht_k-t_k\right),$$

$$P=\frac{1}{3}K_n\sum\delta_k\left(sht_k-8\cdot sh\frac{t_k}{2}+3t_k\right). \tag{3}$$

Here ρc^2 is the energy density. P-pressure in a plasma

$$\delta_k=\left(\frac{m_k}{m_n}\right)^4,\quad K_n=\frac{m_n^4c^5}{32\pi^2\hbar^3},\quad t_k=4arcsh\frac{\left(3\pi^2\right)^{1/3}\hbar n_k^{1/3}}{m_kc},$$

n_k is a density of a k-kind particles. The summation is carried on all kinds of baryons and leptons with negative charge (electron and μ^- meson), which are present in a plasma at given density. In the energy density it is necessary to take into account contribution of pion condensate $n_\pi m_\pi c^2$ also, if it is present. Indeed in the variant of an ideal gas, availability or absence of pion condensate is determined by number of kinds positively and negatively charged baryons. In variant of the same number of kinds positive and negative baryons in hadronic matter pions are not present. The parameters t_k are functions from density of common number of baryons n. It seems, that in approximation of an ideal gas of baryons, we have an analytical state equation of a degenerate matter, but it not so. A situation here considerably more difficult. The point is that before to write equation (3) it is necessary to find out which particles can be in a plasma, to define their thresholds of stabilization and to calculate the concentration of particles depending on baryons density n.

In the second work of Hambartzumian and Sahakian ([5]) the models of neutron stars for variant of a baryons ideal gas were calculated. The masses of configurations were, as in the case purely neutron configurations about mass of Sun; and radiuses of the order of 10km. We shall note that already in these first works ([5,6]) the most characteristic features the stellar configurations from the degenerate matter were established. We have in view the character of curve dependence of mass on density, internal structure and anomal defect of mass. To these questions we yet shall return.

2. The models of the real baryons gas and consequent stellar configurations were investigated by Sahakian and Vartanian ([7,8]). In these works the contribution of nuclear interaction took account by introduction the conception of effective mass for baryons and relevant corrections in the state equation were made. The corrections in the threshold of stabilization and in concentration of particles were made. In the state equation of ideal gas to expression of density the item $\rho_V(n)$ was add, and to expression of pressure- the item $P_V(n)$, which took account the contribution of the nuclear interactions. The calculated masses of stellar configurations in this case were twice more than the masses of configurations of ideal baryons gas and the radius were the order of 10 km again. For the first time in work ([8]) was represented the dependence of mass degenerate stellar configurations on the central density, including the branch of white dwarfs

The further stage in the development of theory of superdense stars was the work of Sahakian and Chubarian ([10]). In it was initiated to investigate envelopes of neutron stars and white dwarfs. The theory of white dwarfs has been existed long ago ([11,12]). In this question some definitions, depending on the new development of degeneracy matter theory, were necessary carry out. In the white dwarfs and envelopes of the neutron stars the matter consist of an atomic nuclei and degeneracy gas of electrons. Such plasma has been called "Ae-plasma" or "Ae-phase" of matter. It had been shown, that in the "Ae" plasma parameters of nucleous A and Z depend on the boundary energy of electrons (mass density). It was shown, that the average value of relation A/Z grows with the growth of density. The circumstance depend on the effect of neutronization. This effect show down rapid growth of electrons boundary energy, with growth of density, i.e. depend on condition of plasma energy minimum at the given density. Following approximation for relation A/Z was found:

$$\frac{A}{Z} = 2 + 1.255\cdot10^{-2}x + 1.376\cdot10^{-6}x^3 + 1.755\cdot10^{-5} \qquad (4)$$

where $x = p_e/m_e c$, and $p_e = (3\pi^2)^{1/3}\hbar n_e^{1/3}$ is boundary impulse of electrons. In our works ([7,10]) the existence so called neA and npeA phases, replacing Ae phase at high densities had been asserted. The later had been shown, that indeed such states in the degeneracy plasma don't exist. It appeared, that Ae phase replacing by phase of continuous nuclear matter and at this transition of plasma mass density experience the jump, changing in

500 times because of high densities (small distance between nuclei) owing to tunnel effect the light nuclei don't stable of the heavy nucleis formation. Hence, minimum value of mass number A_m is a function of mass density, it growth with density growth.

In the Ae plasma the energy density is determined by atomic nucleons, and pressure by electron gas. For state equation we have

$$\rho \approx \frac{32}{3}\left(\frac{m_e}{m_n}\right)^3 \frac{\overline{A}}{Z} x^3, \quad x = p_e / m_e c,$$

$$P = \frac{4}{3}\left(\frac{m_e}{m_n}\right)^4 \left[x\left(2x^2-3\right)\sqrt{1+x^2} + 3\ln\left(x+\sqrt{1+x^2}\right)\right] \tag{5}$$

Using this state equation the parameters of "Ae" envelopes of neutron stars and the models of white dwarfs have been calculated. The masses of "Ae" envelopes were of the order $10^{-6} \div 10^{-5}$ from the star mass, thickness the order several hundred meters. The curve of white dwarfs mass on the dependence of density in the center has the maximum $M_{max} \approx 1.27 M_\odot$ at the central density $\rho_0 \approx 2.1 \cdot 10^9$ g/cm^3, which is much less than Chandrasekhar's maximum. The maximum on the curve $M(\rho_0)$depends on the relation A/Z grows with the growth of density, and the diminution of the curve on the right limited by relativistic effects of gravitation theory.

3.The last important stage in the question of development of the theory of neutron stars was the systematic account of the π-mesons role in the thermodynamic of the degeneracy stellar matter in the works of Sahakian and Grigorian ([13-19]).

The analysis of date about masses of atomic nuclei has been shown that in the isobars of nucleons A>200 with the least atomic number exist several π-mesons in the average. The state of Ae plasma with the account of a possible role of π-mesons was investigated. Here the conditions for existence negative pions in nucleouses more favorable than in the nucleouses of usual matter, as in the Ae plasma at densities are higher than certain value $\mu_\pi = \mu_e > m_e c^2$.

Because of large densities in the Ae plasma the times of relaxation of the establishment of a condition with the least energy (absolute equilibrium) are small in the comparison with cosmogonics. That's why it is meaningful to consider Ae plasma, in which the energy on one nucleon, i.e. with the most probable value of mass number A at the given density. The parameters A and Z and nucleon binding energy b(A,Z) of the most stable nucleous are determined by the boundary energy of electrons, and therefore depend on the mass density. The base state of Ae plasma was determined by the following system of equations

$$\mu_p + \mu_e = \mu_n, \ \mu_\pi = \mu_e$$

$$N_n\mu_n + N_p\mu_p + N_\pi\mu_\pi = A\mu_n - Z\mu_e = Mc^2, \tag{6}$$

$$Zn / A = n_e$$

where N_n, N_p, N_π are number of neutrons, protons and negative pions in the nucleous, $N_n+N_p=A$, n_e is the density of electrons number, n is the density of the nuclons, M is the mass of the most probably nuclei, μ_k is the chemical potential of particles. Though the nucleouses are systems with the small number of degrees of freedom, nevertheless the use of concept of chemical potential for being in nucleouses nuclons is justified by that circumstance, that in the special conditions of Ae plasma they also accept the effective participation in an establishment of thermodynamic equilibrium, in the main though the processes of direct and return β decay. The mass of atomic nuclei is determined by the following specified formula of Weizseker ([19])

$$Mc^2 = N_n m_n c^2 + N_p m_p c^2 - c_0 A + c_1 A^{2/3} + c_2 (N_p - N_\pi)^2 / A^{1/3} +$$
$$+ c_3 (N_n - N_p + N_\pi)^2 / A + c_4 (N_n - N_p + N_\pi)^4 / A^3 + c'_3 N_\pi^2 / A + c_\pi N_\pi \quad (7)$$

where

c_0=15.75; c_1=17.8; c_2=0.71; c_3=23.7
c_4=-3.5; c'_3=17.65; c_π=11.96Mev

The first four numbers $c_0 \div c_3$ represent known coefficients in the usual Weizseker's formula, the other parameters were determined ([14]), by adjustments of binding energy for 200 nucleouses with mass numbers 220≤A≤257 on a way of the least squares. The number of pions N_π was determined by formula ([14])

$$N_\pi = (1+\alpha)\frac{c_3}{c_3 + c'_3}(A - 2Z) - (1+\beta)\frac{c_\pi}{2(c_3 + c'_3)}, \quad (8)$$

where α=-β=0.0088. The formula for pions number was determined from relation

$$\mu_p + \mu_\pi = \mu_n$$

This relation gives us result (8) with α=β=0. However, the parameters α and β were introduced for accounting a possible deflections from a case, when α=β=0 (inaccuracy of thermodynamic approach). Chemical potential of electrons is equal

$$\mu_e = \varepsilon_e = \left(m_e^2 c^4 + a^2 n_e^{2/3}\right)^{1/4}, a = \left(3\pi^2\right)^{1/3}\hbar \quad (9)$$

ε_e is a boundary energy of electrons. The chemical potentials of the other particles were determined by a usual way:

$$\mu_n = m_n c^2 - c_0 + \frac{2}{3}c_1 A^{-1/3} - \frac{1}{3}c_2 y^2 A^{2/3} + c_3\left[1 - (2y - y_\pi)^2\right] +$$
$$+ c_4\left[1 + 3(2y + y_\pi)^2\right](1 - 2y - y_\pi)^3 - c'_3 y_\pi^2,$$
$$\mu_p = m_p c^2 - c_0 + \frac{2}{3}c_1 A^{-1/3} - c_2\left(2 - \frac{y}{3}\right)yA^{2/3} - c_3(3 - 2y - y_\pi)(1 - 2y - y_\pi) -$$
$$- c_4\left[7 - 3(2y + y_\pi)\right](1 - 2y - y_\pi)^3 - c'_3 y_\pi^2,$$
$$\mu_\pi = -2c_2 yA^{2/3} + c_\pi + 2c'_3 y_\pi^2 + 2c_3(1 - 2y - y_\pi) + 4c_4(1 - 2y - y_\pi)^3, \quad (10)$$

where y = Z/A, $y_\pi = N_\pi / A_0$.

Substituting (7),(9) and (10) in the system of the equations (6), we obtain

$$2c_3(1 - 2y - y_\pi) + 4c_4(1 - 2y - y_\pi)^3 - 2c'_3 y_\pi^2 - c_\pi + \Delta m \cdot c^2 = 0,$$
$$y_\pi = \frac{1}{4c'_3}\left(\varepsilon_e + 2c_3 yA^{2/3} - 2c_\pi + \Delta m \cdot c^2\right), \quad y = \left(\frac{c_1}{2c_2 A}\right)^{1/2}, \quad (11)$$

where Δm = $m_n - m_p$. The parameters of the basic state Ae-plasma are determined by this system of the equations. To a question on the state equation we shall return below. The basic results depending on the pressure are given in table 1. It is seen from table that in the Ae plasma beginning with ρ=3.4•10^{10}g/cm^3 the negative pions are in the nucleouses with A≥80. But the more important thing that with promotion of density their concentration monotonously grows, reaching the limiting value y_π=0.22 at the end of this phase of the degeneracy matter. The end of Ae phase is determined by conditions of monotonous growth and continuity of chemical potential of neutrons at transition in a phase of continuous nuclear matter and characterized following dates:

$$\rho=6\bullet10^{11}\ \text{g/cm}^3,\ \mu'_n=\mu_n-m_nc^2=-0{,}63\text{Mev}\ \varepsilon_e=24{,}8\text{Mev},$$
$$A=353,\ y=0{,}595,\ y_p=0{,}405,\ y_\pi=0{,}22 \tag{12}$$

Table 1

The parameters of ground state of Ae-plasma

P ergcm^{-3}	ρ ergcm^{-3}	$x_e=$ $=P_e/m_ec$	A	Z/A	N_π/A	$m_nc^2-\mu$ Mev	μ_e Mev	$(m_n-\rho/n)c^2$
$9.868\cdot10^{22}$	$2.611\cdot10^{6}$	1.065	62	0.450		9.010	0.746	9.049
$9.712\cdot10^{24}$	$6.296\cdot10^{7}$	3.066	63	0.445		8.607	1.648	8.766
$6.570\cdot10^{25}$	$2.573\cdot10^{8}$	4.883	65	0.440		8.209	2.547	8.473
$5.977\cdot10^{26}$	$1.355\cdot10^{9}$	8.428	68	0.430		7.430	4.337	7.887
$2.412\cdot10^{27}$	$3.931\cdot10^{9}$	11.926	71	0.420		6.674	6.115	7.310
$6.704\cdot10^{27}$	$8.657\cdot10^{6}$	15.389	75	0.410		5.942	7.880	6.745
$1.502\cdot10^{28}$	$1.624\cdot10^{10}$	18.819	78	0.400		5.233	9.630	6.192
$2.919\cdot10^{28}$	$2.742\cdot10^{10}$	22.216	82	0.390		4.550	11.36	5.653
$3.830\cdot10^{28}$	$3.410\cdot10^{10}$	23.776	85	0.385	0.001	4.239	12.160	5.407
$4.074\cdot10^{28}$	$3.598\cdot10^{10}$	24.147	86	0.382	0.004	4.167	12.350	5.344
$8.350\cdot10^{28}$	$6.787\cdot10^{10}$	28.888	104	0.347	0.043	3.284	14.771	4.564
$1.468\cdot10^{29}$	$1.146\cdot10^{11}$	33.259	127	0.314	0.080	2.546	17.003	3.879
$2.310\cdot10^{29}$	$1.790\cdot10^{11}$	37.252	157	0.283	0.115	1.937	19.143	3.281
$3.345\cdot10^{29}$	$2.639\cdot10^{11}$	40.862	195	0.253	0.147	1.442	20.887	2.763
$4.531\cdot10^{29}$	$3.721\cdot10^{11}$	44.082	246	0.226	0.177	1.049	22.532	2.318
$5.810\cdot10^{29}$	$5.066\cdot10^{11}$	46.909	314	0.200	0.205	0.741	23.967	1.938
$6.409\cdot10^{29}$	$5.780\cdot10^{11}$	48.074	353	0.188	0.217	0.626	24.571	1.783
$6.409\cdot10^{29}$	$2.846\cdot10^{11}$	100.37	∞	0.0035	0.406	0.626	51.269	0.628

The phase transition in a state of continuous nuclear matter with $\rho=\rho_0=2{,}85\bullet10^{14}$g/cm^3, μ'_n=-0,63Mev is made. The mass density experiences the jump about in 500times in this transition.

Dates, given in the table 1, show, that the process of neutralization of matter proceeds up to $\rho=3\bullet10^{10}$ g/cm^3 (μ_e=12 Mev). Then,the process of nucleuses filling by mesons is begin, which is possible to call by effect of pionization. From this threshold the process of nucleuses neutralization stops: at futher promotion of density, the concentration of neutrons and protons is fixed on values $y_n=N_n/A=0{,}6$; $y=N_p/A=y+y_\pi$ =0,4.

At the end of a Ae-phase there are π-mesons in the atomic nucleus,the number of which is 22% of nucleons number. They will obviously stay and in that case, when the atomic nucleuses merging, will form continuos nuclear matter. Certainly, the electrons should be in such plasma. The nuclear matter on the own thermodynamic properties is most likely similar to liquid that it is well known on an example of atomic nucleuses. Therefore, in the theory of nuclear matter the account of nuclear interactions of particles is important. But how to take into account these interactions? It is rather difficult problem. The nature, however, has presented us a unique possibility for round of this difficulty. We mean the fact of presence of atomic nucleuses and experimental dates about their bound energy, which with satisfactory accuracy are described by the half-empirical formula of Weitzeker. Just in the works ([13-19]) this possibility was successfully used. An essential deficiency on this way is the fact, that is does not allow to investigate properties of a plasma, depending on density of mass. However, incompressible character of nuclear matter in a wide range of hydrostatic pressure largely neutralized this deficiency, and as a result is possible to solve a number of the important questions of the theory of hadronic plasma, in particular, the questions of the state equation.

Using the specified Weitzeker formula (7) for a case of continuos nuclear matter for whole energy of plasma was assumed

$$E = N_n m_n c^2 + N_p m_p c^2 - c_0 N + c_3(N_n + N_\pi - N_p)^2/N + c_4(N_n + N_\pi - N_p)^4/N^3 + \\ + c_\pi N_\pi + c_3 N_\pi/N + (3a/4)V^{-1/3} N_e^{4/3}, \tag{13}$$

where V is volume of some part of plasma, N_n, N_p, N_π, N_e are numbers of neutrons, protons, pions and electrons in this volume, $N=N_p+N_n$ is the number of nuclons, the last item represents the energy of relativistic gas of electrons $a=(3\pi^2)^{1/3}\hbar c$. In this formula were missed surface and coulomb parts.

Calculating the partial derivatives of expression (13) by the number of particles N_k, we obtain corresponding chemical potentials μ_k. State equilibrium is determined by system of equations

$$\mu_p + \mu_e = \mu_n,\ \mu_\pi = \mu_e,\ n_n + n_p = n_0,\ n_e + n_\pi = n_p \tag{14}$$

where $n_k = N_k/V$ is density of particles. Substituting in these relations the expressions of chemical potentials and solving the obtained equations, we shall get

$$y_n=0.591,\ y_p=0.409,\ y_\pi=0.4,\ y_e=0.0035, \\ \mu_n=938.94,\ \mu_p=887.67,\ \mu_\pi=\mu_e=51.29\text{Mev} \tag{15}$$

As we see, $\mu_n - m_n c^2 = -0.63$Mev, $\mu_p - m_n c^2 = 50.63$Mev and average binding energy of nucleon $b_0 = (\rho_0/n_0 - m_n)c^2 \approx \mu_n - m_n c^2 = -0.63$Mev. From brought dates it is visible, that continuous nuclear matter is in a liquid state. Thus,is not correct as representation that at density near to nuclear, plasma is in a gas state, an the fact, that it consist from neutrons mainly.

Brought in(12) dates about the top threshold of Ae plasma were found from a condition of a continuity of chemical potential of neutrons at phase transition $Ae \rightarrow np\pi^- e$. In the phase transition in the interior of neutron stars, the pressure doesn't experience the jump and equal

$$P_0 = 6.4 \cdot 10^{29}\text{erg/cm}^3 \tag{16}$$

It means that neutron star consist of two basic parts: from hadronic nuclei and Ae-envelopes.On surface of division these regions of star the density experience jump about in 500 times. There are the essential changes in the nuclear matter, when it condensation. This take place when hydrostatic pressure $P \geq 5 \cdot 10^{33}$erg/cm^3. In accordance with increase of density at its certain values there are different hyperons, resonance and leptons in plasma. The chemical composition and the concentration of particles in equilibrium hadronic plasma are defined by the equations system (1).

At densities on one order higher than nuclear, we deal with plasma from all kind baryons and π^- mesons. There is a small impurity of electron gas, playing a role of stabilization of the this whole picture of hadronic plasma.

At rather high densities, when hadronic "bags" closely adjoin with each other, plasma consisting from quarks will be formed. But quarks give to know about self already at usual nuclear density during an exchange nucleons by mesons, i.e. the formations from pair quark-antiquark $[q\bar{q}]$. When density in plasma becomes so large, that hadronic bags are density adjoin, to speak about matter consisting from baryons, it is not meaningful, and obviously, the medium consisting from quarks and lepton will be formed. Figuratively speaking, will be formed hadron of macroscopic sizes. To speak about exact value of density, above which we deal with quark phase of degeneracy matter is impossible, so already at densities on one order above nuclear inflination of quarks through a barrier between hadronic bags, occurs so often, that in a scale of densities there is no sudden boundary of division between hadronic and quark phases of matter. Apparently the decay of hadrons occur when distance between their centres reaches value $r_0 \approx 0.5 l_0$, where $l_0 = 0.5$fm is radius, so called, hard core of nucleon, established in experiments by study of elastic scattering of nuclons. Now this peculiarity can be estimated as manifestation of a Pauli principle for quarks. According this presentation the radius of baryon should be about $r_e \approx r_c$ and therefore the threshould of formation of quark plasma approximately equal

$$n \approx \frac{3}{4\pi r_e^3} \approx 1.53 \cdot 10^{40} cm^{-3}, \quad \rho \approx 5 \cdot 10^{16} g/cm^3 \tag{17}$$

From indeterminancy relation follows,that at such density the baryons still nonrelativistic: $cp \approx 400 Mev < mc^2$.

In the quark phase the thresholds of birth and particles concentration are defined by the following system of equations

$$\begin{gathered} \mu_d = \mu_s = \mu_b,\ \mu_u = \mu_c = \mu_t,\ \mu_n + \mu_e = \mu_d,\ \mu_\tau = \mu_\mu = \mu_e, \\ 2(n_u + n_c + n_\tau) - (n_d + n_s + n_b) = 3(n_e + n_\mu + n_\tau), \\ n_u + n_d + n_s + n_b + n_c + n_t = 3n \end{gathered} \tag{18}$$

Here indexes u, d, s, c, b, t are denote tipes (flavors) of quarks, and indexes e, μ, τ concern to leptons, n is the density of equivalent number of baryons Chemical potentials of particles as former is defined by expression (2):$\mu_k = \varepsilon_k + V_k$. For boundary impulses we have

$$p_k = \begin{cases} (3\pi^2)^{1/3} \hbar n_k^{1/3}, \textit{ for letpons} \\ \pi^{2/3} \hbar n_k, \textit{ for quarcks} \end{cases} \tag{19}$$

The difference in the formulas of boundary impulses of leptons and quarks is caused by the presence at the last additional quantum number called colour. Thus,if to consider, that the quark masses are known, the problem of definition of a state of quark plasma is reduced to knowledge of particles interaction energy V_k. For leptons $V_k = 0$. Apparently,it is possible to consider, that the interactions energy of quarks with medium approximately the same for all flavors and in calculations all V_k is possible to omit.

In the region of densities $\rho \geq 10^{23}$g/cm^3, when all particles in plasma becomes ultrarelativistic an asymptotic freedom of quarks becomes important, and the equations (18) are simplified. The solution of this system is

$$\begin{gathered} n_d \approx n_s \approx n_b,\ n_u \approx n_c \approx n_t,\ n_d^{1/3} - n_u^{1/3} = (3n_e)^{1/3}, \\ 2n_u - n_d \approx 3n_e,\ n_u + n_d \approx n,\ n_\tau \approx n_\mu \approx n_e \end{gathered}$$

The solution of this system is

$$n_u \approx n_c \approx n_t \approx 0.335n,\ n_d \approx n_s \approx n_b \approx 0{,}665n,\ n_e \approx n_\mu \approx n_\tau \approx 1.883 \cdot 10^{-3} \tag{20}$$

Thus in ultrarelativistic quark plasma, the concentrations of all kinds quarks about the same, and concentration of leptons on three order less that concentration of nucleons.

4. The most important question for theory of neutron star is finding of the state equation of degenerated stellar matter.It is one of equations defining a distribution of mass and integral parameters of star. Straight more or less precise definition of pressure depending on mass density, by the calculation of particles concentrations practically is impossible. In the work ([19]) this problem was solved by phenomenologically method, using some undebatable facts.

In the basis of inference of state equation were supposed the following well known relations of thermodynamics

$$\frac{dP}{d\mu}=n,\ \rho c^2=n\mu-P,\ \frac{c^2}{v_S^2}=c^2\frac{d\rho}{dP}=-\mu n^2\frac{d^2\mu}{dp^2}, \tag{21}$$

are true for all phase of degeneracy stellar matter. Here n is baryon's density, ρc^2 is density of whole energy, P is pressure, μ-chemical potential of neutrons, or in the case of quark plasma is the electroneutral combination of three particles with barionic charge equal one, v_S is sound velocity. Since on the division boundary of Ae-phase and nuclear matter the density experience the jump, it is convenient the state equation to seek in form $\rho(P)$.

In the Ae phase the pressure determined by electrons and energy density in main by atomic nucleuses. The energy of electronic gas is equal

$$E_e=4VK_e\left[x(2x^2+1)\sqrt{x^2+1}-\ln(x+\sqrt{x^2+1})\right],$$

where V is volume of the system, $x=p_e/m_ec$, $K_e=m_e^4c^5/32\pi^2\hbar^3$. Hence, for pressure we obtain

$$P=\frac{4}{3}K_e\left[x(2x^2-3)\sqrt{x^2+1}-3\ln(x+\sqrt{x^2+1})\right]. \tag{22}$$

Energy density is equal

$$\rho c^2=\frac{n}{A}Mc^2+\frac{E_e}{V}. \tag{23}$$

The mass M and mass number A of nuclei are functions of mass density, so the state equation yet did not find. It is convenient to define it of system (22), but for this is required the knowledge of dependence μ, from x. This dependence in ([19]) was approximated by the formula

$$\frac{\mu-m_nc^2}{m_ec^2}=-18.34+(0.451-4.4\cdot10^{-4}x-3.2\cdot10^{-5}x^2)\times$$
$$\times\sqrt{x^2+1}+4.4\cdot10^{-4}\ln(x+\sqrt{x^2+1}) \tag{24}$$

Thus, the expressions (22), (24) and $\rho c^2=n\mu-P$ together definition the state equation $\rho(P)$ of Ae-plasma in the parametric form. For don't accurate calculations, neglecting in (23) by item E_e/V, taking $M/A=m_p$ and accounting the condition of electroneutrality $nZ/A=n_0$, we obtain

$$\rho c^2\approx nm_pc^2=\frac{1}{32\pi^2}m_pc^2\left(\frac{m_ec}{\hbar}\right)^3\frac{A}{Z}x^3. \tag{25}$$

In the nonrelativistic and ultrarelativistic cases from (22) we obtain

$$P\approx\begin{cases}\frac{32}{15}K_ex^5, & x\ll1\\ \frac{8}{3}K_ex^4, & x\gg1.\end{cases} \tag{26}$$

Excepting from (25) and (26) parameter x, we get

$$P\approx\begin{cases}B_1\rho^{5/3}, at & \rho<2\cdot10^6\,g/cm^3\\ B_2\rho^{4/3}, at & \rho\gg2\cdot10^6\,g/cm^3.\end{cases} \tag{27}$$

The correct derivation of state equation for hadronic plasma spreading the region of densities $3\cdot10^{14}\le\rho\le7\cdot10^{16}g/cm^3$ does not simple problem. For this it is of particles in a plasma their concentrations, partial pressures, partial energy densities

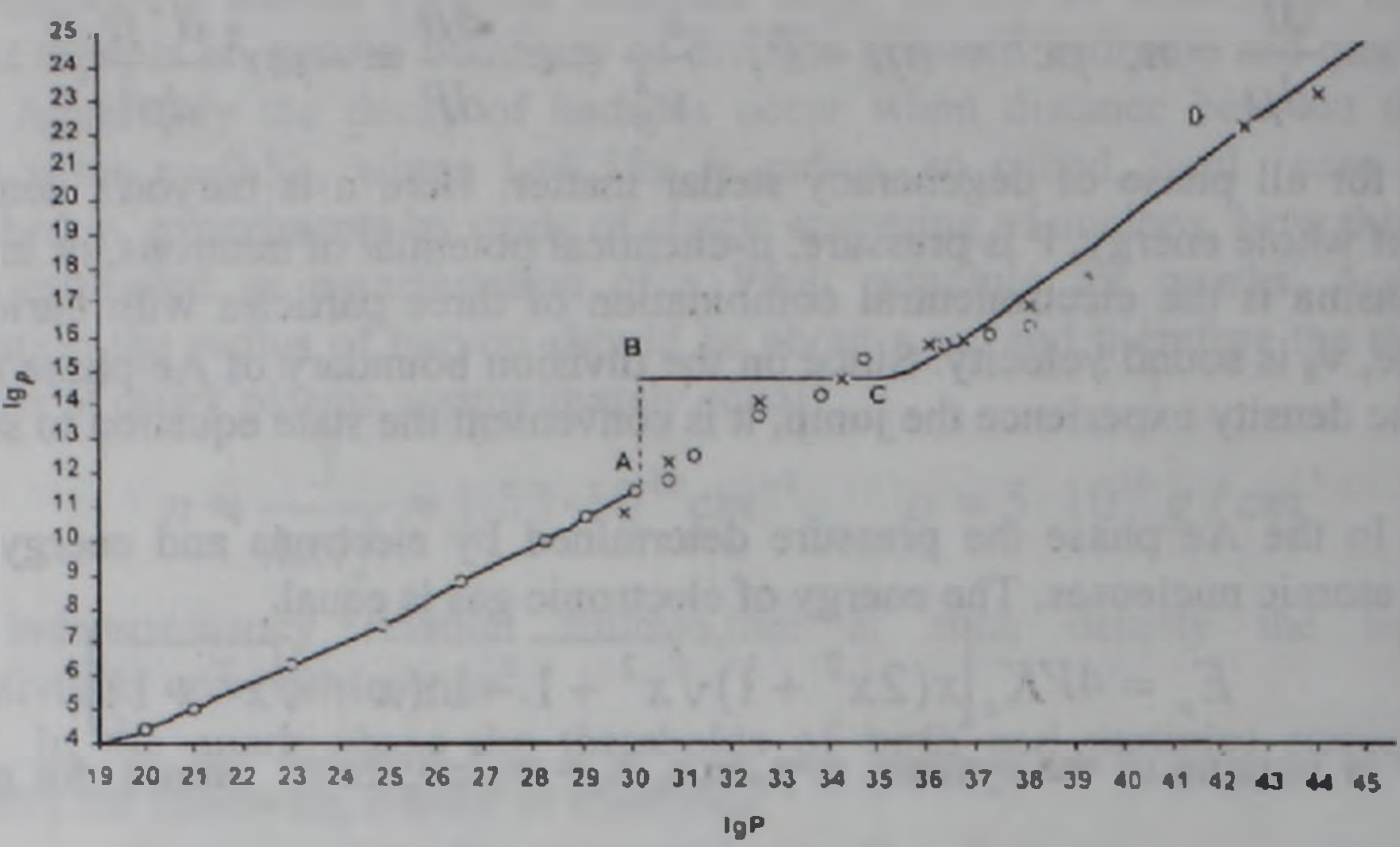

Fig. 1. The diagram of the state equation of degnerate plasma. Pressure P is measured in ergcm^{-3}, and mass density in gcm^{-3}. The initial part of the curve ρ(P) up to point A correspongs to Ae-plasma and the dashed line AB-to the density jump at the transition to the nucleas matter phase (the almost horizontal segment BC), Cdis the hadronic plasma, the quarck matter is beyond point D. The cucles correspond to the equation PBBP, but cross-to state equation SV.

necessary the exact knowledge of chemical potential of plasma, the interactions energy depending on the density of baryons n. This is practically does not solving problem. Therefore in ([19]) the question of state equation was solved round about way, using number physical considerations. But for this purpose first of all it is necessary to consider the quark phase, state equation of that is defined comparatively simple. Relatively simple situation in the quark phase with accounting asymptotic behavior (20) allowed us to find approximation

$$\mu = kP^{1/4}(1 + P^{-\alpha}), \tag{28}$$

between chemical potential and pressure, where α=0.0081, β=300erg$^{\alpha}$cm$^{-3\alpha}$. Now, using (22), we can calculate density of baryons n, density of whole energy and sound velocity.

Further the state equation of hadronic plasma was found by the conclusion of its parameters with phases of nuclear matter and quark plasma. The state equation of hadronic matter, covering the region of pressures $6{,}4 \cdot 10^{29} < P < 2 \cdot 10^{37}$ erg/cm^3, it is possible to obtain with sufficient accuracy, demanding that phase quantities μ, P, n, ρ, v_S changed by a continuous and monotonous manner. Here important is the fact, that at any phase transition chemical potential and pressure should not experience jump, and inside each of phase they should change by monotonous manner. For chemical potential the following approximations ([19]), ([21]) was found.

$$\mu = \begin{cases} a_1 + a_2(1 + P/L)^{\nu}, & at \;\; P_0 \le P \le P_1 \\ a_3(1 + P/D)^{\sigma}, & at \;\; P_1 \le P \le P_2 \end{cases} \tag{29}$$

where a_1, a_2, a_3, L, ν, D, σ are constants. They was determined from boundary conditions for μ, n, v_S at $P=P_0=6.41 \cdot 10^{29}$erg/cm^3 (pressure at the end of Ae plasma), from the requirement of a continuity of this quantities at $P=P_1=5 \cdot 10^{37}$erg/cm^3 (end of phase of

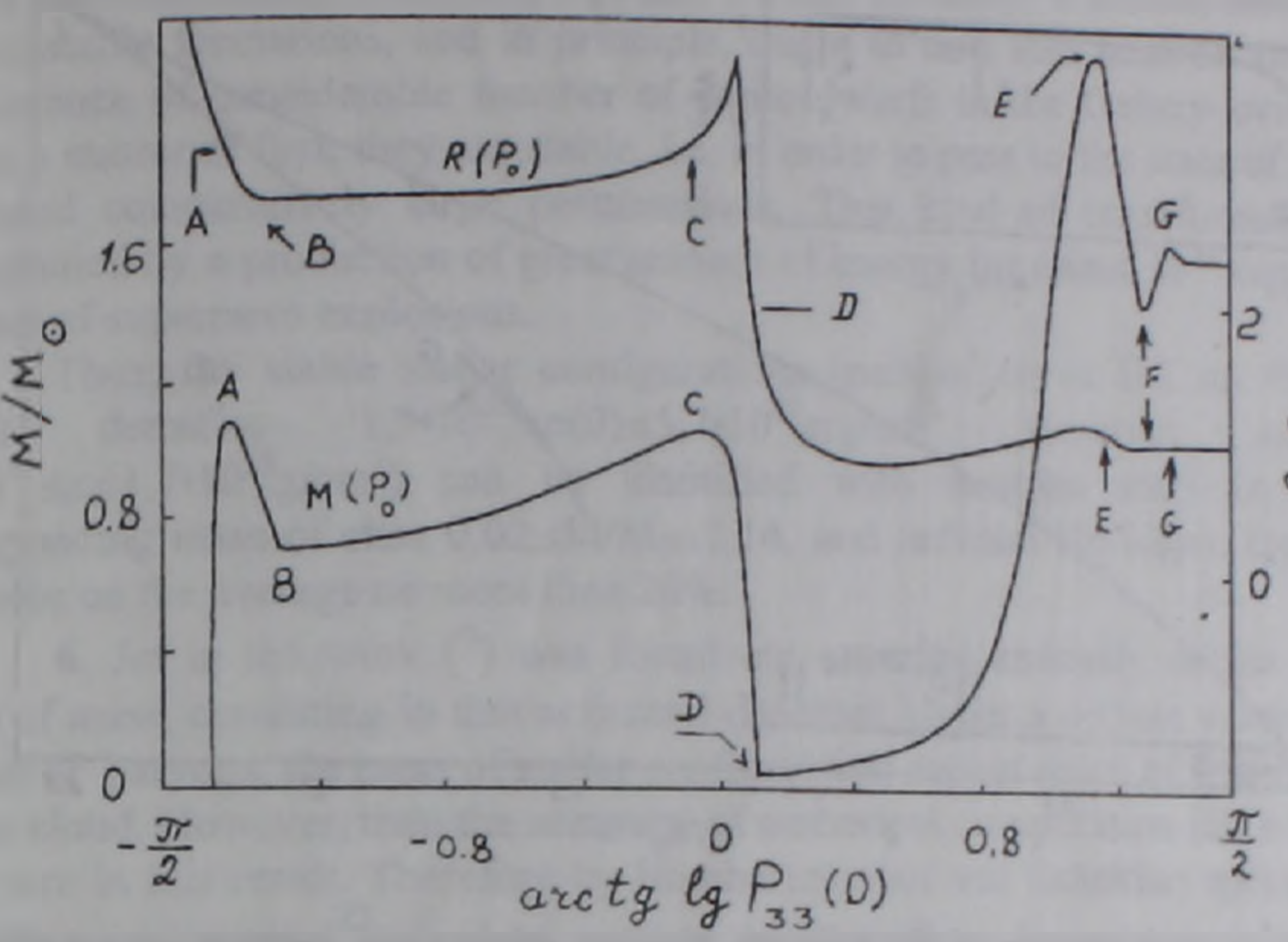

Fig.2. The dependece of the radius R measured in km and the mass of configurations measured in selar on the central pressure P(0). The same letter in this and next figure represent points corresponding to one same configurations.

incompressible nuclear matter) and continuity μ in a point $P=P_2=1.9 \cdot 10^{37}$ erg/cm^3, using the relations (21)

a_1=937.61, a_2=1.312, a_3=939.3, ν=0.9384,
L=$3.352 \cdot 10^{32}$, D=$6.659 \cdot 10^{34}$, σ=0.232

In (29) the pressure is measured in GSS system, and μ-in Mev. So the state equation $\rho(P)$ is found in the whole region of pressure of degenerate Wae stellar matter, it was named the state equation GS (Grigorian-Sahakian). The diagram of this state equation is brought on fig 1.

5. The parameters of stellar configurations from degeneracy matter are determined by following system of equations

$$\frac{du}{dr} = 4\pi\rho r^2, \tag{30}$$

$$\frac{dp}{dr} = -\frac{G(\rho c^2 + p)}{c^2 r(r - \dfrac{2Gu}{c^2})}(u + \frac{4\pi}{c^2} pr^3) \tag{31}$$

$$\frac{dJ}{dr} = \frac{8\pi}{3}\rho r^4 \frac{1 + P/\rho c^2}{1 - 2Gu/c^2 r}(1 - \frac{2GJ}{c^2 r^3})(1 - \frac{GJ}{2c^2 r^3}), \tag{32}$$

Where r is distance from center of configuration, u(r)- accumulated mass, J(r)- accumulated moment of inertia. To these equations it is necessary to add state equation, then system will be complete, the boundary conditions are those

$$P(0),\ u(0)=0,\ J(0)=0,$$
$$p(R)=0,\ u(R)=M,\ J(R)=J, \tag{33}$$

where M is mass and J-moment of inertia of a star. The equations (30),(32) where integrated in work ([19]). On fig 2. the curves $M(P_0)$ and $R(P_0)$ are brought. Part of curve

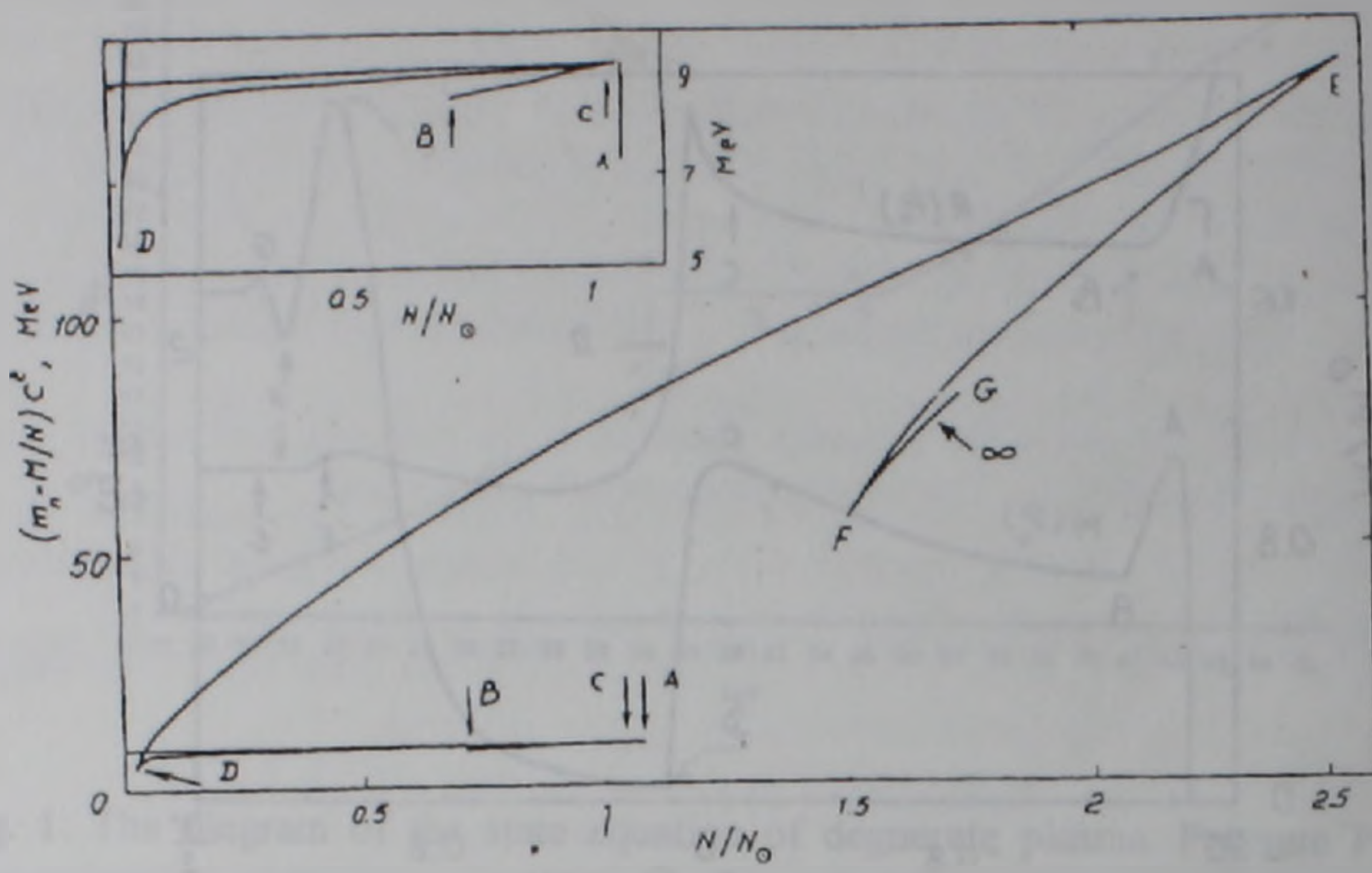

Fig 3. The dependence of the average binding energy per one baryon on the total number of baryons N; $N_{\Theta} = \dfrac{M_{\Theta}}{m_n}$ is the number of nuclear in the Sun, m_n is the neutron mass. In the left hand in upper corner is plotted in the large-scale DCBA part of the curve. The curve point designated by the symbol ∞ corresponds to configuration with an infinitely great central pressure. At the given number of baryons configurations with the greatest binding energy are stable.

$M(P_0)$ to the left of the point B represents the configurations of white dwarfs. The ordinate of curve in the point A has a maximum: $P(0)\approx10^{27}$erg/cm^3, $M=1.08M_{\Theta}$, that noticeably is less than Chandrasechar's maximum. Part ABCD corresponds to unstable configurations. Their existence is not excluded in depts more massive celestial bodies. The stable are only the configurations appropriate the part DE. They represent models of neutron stars and are characterized by central pressures $1.3\cdot10^{29}\leq P(0)\leq 3{,}7\cdot10^{35}$ erg/cm^3,masses $0.2\leq M/M_{\Theta}\leq 2.15$ and radiuses $100\geq R\geq 10$km. Beyond the point E the ordinate of the curve oscillating with the attenuating amplitude goes down, to the value $M=1{,}5M_{\Theta}$ in the limit $P(0)=\infty$. All the configurations corresponding to this part of he curve are unstable. The others important parameters of the degeneracy stellar configurations are baryons number N and the average binding energy of nucleon $b=(m_n-M/N)c^2$. The baryons number in a star was calculated by formula

$$N = \int_0^R n(r)\sqrt{-g_{11}}\,r^2 dr\,,$$

where

$$g_{11} = -\left(1-\frac{2Gu}{c^2 r}\right)^{-1}$$

is a radial component of metric tensor.Fig 3 shows the dependence of the average binding energy of nucleon b(N) from baryons number. From the curve b(N) it is seen, that at same number of baryons N, binding energy b(N) (i.e. mass) can two and, even in some parts (to right of point E) three and more values. Obviously, at the same number of baryons stable is that configuration of which the binding energy b(N) large, i.e. mass is less. In particular, at the same number of baryons average binding energy nucleons in neutron stars essentially more than in white dwarfs. Actually, it means, that white dwarfs are metastable formations, and in principle, ought to turn into neutron stars. However, the existence of considerable number of white dwarfs in the Galaxy evidently shows, that, as a matter of fact, they are stable, i.e. in order to pass to the state of neutron stars, they need comparatively large pertubartions. This kind of transformation will be accompanied by a production of great amount of energy for about 10^{51} erg, which enters the class of supernova explosions.

Thus, the stable stellar configurations (part of curve DE on the fig.2) with central densities $1,3\cdot10^{35}\leq p(0)\leq5,7\cdot10^{35}$erg/cm^3 (interval of densities $3,1\cdot10^{14}\leq\rho\leq1,7\cdot10^{15}$g/cm^3) can be identified with neutron stars in pulsars. The corresponding mass of stars $0,02\leq M/M_0\leq2,14$, and radiuses $R\approx10$km. Deviations from this value on the average no more than 20%.

6. Jet in the work ([6]) was found out amazing anomaly in the gravitational defect of mass, consisting in that at central densities higher a certain value, for the same number of baryons, the mass of stellar configuration appear more of appropriate mass of diffuse cloud. However, then the accuracy of numerical computation did not rather large, to be sure in this result. Therefore by Hambartzumian and Sahakian special research of this effect was spent ([22]), and to exclude the distortions brought certainly by nuclear interactions and complications in chemical structure, the configurations, consisting of

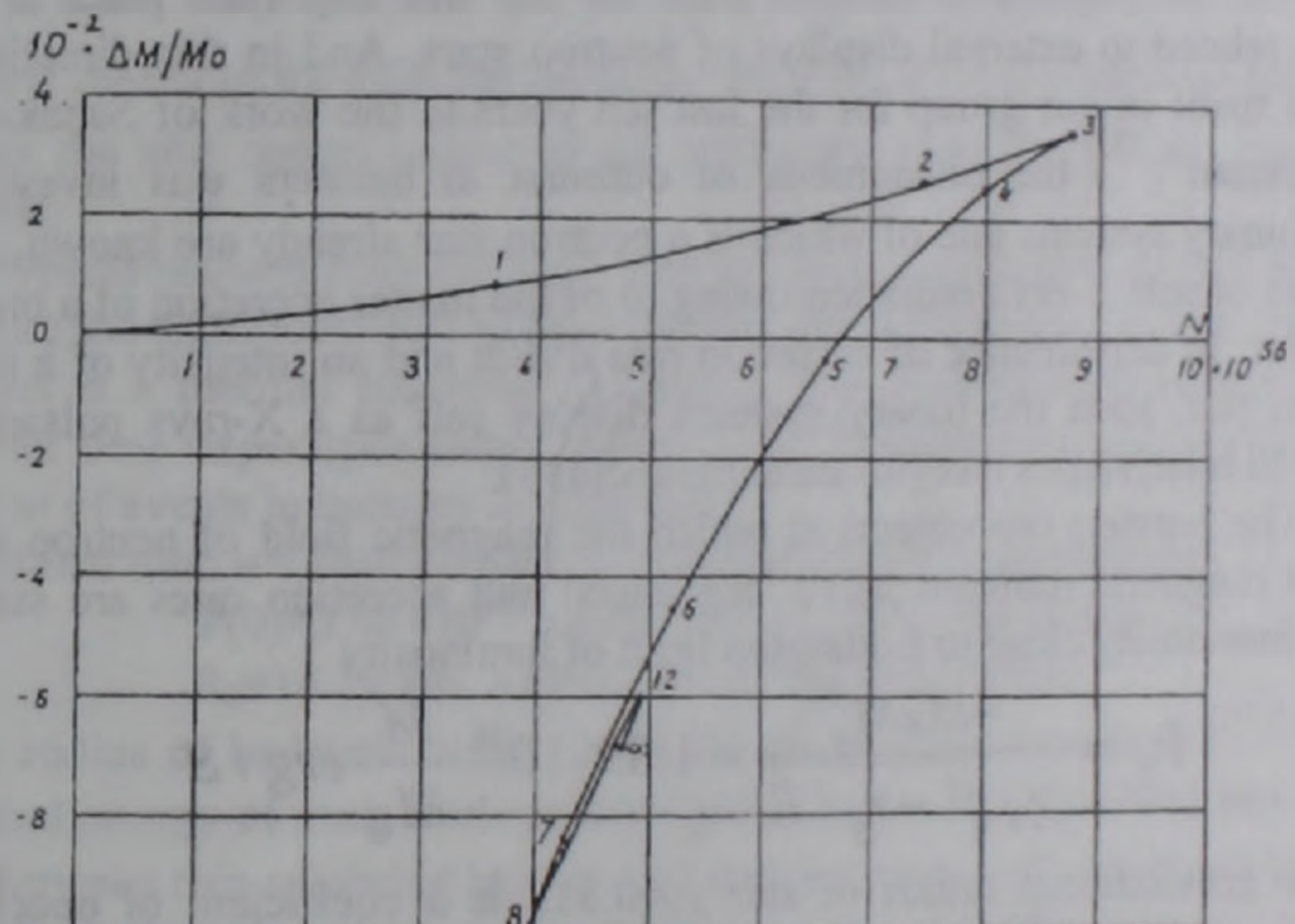

Fig 4. The dependence of packing fraction $\Delta M/M_\odot$ on the total number of neutrons N, $\Delta M = M_0 - M$, $M_0 = m_n N$ The figures on the curve indicate values of parameter $t_n(0) = 4arcshP_F/m_nc$ for corresponding configurations

ideal gas of neutrons, were considered.

The dependence of packing fraction of neutrons is

$$\frac{\Delta M}{M_0} = \frac{M_0 - M}{M_0} \quad (34)$$

depending on the total number of neutrons in the configurations was calculated. Here

$$M_0 = Nm_n\,, \quad M = 4\pi \int \rho r^2 dr,$$

The result is represented on fig. 4.

For usual stars $\Delta M>0$ always. In the linear sequence $M(\rho_0)$of neutron configurations the curve of pacing fraction at first has a normal behaviour with the growth of central density monotonously grows and at value $n(0)=3.10^{39}cm^{-3}$ reaches a maximum,then the curve sharply changing own direction, goes down. At $n(0)\geq 1.12\cdot 10^{40}cm^{-3}$ the packing fraction becomes negative, in the point with $n(0)=2\cdot 10^{41}cm^{-3}$ has a deep minimum,approximately equal -0.1, then it oscillating with a small and strongly damping amplitude, and continuing to remain negative, at $\rho(0)\to\infty$ it goes to limit -0.69. In the models of configurations from real gas of baryons $\Delta M>0$ always.But never the less here absolute gravitational defect of mass has abnormal behavior too. Indeed at central pressures, exceeding $P(0)\approx 4\cdot 10^{35}erg/cm^3$ the growth of nucleon gravitational binding energy is replaced by its appreciable ccc.

The circumstances analysis of the above mentioned peculiarities of mass gravitational defect of degeneraces stellar configurations brings us to conclusion that its abnormal behaviour is caused by two reasons, by strong growth of particles boundary energy in the appropriate configurations, and by catastrophic violation of additivity of internal energy in the inside the most dense baryonic configurations caused by strong curvative of space metric. The cosmogonical aspects of abnormal gravitational mass defect until do not investigated.

7.In the physics of neutron stars the not less important place is occupied the questions related to external displays of neutron stars. And in this direction the certain work was spent in our group for the last ten years.In the work of Sahakian, Alodjantz and Sargissian ([23]) the phenomena of outburst in bursters was investigated. Many compact binary systems one of which is a neutron star already are known. Such systems are sources of soft X-rays radiation owing to of the matter accretion of a ordinary star on neutron star. In dependence of accretion rate dM/dt and an intensity of a magnetic field of neutron star, such the binary systems display self as a X-rays pulsars, bursters or objects with relativistics jets (for examples SS433).

The bursters are objects at which the magnetic field of neutron star relatively weak (the magnetic moment $\mu\leq 10^{28}$erg/gauss) and accretion rates are such, that them radiation luminosity close to Eddington limit of luminosity

$$L_E = \frac{4\pi GM_c}{\chi_e\sqrt{1-r_g/R}} = 1.89\cdot 10^{38}\frac{M}{M_\odot} erg/S \tag{35}$$

Here r_g is gravitational radius of star χ_e=0.312 is a coefficient of opacity, cosed by Tomson scattering in the assumption, that of a surface of the neutron star, the mass of hydrogen part is x=0.64. At quasistationar regime the accretion rate of appropriate luminosity (35) is equal

$$\dot{M}_E = \frac{4\pi GM}{c\chi(1-\sqrt{1-r_g/R_0})} = 1.21\cdot 10^{18}\frac{M}{M_\odot} g/S \tag{36}$$

At the constant rate of accretion $\dot{M} < \dot{M}_E$: the neutron star is in a stationary state and in dependence on that, what the magnetic field it keep self or as X-ray pulsar (at a strong magnetic field $H(R)\approx 10^{12}$gauss) or as compact nonpulsating source of X-radiation (when a magnetic field relative weak). The bursters are objects with the accretion rate close to

(36), $\left|\dot{M} - \dot{M}_E\right| << \dot{M}$. In this case in the neutron star qunsistationary thermal state is established, in which calm long periods ($\tau \sim 10^4 \div 10^5$S) of slow heating are interrupted by short outbursts, accompanying by ejection of certain quantity of matter and radiation energy, when the luminosity reached of Eddington limit, hardly only small part of its thermal energy, which necessary in order to the luminosity of a star was lowered hardly below of Eddington limit, therefore its shown in ([23]), that just these small fluctuation in the thermal reserve of neutron star bring in the phenomena outbursts in the bursters. Indeed, at accretion process, as soon as the luminosity, reaching the value L_E hardly exceeds it, a radiating force (a radiation pressure) on the star surface preponderate over gravitational, inevitably bring to flow of matter from a star and on the contrary. when during the outburst, the luminosity little bit lowered below L_E preponderate over the gravitation force and at once renews the process of accretion. At a outburst the star loses a energy, as in comparison with the calm period the luminosity on the order is higher, therefore for this short interval of time $\dot{T}(r,\tau) < 0$, however extenuation of temperature is rather insignificant. In an interval of time between outburstrs the star is in an accretion regime, on its surface huge energy is allocated, but the main part of this energy is radiated from a surface, and only the small part it is transferred in interior on heating of hadronic nuclei. The star slowly fills up that small energy, which has lost during the outburst therefore for calm period of its life $\dot{T}(r,\tau) < 0$. But with time $\dot{T}$ is decreasing, so as the star comes nearer to the stationary state, then there comes the following outburst.

At the typical neutron stars the main part of mass is concentrated in a hadronic nucleous, therefore the significant part of their thermal energy is reserved here. Thus, the hadronic nucleous is a thermal container of neutron star. The thermal energy of this container with the time experiepce rather insignificant fluctuations $|\Delta Q|/Q<<1$, than the regulnr motion of events in bursters in provided.

For an illustration we shall consider a configuration with parameters

$$P(0)=3.96 \cdot 10^{34}\ \text{erg/cm}^3,\ R=11.73\ \text{km},\ M=1.08M_\odot,$$
$$R_0=11.19\ \text{km},\ \Delta M=3.24 \cdot 10^{-5} M_\odot,\ r_g=3.21\ \text{km}, \qquad (37)$$

where R_0 is the radius of hadronic core, ΔM is the mass of Ae-envelope ([19,21]). In a burster the thermal energy of such neutron star approximately is equal $Q=3.96 \cdot 10^{44}$erg ([21,23]). At time between two adjoining bursters the thermal energy of star grows by value $\Delta Q=2 \cdot 10^{38}\tau_1$, where $\tau_1 \approx 10^4 \div 10^5$s is a time between two bursts.This energy about on three order is more than that energy ($E \sim 10^{39}$erg) which is sheart out in one burst. While all energy ΔQ, accumulated during the time T_1, should be spent in burst time, in order to supply regular action of burster's mechanism. The fact is that the energy ΔQ in main is spent on accomplishment of work at mass eruption during the burst, The mass m_B ejecting during the burst is defined by relation

$$\frac{\gamma G M_m}{R} \approx \Delta Q ,$$

where γ is Lorentz factor for ejected mass. For a star with parameters (37), the mass m_B is the order 10^{22}-10^{23} g. We shall at last note, that in bursters the temperature of neutron stars along radius with the accomt of the relativistic factor practically is do not change and equal $\sqrt{g_{00}}T(r) \approx 2 \cdot 10^7$.

8. At the beginning of 90-s, developing the pioneer works of Sturrok ([24]), Ruderman and Sutherland ([25]), by Sahakian was developed a new variant of radioradiation of pulsars([26-28]). In the work ([26]) the problem of radioradiation of pulsar was considered for aligned rotator, but in ([27,28]) for real case, when the magnetic axis of star does not coincide with a rotation axis (inclined rotator).

The electrical field in the neutron star, in its magnetosphere (region of closed field lines) and in the radiating channel (region of open field lines) is determined. For radioradiation of pulsar an electrical field in the radiating channel is important

$$E_B \approx -\frac{\Omega B_s R^5}{cr^4}\cos\alpha \tag{39}$$

where B_S is magnetic induction near to a pole, Ω is angular velocity of rotation, R-radius of the star, α-the angle of inclination of the vector of magnetic moment of star relatively rotation axis, r-distance from center of a star. It is shown, that over a magnetic cap of a star the special region, named the magnetic funnel will be formed, where there are the vigorous processes of multiplication of high energy quants and electron-positron pairs. The altitude of the magnetic funnel over a magnetic cap $h\approx 8\cdot 10^6\Omega^{0.2}\mu_{30}^{1/3}R_6^{1/3}$cm, and it radius $r(\Omega r/c)^{0.5}$ slowly depends of inclination angle α,and μ is magnetic moment of star.It is shown, that radioradiation of pulsar is formed in magnetic funnel. Here during the active radiating processes two main flaws of high ultrarelativistic energy particles are formed: going upward a flow of electrons, and falling on magnetic cap of a star flow of positions. This main flows are accompanied by separate narrow strips of positrons and electrons streams rather small energy, being sufficient powerful coherent sources of radioradiation. These stripes of secondary particles streams are formed at once after the annihilation of quants of (curvative) radiation (emitting) by particles of main streams.

Estimation of pulsar radioluminosity is made

$$L \approx 7.4\cdot 10^{23}\Omega^{3.52}\mu_{30}^{8/3}\psi(\alpha) \tag{39}$$

where $\psi(\alpha)$ is known function, at $\alpha < 50^0$, $\psi(\alpha)\approx 1$. Equating the theoretical L_0 and the observable radioluminosity L_0, we are obtained

$$\mu = 10^{30}p^{1.32}R_6^{0.4}(2{,}1\cdot 10^{27}L_0/\psi)^{3/8} \tag{40}$$

for a magnetic moment of pulsar, p is period of pulsar. The magnetic moments of slow pulsars calculated by this formula expressed appreciably larger, than magnetic moments of fast pulsars. It means, that in average the mass of slow pulsars there are more of fast pulsars masses.

Magnetic funnel works with interruption, periodically experiencing discharge, hence the process of formation of pulsar radioradiation works with interruption. Duration of process of radioradiation on formation and interruption between these processes are order

$$\frac{h}{c} \approx 2.7\cdot 10^{-4}\Omega^{0.2}\mu_{30}^{1/3}s \tag{41}$$

i.e. radioimpulse of pulsar has microstructure. Hence the radiation of microstructure profiles of observed radioimpulses of pulsars will allow to obtain the additional valuable information about magnetic moments of neutron stars.

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Է.Վ.ՉՈՒԲԱՐՅԱՆ

Նեյտրոնային աստղերի ֆիզիկա

Աշխատանքը հանդիսանում է նեյտրոնային աստղերի ֆիզիկայի բնագավառում վերջին 30 տարում Երեւանի պետական համալսարանում կատարված աշխատանքների համառոտ ակնարկ: Մասնավորապես քննարկվում են հարցեր կապված 10^6 գ/սմ^3 եւ ավելի խտության դեպքում նյութի վիճակի, նյութի նեյտրոնացման եւ պիոնացման էֆեկտների հետ: Գտնված է այլասերված պլազմայի վիճակի հավասարումը, որոշվել են այլասերված աստղային կոնֆիգուրացիաների ինտեգրալ բնութագրիչները, նկատվել է գերխիտ աստղերում զանգվածի անոմալ դեֆեկտ, որը պայմանավորված է տարածության խիստ կորացման հետեւանքով ներքին էներգիայի ադիտիվության խախտման հետ: Դիտարկվել են նաեւ նեյտրոնային աստղերի արտաքին դրսեւորումներին առնչվող հետեւյալ հարցերը՝ բռնկումները բարստերներում եւ պուլսարների /բաբախիչների/ ռադիոճառագայթումը:

Академик НАН Армении Г.С.Саакян, Академик НАН Армении Э.В.Чубарян

ФИЗИКА НЕЙТРОННЫХ ЗВЕЗД

В работе приводится краткий обзор результатов, полученных в Ереванском университете по физике нейтронных звезд за прошедшие 30 лет. В частности обсуждаются вопросы, связанные с состоянием вещества при плотностях от 10^6 г/см^3 до сколь угодно больших, эффекты нейтронизации и пионизации вещества. Найдено уравнение состояния вырожденной плазмы, получены интегральные параметры вырожденных звездных конфигураций, обнаружен аномальный дефект массы в сверхплотных звездах, обусловленный нарушением аддитивности внутренней энергии, вызванной сильным искривлением пространства. Наконец, рассматриваются вопросы, относящиеся к внешним проявлениям нейтронных звезд: вспышки в барстерах, радиоизлучение пульсаров.

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ДОКЛАДЫ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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PHYSICS

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THE ROTATING SUPERDENSE CONFIGURATIONS.

From astrophysical observations it is possible to conclude, that the rotation of cosmical objects is general property of celestial bodies. The problem of rotation, as one of unsolved problems of astrophysics, represents purely theoretical interest. It was necessary to offer a technique for its solution, as within the framework of the Newton theories as and in the Einstein theory. The solution of this problem is would given possibility to estimate the angular velocity of rotation, to calculate the internal and integral characteristics of rotating superdense configurations, to define a role of rotation for stability and energetic reserves of celestial bodies. We shall notice also, that rotation of superdense configurations makes their "lively", hence more "observable" in comparison with a static configuration, the detection of which almost is impossible because of their small luminocity (small radiuses and low surface temperatures). If to take into account, that rotation of celestial bodies, must be accompanied by generation of strong magnetic fields, then the rotating magnetic star becomes not only source of electromagnetic waves, but and the generator of fast charged particles. These particles, moving in a magnetic field of a star, can transform rotation energy of star to electromagnetic radiation in a rather wide region of frequencies. As have shown the observations, such logic scheme has qualitatively justified self, as neutron stars were found out as the radio and optical telescopes (pulsars), by X-ray and γ-receivers (pulsing X-ray sources, bursters, sources of γ-radiation).

The problem of rotation of superdense configurations is so complicated, that the exact solution of this problem is impossible even within the framework of Newton theory ([1-5]). The account of effects the general theory of gravitation complicate much more the problem. It is enough to tell that there are not exact general axial-symmetric external solutions of the Einstein equations. Therefore does not surprised, that the methods of solution the problems of rotation arising in the last decades the are approximate [8-10]. Below we shall elucidate the basic ideas of a method of the superdense celestial bodies rotation problem solution and give comments some on our sight the important physical results [6-7].

The method is that the rotation is considered as perturbation to a static spherical problem. All unknown functions are expanded by nondimensional parameter $\beta=\Omega^2/8\pi G\rho_c$ where G is the gravitational constant, ρ_c is the density in the center of star, Ω is the angular velocity, coinciding with relation of rotation energy to gravitational energy, which for superdense configuration is much less than unit. A successful choice of parameter of the perturbation theory allows to be limited even by linear approximation on β, and as show calculations the received results are satisfactory in the whole range of angular velocities, since zero up to limiting value $\Omega_{max}=(GM/R_e^3)^{1/2}$ (here M is the mass of star, R_e - is its equatorial radius), determined from a condition of absence of the matter flow from equator. As show calculations within the framework of Newton theory, corrections, caused by the members of expansion β^2, in order to one percent, that means fast convergence of series [10]. It is natural to expect that the corrections in the general theory of relativity are the same order that testify about good convergence of expansion by β within the framework of Einstein theory. As we know in the USA (Cornell University) and in France (Medon observatory) a new methods of calculations rapidly rotating neutron stars on a supercomputer now was developed. The preliminary dates confirm our statement.

So, being limited linear approximation on β, we shall consider a problem of rotation within the framework of the Einstein theory [6] Four-dimensional interval in a case of axial symmetric mass distribution can be written down as

$$ds^2=e^{\nu}dt^2-e^{\lambda}dr^2-e^{\mu}[d\theta^2+\sin^2\theta(d\varphi+\omega dt)^2] \qquad (1)$$

System of units G=c=1 is here chosen. Unknown functions ν, λ, μ and ω depend on r, θ and parameter $\Omega=d\varphi/dt$.

Function ω is angular velocity of "entrainment" of freely falling system relatively motionless observer.

Hereinafter conveniently to use following combinations of Einstein equations

$$G_1^{\ 1}-G_0^{\ 0}=8\pi(T_1^{\ 1}-T_0^{\ 0}),\ G_2^{\ 2}+G_3^{\ 3}=8\pi(T_2^{\ 2}+T_3^{\ 3}),\ G_3^{\ 0}=8\pi T_3^{\ 0},\ G_2^{\ 1}=0 \qquad (2)$$

where

$$G_\iota^{\ \kappa}=R_\iota^{\ \kappa}-1/2\delta_\iota^{\ \kappa}R,\ T_\iota^{\ \kappa}=(\mathcal{P}+\rho)U^\kappa U_\iota+\mathcal{P}\delta_\iota^{\ \kappa},$$

Integral of hydrodynamic equilibrium equations $T_{\iota;\kappa}^{\ \kappa}=0$ looks like

$$M=2\int\frac{dP}{P+\rho}=2\ln U^0-\nu+const \qquad (3)$$

Here $\mathcal{P}$ is the pressure, ρ is the energy density of matter, and U^ι - four-dimensional velocity of matter. The system equations and state equation of the degeneracy matter $\mathcal{P}=\mathcal{P}(\rho)$ represent closed system of equations for a problem of rotation. We shall note also, that at a movement of matter, at given number of baryons N= $\int d^3x\sqrt{-g}nU^0$ and fixed moment of inertia $I_z=\int d^3x\sqrt{-g}\,T_3^{\ 0}$ we have $U^3=\Omega U^0$ and

$$U^0=[e^{\nu}-e^{\mu}(\omega+\Omega)^2\sin^2\theta]^{-1/2} \qquad (4)$$

From invariance under transformation t → -t, at which the angular velocity changes a sign, follows, that nondiagonal components of metric tensor should be odd, diagonal components - even functions on Ω. As to pressure and mass density, it is obvious, that they are even functions of angular velocity of rotation. Hence follows, that the expansion of unknown functions of small parameter β, can be presented as

$$\omega=\sqrt{\beta}\, q\,,\ e^{-\lambda}=e^{-\lambda 0}\,(1+\beta f),\ e^{\nu}=e^{\nu 0}(1+\beta\Phi),\ e^{-\mu}=R^2\,(1+\beta U),$$
$$\mathcal{P}=\mathcal{P}_0+\beta\mathcal{P}_1,\ \rho=\rho_0+\beta\,\rho_1,\ M=m+\beta N \tag{5}$$

where q, Φ, u, $\mathcal{P}_1$, ρ_1 and N are unknown functions from r and θ, and $e^{\lambda 0}$, $e^{\nu 0}$, $\mathcal{P}_0$, ρ_0 and m are the components of metric tensor, pressure and density of the appropriate spherical problem.

The system of the equations obtaining after substitution (5) in (2) and (3) admits separation of variables. Indeed, the angular operator, included in these equations satisfies to a condition

$$\frac{\partial}{\partial\gamma}\left[\left(1-\gamma^2\right)\frac{\partial}{\partial\gamma}P_l(\gamma)\right]=-\frac{l(l+1)}{r^2}P_l(\gamma)\ ,\ \gamma=\cos\theta,$$

that allows to search solution all unknown functions as expansions by Legandre polinom

$$\Psi(r,\theta)=\sum_{l=0}^{\infty}\Psi_e(r)P_e(\cos\theta)\cdot \tag{6}$$

In result system of the ordinary differential equations for unknown radial functions $\Phi_l(r)$, $U_l(r)$, $f_l(r)$, $N_l(r)$ and $q_l(r)$ as outside of mass distribution as and inside of configurations are obtained. For a finding of the internal solutions and integral characteristics of rotating configurations, knowledge of the general solutions of the equations obtained by us for radial functions in outside of mass distribution is necessary, i.e. at $\mathcal{P}=\rho=0$. It is wonderful, that the equations obtained by us allow the general analytical solutions as a series of hypergeometric functions in terms of r_g/r, where r_g is gravitational radius of static configuration. The equations inside of mass distribution have not the analytical solutions and them we can integrate only numerically, if unknown functions and their derivatives in the center of configuration are given. One of these conditions be found from behavior unknown functions at r→0, and other is determined together with constants entering in the external solution from joint conditions on a surface of a star,

$$R(\theta)=R_0+\beta\sum_{l=0}^{\infty}d_eP_e(\cos\theta). \tag{7}$$

The joint conditions require a continuity of unknown functions and their derivatives on R = R(θ). If to them to add a condition ρ=0 on R=R(θ),will be simultaneously determined and constants d_e, i.e. the equation of a surface of a star.

Here we don't reduce the equations for unknown functions, the external general solutions and formulas, connecting the integral characteristics of a star with results, of the numerical integration, shall only note, that constants, entering in the external solution are expressed trough values of specially chosen functions in a point r =R_0, where R_0 is radius of static configuration. These functions chosen so, that they satisfy to the same equations, as unknown functions, but already with the given initial conditions in a point r =0, as allows to perform the numerical integration and to define all integral characteristics of a star. The possibility of introduction of such functions is connected with linearity of equations comparatively of unknown functions.

We shall pass now to the discussion of results obtained from numerical integration. Results of numerical accounts made for central densities in an interval $1.7 \bullet 10^6$ g/cm^3<ρ<∞ are the internal and integral characteristics of white dwarfs and baryonic stars, state equation of which are taken from work ([11]). For these central densities the dependencies of metric coefficients, and also density and pressure of matter from coordinates are found. The behavior of these functions mainly same as in the static configurations, only occurs additional dependence from polar angle θ. In particular, the metric coefficient g_{03}, which is absent in a static configurations, grows inside of configuration at approach to the center of stars. The calculation of g_{03} gives the quantitative description of Lense-Tirring effect, according to which freely falling body entrained by rotation of a star. Limiting the discussion of behavior of the internal characteristics with remarks made, we shall stop in more detail on the analysis of the integral characteristics of superdense configurations.

The maximum angular velocities, which are determined from a condition of equality of centrifugal forces with gravitational forces on equator of a star, for baryonic stars are greater than for white dwarfs. With increase of central density of baryonic stars the maximum angular velocity grows from value 50s^{-1} up to $1.6 \bullet 10^4$ s^{-1}. Despite such huge angular velocities (baryonic stars the most rapid rotating objects our Galaxy) their average deformations do not exceed 10% of static configurations radius. The section, perpendicular to rotation axis represents a figure like ellipse. Depending on value of central density (at N=const) some configurations inflated so, a polar R_p and equatorial become more of corresponding static R_0, others flatten in a direction of an axis of rotation and inflated in a direction of equator, and some are compressed in all directions (in comparison with R_0) The parameter determining deformation of a star from spherical is eccentricity $e=(1-R_p / R_e)^{1/2}$, it weekly depends on the central density and at $\Omega=\Omega_{max}$ the order 0.7.

Others important integral characteristics are relativistic moment of inertia and quadruple moments of baryonic stars. As show accounts, if the moment of inertia changes from value $1,6 \bullet 10^{44}$ gcm^2 for $\rho_c=1,8 \bullet 10^{14}$gcm^{-3} up to value $1,15 \bullet 10^{45}$ gcm^2 for ρ_e $=1,44 \bullet 10^{15}$ gcm^{-3}, the quadruple moment for the same central density varies from value $2,3 \bullet 10^{43}$ gcm^2 up to $3,65 \bullet 10^{44}$ gcm^2. The both quantity grow with growth of central density. In spite of that with growth of central density the average sizes of a star decrease, but the growth of mass and deformation of a star brings to increase as a moment of inertia, as a quadruple moment of a star.

The most important integral characteristics of baryonic stars is their mass. Results of accounts allow to find dependence of masses rotating baryonic stars from central density and angular velocity of a star. For all configurations with the same ρ_c rotation brings to increase of mass. Therefore the mass grows proportionally to a square of angular velocity. The relative increase of mass configurations, rotating with the maximum angular velocity, depends on central density so, in a maximum of mass of white dwarfs, it compose 7% and near to a maximum of mass of baryonic stars it equal 14% of mass of static configurations.

By other important integral characteristic of superdense configurations is absolute gravitational defect of mass, determined by relation:

$$\Delta M = mN - M,$$

where m-baryonic mass, N-number of baryons, and M - is the mass of configuration. The quantity ΔM actually is energy, which would be allocated at formation a dense star from rarefied gas. The dependence of packing fraction $\alpha = \Delta M/mN$ from number of baryons N and angular velocity were calculated. The packing fraction α at $\Omega=0$ always is more from α at $\Omega=\Omega_{max}$ for all N. It means, that state of rotation for star in certain sense metastable and the transition to small angular velocities will be accompanied by allocation of energy. A difference of packing fraction $\Delta\alpha$ for a rotating and static configuration with fixed value of whole number of baryons $N=8\bullet10^{56}$ is about $\alpha=0.01$.

It means, that rotating star in comparison with static has a store of energy as energy of rotation and deformation of the order 10^{52} erg. Such energetic reserves are sufficient for supply of radiation of compact galactic objects.

In the last decade the interest to the problems of rotation of the celestial bodies increased connected with the discovery of the millisecond pulsars and observations of the period irregularities of the radio pulsars.

To calculate of the integral parameters of the rapidly rotating superdense stars (millisecond pulsars) there have been suggested special code for integration of the system of the differential equations. The calculation has been done for the arbitrary value of the angular velocity ([12]). For the investigation of the pulsar period irregularities it is necessary note, that the neutron star consist from two components: normal and superfluid. The differential equations describing the rotation of the stars have been written in the frame of Newtonian theory and their solutions have been applied for construction of the theory of pulsar and postjump relaxation. The comparison of the theory with observations gives good qualitative agreement.

The system of equations describing the rotation of the two components model of the neutron star have been considered in the paper ([13]) in the frame of Einstein theory of gravitation. Investigation of the solutions of these equations and its applications to the theory of angular evolution and theory of glitches and postjump relaxation have been just started.

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Պտտվող գերխիտ կոնֆիգուրացիաներ

Էյնշտեյնի տեսության շրջանակներում դիտարկվել է պտտման խնդիրը ըստ պտտման անկյունային արագության քառակուսային մոտավորության շրջանակներում: Ստացվել են Էյնշտեյնի հավասարումները և կառուցվել նրա լուծումները՝ ինչպես զանգվածների բաշխումից դուրս, այնպես էլ՝ ներսում: Որոշել են պտտվող կոնֆիգուրացիաների ինտեգրալ բնութագրիչները:

Д.М. Седракян, Э.В. Чубарян

Вращающиеся сверхплотные конфигурации

Рассмотрена задача вращения в рамках теории Эйнштейна в квадратичном по угловой скорости вращения приближении. Получены уравнения Эйнштейна и найдены их решения как вне, так и внутри распределения масс. Определены интегральные характеристики вращающихся конфигураций.

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Application of Ambartsumian Principle of Invariance to Problems of Radiation Transfer in Solids

(Submitted 3 July 1998)

The theory of radiative transfer ([1-3]) developed for multiple scattering of light in gaseous media and the scattering of neutrons may be applied for the solution of a member of new problems of transfer in solids. The present work is a brief review of works carried out in IAPP, NAS, RA. The problem of γ-radiation in crystals containing Mössbauer isotopes is an important example of the aforesaid. Of great interest in the gamma resonance spectroscopy are angular and frequency distribution of the of γ-radiation scattered in the crystal that gives information about the properties and structure of the medium.

The Mössbauer γ-quantum noncoherent scattering has been theoretically analysed in many papers ([4-7]). But they take into account only a single quantum scattering which is true either for a thin scatterer ($\tau_0<<1$, τ_0 is the scattering layer effective thickness) or for scattering with a small probability of re-emission $\lambda=1/(1+\alpha)$ (α is the inner conversion coefficient). But in the experimental particle we often have to deal with thick samples $\tau_0>1$ and Mössbauer nuclei with a small conversion coefficient in which the single scattering approximation is a priori wrong.

The first attempt to take the multiple scattering into account in a one-dimensional medium was made in ([8]) where the theory of the optical radiation transfer was applied to solve the problem of the γ-radiation transfer in isotropic media containing Mössbauer nuclei.

The intensity of radiation at multiple isotropic scattering with complete redistribution in frequencies on crystals of finite thickness is determined. It is shown that the width of resonance line in the energy spectrum of γ-radiation increases as the thickness of scatterer.

For homogeneous semi-infinite media the problem of radiation transfer in the simplest models of a scattering act, i.e., the monochromatic scattering with complete redistribution in frequencies (in the isotropic case) admits exact analytical solutions ([1,2,9,10]).

In Refs ([11-12]) approximate analytical solutions have been obtained for general problems of radiation transfer in a layer of finite thickness in case of complete redistribution in frequencies inside the spectral line (both in the one-dimensional approximation and in the three-dimensional case). The accuracy of these solutions increases with increasing layer thicknesses. The calculations were carried out for the case of isotropic scattering. Using the method of ([14-16]) that permits to solve the transfer problem in the slab of finite thickness with the help of the Ambartsumian function for half-space. The Ambartsumian functions φ and ψ were calculated for cases of Doppler and Lorentz profiles of the coefficient. In the three-dimensional case these functions have the following form([12]):

$$\varphi(\tau_0,z)=\varphi(z)-\frac{2}{\lambda}\cdot\frac{\widetilde{F}(\tau_0,z)}{\varphi(z)}\beta(\tau_0,z)\left(1-\frac{\lambda}{2}\varphi_0\right)-\frac{\lambda}{2}\varphi(z)\widetilde{F}(\tau_0,z)\psi_0,$$
$$\psi(\tau_0,z)=\varphi(z)\widetilde{F}(\tau_0,z)\left(1-\frac{\lambda}{2}\varphi_0\right)+\frac{2}{\lambda}\cdot\frac{\beta(\tau_0,z)}{\varphi(z)}\left(1-\frac{\lambda}{2}\psi_0\widetilde{F}(\tau_0,z)\right), \quad (1)$$

On the basis of results ([11-13]), in Refs ([17,18]) the problems of γ-radiation transfer were studied in cases of both isotropic and anisotropic scattering with complete redistribution in frequencies. The problems of diffuse reflectectia and transmission of Mössbauer radiation were solved.

In ([17]) deals with the problem of the γ-quantum transfer in the three-dimensional case of plane-parallel layers taking account of the multiple scattering at various elementary processes of scattering.

In the media containing Mössbauer nuclei there are two main competing channels scattering, the nuclear resonance scattering and the electron scattering ([6]) (Rayleigh and Compton scattering). However, since the cross-sections of Rayleigh and Compton scattering are much less than that of the resonance scattering, their contributions are not taken into account.

In the case of the nuclear resonance scattering we deal with nonconservative scattering, i.e., $\lambda<1$.

Another important feature of the scattering process is the probability $g(x_1, x_2)$ that after absorbing a quantum with the frequency x_1 the nucleus will emit a quantum with the frequency x_2, where $x=(\omega-\omega_0)/(\Gamma_a/2)$ is the deviation of the γ-quantum frequency ω from the resonance frequency ω_0 in units of the half-width of the nuclear excited state $\Gamma_a/2$. In the first part of ([17]) paper deal with the model already proposed in ([4-5]) which assumes that $x_1=x_2$ (monochromatic scattering), i.e., $g(x_1, x_2)$ is a δ-function $g(x_1,x_2)=\delta(x_1-x_2)$. The second part is devoted to the model of complete frequency redistribution $(g(x_1,x_2)=(1/\pi)\alpha(x_2))$, corresponding to the case when the nucleus forgets the absorbed quantum frequency (a nucleus "without memory").

The resonance scattering angular distribution, which is non-spherical in the general case due to the crystal structure and its physical properties, is another characteristic quantity of the elementary process of scattering. Below consider the case of an isotropic medium corresponding to the case of a spherical distribution $x(\gamma)=1$.

In addition to these assumptions, using the so-called probability method ([9]) for calculating the intensity $I_\eta^+(\upsilon)$ of the transmitted resonant γ-quantum beam and for the intensity $I_\eta^-(\upsilon)$ of the reflected one, we have

$$I_\eta^+(\upsilon)=\frac{1}{\eta}\int_0^{\tau_0}d\tau\int_{-\infty}^{\infty}p(\tau_0-\tau,\eta,x_1,x_2,\upsilon)dx_1dx_2\,,$$
$$I_\eta^-(\upsilon)=\frac{1}{\eta}\int_0^{\tau_0}d\tau\int_{-\infty}^{\infty}p(\tau,\eta,x_1,x_2)L(\tau,x_1,\upsilon)dx_1dx_2\,, \quad (2)$$

where $p(\tau,\eta,x_1,x_2)dx_2$ is the probability that the quantum with frequency x_1 absorbed in the effective thickness τ leaves the medium with the frequency in the range (x_2, x_2+dx_2), $L(\tau,x_1,\upsilon)dx_1$ is the energy absorbed in the medium per second, υ is the source velocity, and $\eta=\cos\vartheta$.

It should be noted that in our treatment the radiation sources may be found both inside and outside the scattering medium.

We discuss the case when the γ-radiation incident on the absorber at an angle ϑ_0 has the intensity $I(x_1, \upsilon)=I_0\alpha_0(x_1-\upsilon)$; then the energy absorbed at the depth τ from the surface may be expressed by

$$L(\tau,x_1,\upsilon)=I_0\alpha_0(x_1-\upsilon)\alpha(x_1)e^{-\alpha(x_1)\tau/\eta_0}, \tag{3}$$

where $\alpha_0(x)$ is the radiation line-shape and $\alpha(x)$ the absorption lineshape (both lines are assumed to be Lorentzians $\alpha(x)=\alpha_0(x)=1/(x^2+1)$), $\eta_0=\cos\vartheta_0$, and πI_0 is the resonant quantum complete intensity.

For the probability p it is easy to find a certain integral relation. Actually, the emission probability of a quantum absorbed at the depth τ is $(\lambda/4\pi)g(x_1, x_2)$, while the probability that a quantum leaves the medium with the frequency x_2 at the angle ϑ after multiple scattering is $p(\tau,\eta,x_1,x_2)$. But the quantum may also leave the medium without scattering, the probability being $(1/4\pi)e^{-\alpha(x_2)\tau/\eta}$, or it may first be absorbed at the point τ', then re-emitted and leave the medium, the probability being the same p. Summing over all those points τ' we obtain

$$p(\tau,\eta,x_1,x_2)=\frac{\lambda}{4\pi}g(x_1,x_2)e^{-\alpha(x_2)\tau/\eta}+\frac{\lambda}{2}\int_{-\infty}^{\infty}g(x_1,x)\alpha(x)dx\times$$

$$\times\int_0^{\tau_0}p(\tau',\eta,x_1,x_2)d\tau'\int_0^1 e^{-\alpha(x)|\tau-\tau'|/\eta'}\frac{d\eta'}{\eta'}. \tag{4}$$

In ([17]) the solution of this integral equation is discussed for two (monochromatic, complete frequency redistribution) cases of the function $g(x_1, x_2)$.

In the general case, however, the scattering distribution is nonspherical and, consequently, the problem must be solved taking into account the anisotropy of scattering. In ([18]) is devoted to anisotropic incoherent scattering of Mössbauer radiation taking account of its multiple interaction with the nuclei. It should be noted that in ([5,6]) only single scattering has been considered, which obviously leads to wrong results.

In the ([18]) paper is also devoted to the study of the frequency-modulated γ-radiation scattering in a plane parallel layer.

The angular distribution of the nuclear resonance scattering of γ-quanta (the scattering indicatrix) described by the general theory of angular correlations ([19,20]) is only of the main features of the elementary scattering process. In case of quadrupole interaction between the nuclei and crystalline fields in a polycrystalline medium the angular distribution is written as

$$\chi(\theta)=\sum_{i=0}^{n}A_iC_iP_i(\cos\theta) \tag{5}$$

where A_i are constants, C_i the coefficients of the angular distribution attenuation, and $P_i(\cos\theta)$ Legendre polynomials.

For the spin transition 1/2→3/2→1/2 (L = 1, 2) the angular distribution has a relatively simple form,

$$\chi(\theta) = 1 + \frac{C_2}{4} P_2(\cos\theta) \tag{6}$$

where the coefficient C_2 changes from 0.2 (for strong quadrupole or magnetic interaction) up to 1 (for weak quadrupole or magnetic interaction).

The anisotropy of the factor f_a ([22]) is also taken into account in ([21]) and it is shown that in this case the value C_2 is found in the same range.

Thus, the anisotropic scattering of Mössbauer radiation in finite layers will be considered in the indicatrix approximation (5).

In calculations the Rayleigh channel of γ-quantum scattering is neglected. The terminology and definitions are taken from ([8]).

Let us introduce $p(\tau, x_1, x_2, \eta', \eta, \varphi) d\Omega dx_2$ denoting the probability that a quantum with the frequency x_1 (in units Γ/2), incident at an angle arccos η′ to the external normal of the medium surface τ = 0 and absorbed at the optical depth τ, will emerge from the medium through the surface τ = 0 at the angle arccos η to the normal, at the azimuth φ within the solid angle dθ, and frequency in the range x_2, $x_2 + dx_2$. Taking into account multiple scattering we have the equation for the quantum emergence probability from the medium (see e.g. ([23]))

$$p(\tau, x_1, x_2, \eta_0, \eta, \varphi) = \frac{\lambda}{4\pi} g(x_1, x_2) \chi(\gamma) e^{-\alpha(x)\tau/\eta} +$$

$$+ \frac{\lambda}{4\pi} \int_{-\infty}^{\infty} dx \int_0^{2\pi} d\varphi' \left[\int_0^1 \chi(\gamma') \frac{d\eta'}{\eta'} \int_0^{\tau} e^{-\alpha(x)(\tau-\tau')/\eta'} g(x_1, x) \alpha(x) p(\tau', x, x_2, \eta', \eta, \varphi - \varphi') d\tau' - \right.$$

$$\left. - \int_{-1}^{0} \chi(\gamma') \frac{d\eta'}{\eta'} \int_{\tau}^{\tau_0} e^{-\alpha(x)(\tau-\tau')/\eta'} g(x_1, x) \alpha(x) p(\tau', x, x_2, \eta', \eta, \varphi - \varphi') d\tau' \right]$$

where γ and γ′ are the scattering angles, $g(x_1, x_2)$ is the probability of the frequency redistribution (see ([24]), λ the probability for the quantum to survive in the process of interaction with the nucleus, and α(x) the line contour ($\alpha(x) = 1/x^2 + 1$).

Further simplification of this equation is connected with the substitution of physical models for the scattering indicatrix χ(γ) and the probability of the frequency redistribution $g(x_1, x_2)$. The scattering indicatrix may be always expanded by Legendre polynomials (see ([8])), therefore, the property of these polynomials may be applied to further calculations.

For the frequency redistribution probability there may evidently exist two models: the model of monochromatic scattering and the model of complete frequency redistribution .

The fact that the redistribution function $g(x_1, x_2)$ does not depend on the absorbed quantum frequency simplifies the equation for the quantum emergence probability which now will not depend on the absorbed quantum frequency x_1.

In the Mössbauer spectroscopy the scattering given the intensities of the radiation emitted from the medium through the surfaces τ = 0 and τ = τ_0 depending on the source velocity v (in the units Γ/2).

$$I_{\eta}^{-}(\nu)=\frac{1}{2\pi\eta}\int_{0}^{2\pi}d\varphi\int_{-\infty}^{\infty}dx_2\int_{0}^{\tau_0}p(\tau,x_2,\eta_0,\eta,\varphi)L(\tau,\nu)d\tau,$$

$$I_{\eta}^{+}(\nu)=\frac{1}{2\pi\eta}\int_{0}^{2\pi}d\varphi\int_{-\infty}^{\infty}dx_2\int_{0}^{\tau_0}p(\tau_0-\tau,x_2,\eta_0,\eta,\varphi)L(\tau,\nu)d\tau, \tag{8}$$

where $L(\tau, \nu)$ is the integral intensity of the γ-quanta absorbed at the optical depth τ,

$$L(\tau,\nu)=\int_{-\infty}^{\infty}\alpha_0(x_1-\nu)\alpha(x_1)e^{-\alpha(x_1)\tau/\eta_0}dx_1, \tag{9}$$

where $\alpha_0(x)$ is the shape of the radiation line.

Therefore, to calculate the intensities $I^{-}(\nu)$ and $I^{+}(\nu)$ it is necessary to fined the probability $p(\tau, x_2, \eta_0, \eta, \varphi)$. On the other hand, this problem is of independent interest, particularly in the theory of the optical radiation transfer. In ([18]) taking account of the specific nature of the Mössbauer measurements and further considerations are similar to those in ([25,26])

In the ([18]) paper is also devoted to the study of the frequency-modulated γ-radiation scattering in a plane parallel layer.

. In this case also have to deal with another type of Mössbauer spectroscopy, namely with the modulation Mössbauer spectroscopy for scattering, which may suggest new possibilities for the investigation of physical and chemical properties of condensed media.

All the results given above are also valid for this case. The only difference is the incident radiation intensity. If the Mössbauer radiation in the source is frequency modulated by acoustic oscillations, then the lineshape of the incident radiation has the form ([26,27]).

$$\alpha_0(x-\nu)=\sum_{n=-\infty}^{\infty}\frac{I_n^2(\alpha)}{(x-\nu-n\Omega)^2+1}, \tag{10}$$

where $I_n(\alpha)$ is the Bessel function, $\alpha = A/\lambda$ the modulation index, A and Ω are amplitude and frequency of ultrasonic oscillations (in units $\Gamma/2$), and λ is the given γ-quantum wavelength.

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Հայաստանի ԳԱԱ ակադեմիկոս Ա. Ռ. ՄԿՐՏՉՅԱՆ, Խ. Վ. ՔՈԹԱՆՋՅԱՆ

Համբարձումյանի ինվարիանտության սկզբունքի կիրառումը պինդ մարմնում ճառագայթման տեղափոխման խնդիրներում

Աշխատանքը հանդիսանում է Համբարձումյանի ինվարիանտության սկզբունքի կիրառմամբ, պինդ մարմնում ճառագայթման տեղափոխման խնդիրների ուղղությամբ ՀՀ ԳԱԱ ՖԿԳԻ-ում կատարված աշխատանքների համառոտ ակնարկ:

Академик НАН Армении А. Р. МКРТЧЯН, Х. В. КОТАНДЖЯН

Применение принципа инвариантности Амбарцумяна к задачам переноса излучения в твердых телах

Работа представляет собой краткий обзор исследований, выполняемых в ИППФ НАН РА по проблеме переноса излучения в твердом теле с применением принципа инвариантности Амбарцумяна.

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ДОКЛАДЫ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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ASTROPHYSICS

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On the Ambartsumian's Concept on Activity of Nuclei of Galaxies

(Submitted 13/VII 1998)

This year is special year not only for the scientiest of Armenia, but also for the all astronomical community over the world. 90 years ago on the Armenian sky new star was borne named Victor. I do not like compare it with flare of Nova or Supernova stars, because their enormous brightness quickly decrease during some monthes or year. I prefer to compare it with flare of FU Ori type stars, Ambartsumian called them "Fuors", because they keep the activity and high brightness after flare constant during many decades of years, as Victor Ambartsumian did.

All his long life he devoted studing and explaining the mysterious physical phenomena, which take place in Universe. Infriquent are the cases in the history of science when during his lifetime a scientist advences one after another ideas that are basically at variance with established scientific consepts and traditions. Still rarer are the cases when those ideas thrust their way through the endless zigzags of science and prove there viablity in the course of its development. Victor Ambartsumian belongs no doubd to this class of scientists. His ideas on the decay of stellar clusters, on the statistics of double stars and the age of the Galaxy, on stellar associations, explosive processes connected with flare stars and fuors and, finally, on the activity of nuclei of galaxies were as unexpected in those days as original and served the cause of heated arguments and discussions. Often they even seemed unreal. However most of them gradually won recognition through the world. In this paper we shoud like to touch briefly upon some aspects of V.A.Ambartsumian's idea on the optical activity of nuclei of galaxies.

Some 45 or 50 years ago the galaxies were conceived of as thoroughly formed steady systems with a rich past and with no prospect of radical changes in the future. Therefore the investigation of the structure of galaxies was in most cases confined to their classification and general photometry, based solely on their external morphological characteristics, setting little store by the composition of their central regions. V.A.Ambartsumian was the first to pay particular attention to the significance of galactic nuclei which form a typical particular of nearly all galaxies with high luminosity. This wonderful peculiarity of galaxies had for long been neglected by astronomers. The role of nuclei in the evolution of galaxies was manifestly underestimated.

In the second half of the 50's a new phase in extragalactic exploration set in, when a new concept on the basic role of the nuclei of galaxies in their life and evolution had been advanced by V.Ambartsumian ([1-9]). Of course the origin of this concept was not unfounded. It was preceded by a number of wonderful discoveries resulting in the revision of our notions on the world of galaxies. In the first place this was the identification of one of the powerful radio source Cygnus A by Baade and Minkowski with the galaxy cotaining two nuclei ([10]). A similar picture was observed also in radiosource Perseus A (NGC 1275). The role of two papers was also significant: Haro ([11]) in 1956 discovered 44 galaxies, unusually blue in colour; and, specially, in 1943 Seyfert's paper ([12]), now regarded as classical, shoud be here singled out for mention. The galaxies which he investigated are distinuished by the high luminosity of their nuclei, and, more importently, by the width of the Balmer emission lines. The great width of the emission lines indicates that the turbulent motions of gas clouds in the nuclei of those galaxies, subsequently termed "Seyfert",at times attain a velocity of over 3000 km/s. Now it seems quite strange and surprising that this very important paper of Seyfert was not duly taken into account in the succeeding twenty years or so. It was only after Ambartsumian's idea concerning the activity of nuclei of galaxies had been made public that the astrophysicists returned to that paper, and a regular study of the Seyfert galaxies was started.

On the basis of the analysing of these facts, Ambartsumian came to idea of activity of nuclei of galaxies,which manifests itself mainly in the following forms :

1. Outflow of ordinary gas matter (in form of jets or clouds) from the nuclear region at the velocity of up to hundreds of kilometers per second).
2. Continuous emission of the flux of relativistic particles or other agents, producing high energy particles, as a result of which a radio halo may form around the nucleus.
3. Eruptive ejections of gas matter (M 82 type).
4. Eruptive ejections of concentration of relativistic plasma (NGC4486, 5128, etc.).
5. Ejection of compact blue condensations with an absolute magnitude of the order of luminosity of dwarf galaxies (NGC 3561, IC 1182). Here the division of the nucleus into two or more comparable components is also presumed, initiating the formation of multiple galaxies.

The presence of one or several of these phenomena allows us to call a galaxy active. At present a number of types of objects are considered as active: radio galaxies, QSOs, Seyfert galaxies, Lacertides, UV-excess galaxies, blasars,liners.

Let us come back to the phenomenon of radiogalaxies. Above mentioned well-known american astronomers Baade and Minkowski explained this phenomenon as a result of accidental collision of two galaxies. V.A.Ambartsumian was the first and only astronomer, who in his works ([2-4,7,8]), convincing shown, that in the case of radio-galaxies we have not collision, but just activity of nuclei of galaxies, which brings to ejection of matter from necleus, and in some cases as a result of this activity the radio-galaxy is originated.

I am lucky, that I have had chance in 1961 in Berkeley to be present at XInd General Assambley of IAU, were V.Ambartsumian gave a puplic talk on the idea of activity of galaxies. The interese to his talk was so high, that the conference hall and corridors were full by peoples (even outside, where the microphones have been mounted). During this Assembley he was elected as a President of IAU.

It was really revolutionary and extraordinary idea. It is said, that in Solvey conference in 1958 W.Baade even accused him of idealism and noted that for the scietist from Sovet Union speak about ejection and activity of nuclei of galaxies looks very strange.But just couple of years later american astronomers Sandage and Lynds ([13]) published very important paper under the telling title "Evidence of an explosion in the center of galaxy M 82",where have been shown, that some million years ago in galaxy M 82 a very powerfool explosition have been taken place with ejection of enormous mass: equel to about million of Solar masses.

Almost the same time in 1963 QSO-s have been discoverd. Actually, as Ambartsumian noted,they were just the naked active nuclei,which radiate unusual high quantity of energy, highest amongs known cosmical objects. Now we know that many of QSO-s realy surrounded by stellar population, which are called "Host galaxies".

Thus, the Ambartsumian's idea has been confirmed at first by observations of american astronomers.

The new stage in extragalactic field has been started: the era of active galaxies. Majority of large observatories over the world started to find a new active galaxies (AG).

By the leading of V.Ambartsumian the Byurakan observatory also engage itself in activities aimed at discovering galaxies with active nuclei. Ambartsumian and Shahbazian ([14]) were the first to show the existence of blue ejections and condensations associated with contiguous active elliptic galaxies. Subsequently Stockton ([15]) showed that those objects are in fact associated with galaxies and display emission spectra similar to the associations.Then on the initiative of V.A.Ambartsumian,B.E.Markarian started in Byurakan in the mid-sixties observations of the sky with a view to detecting galaxies with anomalous spectra, using the 40" Schmidt telescope with an objective prism of the same diameter.

The first Byurakan survey (FBS) is the most famous work done with this telescope. More than 2000 photographic plates, cover about 17000 square degrees of sky, were obtained.Each plate contains low dispersion spectra (2500A/mm near H-beta) more than 15 000 objects. As a result, 1 500 galaxies with strong UV-excess have been discovered. In 1978 Markarian with his co-workers began Second Byurakan Survey (SBS). The limiting magnitude of objecst have increased from 17^m (for FBS) up to about $19^m.5$ (for SBS). SBS covers about 1 000 square degrees.

But small dispersion of spectra on the Schmidt plates did not give us the possibility to understand in detail the physical structure of this galaxies with strong UV-excess. It was necessary to observe them with the high dispersion slit-spectrograph, which that time not possible in Byurakan observatory. In 1967-69 I was lucky to be first to observe almost all galaxies from the first Markarian list of UV-galaxies with the largest optical telescopes of USA.

I would like to emphasize once more that the detailed spectral investigations of these objects indicated ([16-18]), that over 85% of them turned out to have emission lines, their intensity being directly dependent on the value of UV-excess. One can conclude that the presence of a strong ultraviolet continuum is closely associated with the formation of the emission spectrum and the more intense the continuous spectrum in the visible ultraviolet is, the more intense are the emission lines.

It became also evident that the spectra of those objects differ, nevertheless, essentially from each other as to the excitation degree of the emission lines and their widths. Moreover, they turned out to differ sharply in morphological characteristics as well: one can come across the blue galaxies of Haro, the compact galaxies of Zwicky, the N type galaxies, spiral and irregular galaxies among the Markarian objects. Quite important is the discovery of the Seyfert galaxies and quasers among those objects.

As far back as 1968 I also demonstrated that on the basis of slitspectra, UV-galaxies can be classified in five groups ([16]):

1. Narrow line spectra both in emission and absorption.
2. Narrow, strong emission lines only.
3. Strong and diffuse emission lines; [0III] lines much stronger than the hydrogen lines (Seyfert type 2).
4. Very broad hydrogen lines,narrow forbidden lines (Seyfert type 1).
5. No strong emission lines (BL Lac).

No new type of spectra of UV-galaxies has since been observed, except for galaxies with pure absorption line spectra ([23,24]).

These results have been presented at first international conference on Seyfert galaxies and related objects in 1968 (Tuson,Arizona,USA). Here I called these objects "Markarian galaxies". The further spectral investigations of Markarian galaxies from both Byurakan Surveys have been carried out intensively in Byurakan (M.Arakelian, A.Petrosian, K.Sahakian, J.Stepanian,H.Abrahamian, S.Hakopian, V.Chavushian, L.Erastova, A.Yegiazarian, N.Andreasian and others).

These searches show that among Markarian objects there are representives of all formes of activity predicted by V.Ambartsumian: QSO-s, Seyfert galaxies, BL Lac objects, galaxies with jets, blue compact galaxies, dauble nuclei galaxies and so on.

M.Kazarian, using the same methode of observation found more than 600 new UV-excess galaxies also showing different form of activity. But the most important is, as it was shown by Weedman and Khachikian, that 10% of Markarian galaxies turned out to be Seyfert galaxies.

The number of Seyfert type galaxies was extremely increased thank to study of Markarian objects. In the original paper of K.Seyfert there are only 6 galaxies of Seyfert type. But now more than 1 000 these type of galaxies are known!

On the base of detailed spectral investigations of namber of Seyfert type galaxies Weedman and Khachikian ([20]) have shown, that Seyfert galaxies dearly are divided to two types:

1.Galaxies with very broad hydrogen lines, and narrow forbidden lines (Seyfert type 1).

2.Galaxies with very broad both hydrogen and forbidden lines (Seyfert type 2). This classification of Syefert galaxies is generally accepted in scientific literature and is included in "Glossary of Astronomy and Astrophysics" (with foreword by Nobel Prize S.Chadrasekhar). There are many astrophysical objects and events have been studied in Byurakan observatory and in abroad speaking in favour of Ambartsumian idea.

I would like to dwell here upon two subjects: a) double nuclei AG, and b) variability in the spectrum of AG.

a) As it was mentioned above, the radio galaxy Cygnus A has two nuclei. It is interesting to note that majority of active galaxies (AG) turn out to be double nuclei. It is necessary to stress that in addition to double nuclei there are galaxies with three and more nuclei(or nuclear type formations). It is known also that each of the nuclei of double ncleus galaxies can themselves consist of two components. Therefore the opinion conserning the nature of double nucleus AG are relevant to the multinuclei AG as well. It seems unimportant to use the term "multinuclear AG" or to say "central part of AG cosists of number of condensations". The terminology is not important, because I believe that these objects have been formed as a result of division of single maternal body.

From the time of Kant and Laplace up to the present, the majority of heorists,as well as observers, believe that the Universe develops in a direction from concentrations of diffuse matter to the denser states. Perhaps V.Ambartsumian was the first who declared the opposite point of view. As far back as the end of the 40's, he stated the revolutionary idea that evolution in the Universe goes from the dense condition of matter to the rarefied one.

Unfortunately, very few scientists are attempting the construction of a physical theory for this concept, although there are fairly successful attempts in this field.It seems to me that observational data speaks in favour of this point of view. The existence of double and multinuclei galaxies is the good confirmation of this idea.

The number of double and multi-nucleus AG is increasing all the time. Zwicky compact galaxies with emission spectrum, many radio galaxies, so- called isolated gaint HII regions or Superassociations(SA) are double nucleus objects. Among the galaxies from FBS more than 100 double nucleus galaxies arediscovered. No doubt, that many of them (if not majority) are real double nucleus galaxies. That is, there are not a result of mergering or interaction of two independent galaxies.

Note that following observational data are difficult to reconsile with the hypothesis of gravitational merging:

1) the discovery of the Seyfert type double nuclei galaxies because of their most rarity between galaxies ([21,22]);

2) the discovery of the twin-objects with quite identical spectra and morphology similarto two isolated SA being considered as one galaxy with double nuclei each of which is SA ([23,24]);

3) the discovery of numerous double nuclei galaxies.

On the Fig.1, examples of well-known double nucleus active galaxies are presented.

The existance of galaxies with two condensations in the centre having Seyfert type spectra (Mark.266, 463, 673, 789) is the most definite evidence of the possibility of galaxies with double nuclei in general. However, double nuclei galaxies are not an unusual phenomenon among AG while they are particulary common among UV galaxies. In the following table some physical parametrs: the apparent and absolute photographic magnitudes of the components of the nucleus, distance between components in arcseconds and kiloparsecs (H=75 km/s.Mpc) and differences of their radil velosites for six AG with double nuclei are presented.

Mark 212 (1 mm ~ 2";1 ~ 5 kpc)

Mark 480 (1 mm ~ 1.4";1 ~ 3 kpc)

Mark 930 (1 mm ~ 2.2";1 ~ 1.6 kpc)

Mark 266 (1 mm ~ 1.7";1 ~ 6.5 kpc)

Mark 324 (1 mm ~ 0.6";1 ~ 0.1 kpc)

Mark 463 (1 mm ~ 1.3";1 ~ 4.3 kpc)

Fig.1. Photos and isodenses of some Markarian galaxies
I is a distanse between of nuclei

Parameters of 6 representative of AG with double nuclei

Mark.No	m(pg)	M(pg)	d"	d(kps)	V(km/s)
266	17.5	-17.8	12	6.5	127
	17.8	-17.5			
273	17.5	-18.4	4.3	3.2	–
	18.2	-17.7			
463	17.0	-19.5	4.5	4.3	50
	17.2	-19.3			
673	16.2	-19.6	5.3	3.7	166
	16.2	-19.6			
739	16.2	-19.1	6.6	3.8	85
	17.0	-18.3			
789	16.0	-19.5	4.1	2.5	2
	18.0	-17.5			

b) As it was mentione above one of the form of activity is the ejection from the nuclei of active galaxies isolated clouds with different contents. The further both spectroscopic and morphological investigations are shown that the nuclei of some AG are variable and are underwent irregular changes in brifhtness and in spectrum.In the end of

60-th and the beginning of 70-th it became clear that in central parts of same AG taken place the phy sical processes leading to ejection of huge amount of matter from the nucleus of AG. The small sizes of AG nuclei (AGN) about 10 arcsec don't permit to detect such a gas-newformation by means of derect observation in the near surroundings of them even in radio wavelengths. Therefore the only possibility for the investigation of these physical events is detailed spectrophotometrical observations with comparatively high dispersion. The most effective are investigations in optics.

The appearance of additional new emission components of Hydrogen lines in the spectrum of AGN first was discovered in 1969 by Khachikian and Weedman ([25,26]). During one year (between February 1968 and January 1969) in the spectrum of Markarian 6, wich was Sy2 galaxy, new broad emission components of Hydrogen lines H-alfa, H-beta and H-gamma have been detected.Their blue- shift velocity is correspo-und to 3 000 km/sec.

In January 1970 the intensity of H-beta component was equal to 50% that of basic H-beta line. These observations have been conformed by many authors ([27-32]).

It is known now many active objects with double Hydrogen lines structure in the spectrum 3C 390.3, NGC 1097, NGC 1566 ([33-35]). There are some models,which have been suggested to explane this phenomenon ([26,27,36-39]).

All these models do not give complete explanation of phenomenon. In ([40]) the new fairly simple model is suggested, which gives quantitative accordans with observational data. In Fig. 2 the schematic pitcure of the model is shown.

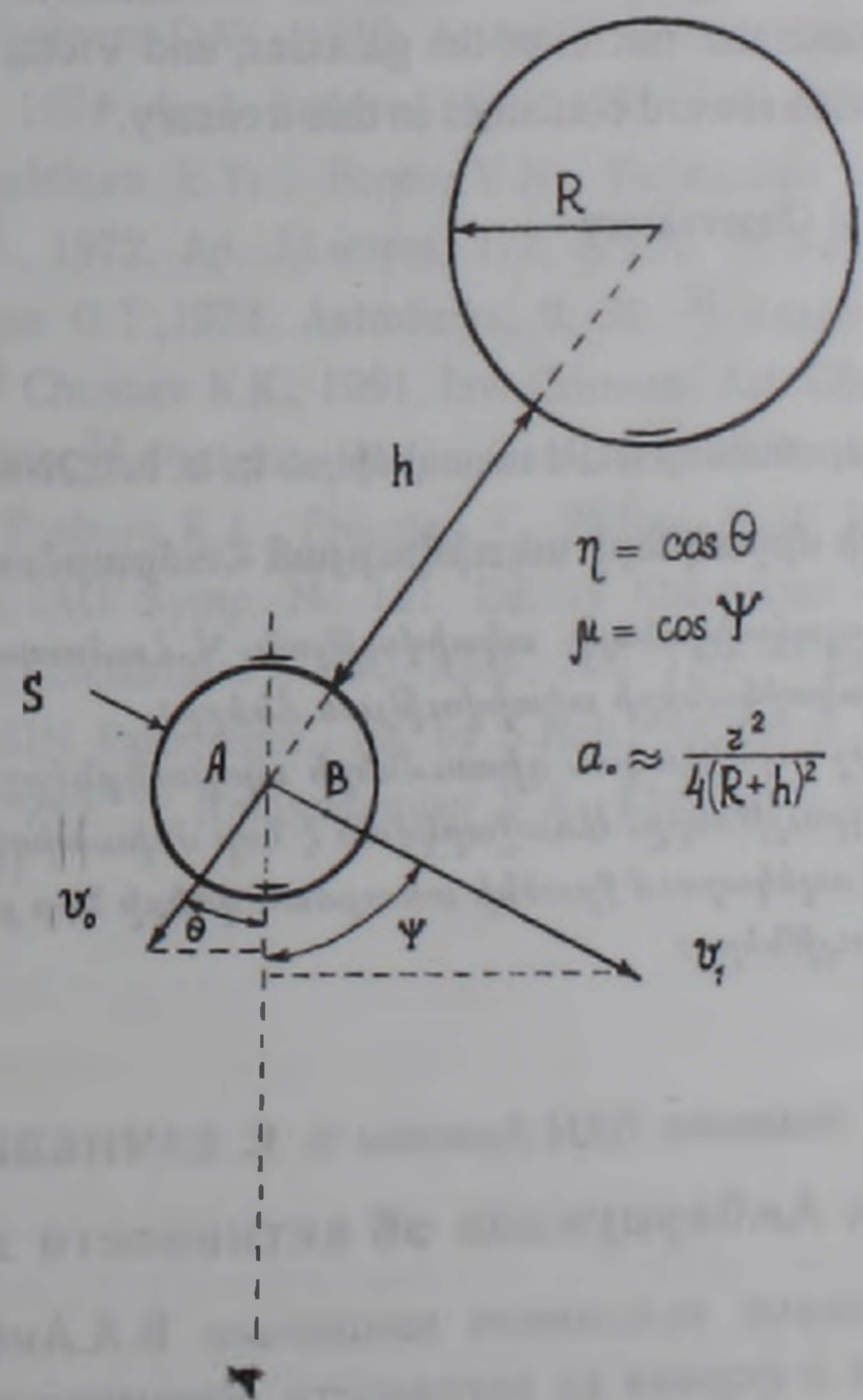

Fig.2

From the nucleus of AG a compact formation S is ejected with the velocity V_o. On the some distance from the surface of AG it exploded. As a result of this explosion globular gas cloud S, mainly consisted of Hydrogen atoms, is formed. Similar to situation in planetary nebula the Lc-quanta of AG lead to ionization of Hydrogen atoms in S. As a result of recombination and following cascade transitions the subordinate Hydrogen lines are formed. On the whole the additional components of subordinate lines are arising. The shift of additional lines relatively to that of nucleus of AG is explained by speed of ejection of S from AGN. As for the widths of additional lines they depend on velocity of expansion of S. The simple estimation presented in this work show that comparable small size and mass of S cloud and acceptable value of speed of his expantion can explain the phenomenon.

The terms "active galaxies", "active nucleus", introduced by Ambartsuman, is now generally accepted in science, although some scientiests (mostly younger one) have no idea about that. Therefore, his followers sometime has to remaind about tremendous inpact of V.Ambartsumian in science, in particular in extragalactic astronomy.

Ambartsumiann's idea on the activity of galaxies exerted tremendous influence on the further development of extragalactic astronomy and stimulated numerous studies in many observatories of over the world.In well-known Volume of the U.S. Academy of Sciences "The Heritage of Copernicus", commemorating the quinquecentennial of the birth of Copernicus,this Ambartsumian's idea is considered as a Copernicous type revolutionary idea, which has changed our notion about the nature of the galaxies. American astronomer Sandage said "Nobody of astronomers would deny today, that mystery indeed surrounds the nuclei of the galaxies, and Victor Ambartsumian was first who understood how rich reward containes in this treasury."

Byurakan Astrophysical Observatory

Հայաստանի ԳԱԱ ակադեմիկոս **Է. Ե. ԽԱՉԻԿՅԱՆ**

Գալակտիկաների միջուկների ակտիվության Համբարձումյանի թեզի մասին

Շարադրված է գալակտիկաների ակտիվության Վ.Համբարձումյանի կողմից առաջարկված թեզը և գալակտիկաների ակտիվության ձևերը:

Քննարկված է որոշ օպտիկական դիտումների արդյունքները, որոնք հաստատում են այդ թեզի հիմնական դրույթները: Առաջարկված է նոր մեխանիզմ, որը բացատրում է ակտիվ գալակտիկաների սպեկտրում ջրածնի առաքման գծերի նոր բաղադրիչների առաջացման ֆիզիկական երևույթները:

Академик НАН Армении **Э. Е. ХАЧИКЯН**

О концепции Амбарцумяна об активности ядер галактик

Изложены основные положения концепции В.А.Амбарцумяна об активности ядер галактик и формах их активности. Рассмотрены некоторые результаты оптических наблюдений галактик, подтверждающие основные положения этой концепции. Предложен новый механизм, который объясняет феномен по-

явления новых дополнительных эмиссионных компонентов у водородных линий в спектрах ядер активных галактик.

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ДОКЛАДЫ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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ASTROPHYSICS

УДК 524-33+524.352

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Flare Stars in Star Clusters and Associationes

(Submitted 26/VI 1998)

Introduction.

The most significant investigations on the study of flare stars and phenomenon of stellar flares have been fulfilled during last two decades. They brought much news in the problem and put it forward on one of the first places in modern astrophysics.

The most important consequence of these investigations, perhaps, is the establishment of the flare activity stage in stellar evolution ([1]).

Highly significant results were obtained on the physical properties of stellar flares. At the present time there is no doubt, that the flare stars in star clusters and associations and the UV Ceti stars in the solar vicinity represent the same class of non-stable stars and the differences observed between them are explained by the differences of their ages ([2]).

In this paper an attempt is made to present the nowadays state of the flare stars problem on the basis of flare star observations.

Flare Stars in Systems.

There are some grounds to beleive that all young flare stars, possessing in the average higher luminosities are observed in star clusters and associations. By photographic observations with wide-angle cameras, carried out during 7500 hours, approximately 1300 flare stars were found in the nearest systems.

In Table 1 the striking fact is, that the majority of known flare stars (about 70%) during the observations showed only one single flare-up. It can be explained by the too low mean frequency of flares for their majority. Really, as the distribution function of mean flare frequency for the stars of Pleiades cluster, derived by Ambartsumian ([5]) shows that the mean flare frequency for the majority of stars is very low. This conclusion is true for other systems too. For example, as the mean flare frequency function shows it is true for flare stars of the Orion association ([6]). As a result of this less than one third of all flare stars is discovered up to now in the systems studied.

Flare Stars in some Nearest Star Clusters and Associations

System	t (hours)	n	n_1	N
Pleiades	3175	546	287	994
Orion I	1406	482	380	1471
Taurus Dark Clouds	937	102	88	532
Cygnus (NGC 7000)	938	67	58	403
Praesepe	698	54	44	215
Monocerotis 1	105	42	40	442
(NGC 2264				
Around Cygnus	324	16	15	129
Total	7583	1309	912	4186

If we take into account that the estimations of total number of flare stars in systems N, presented in Table, correspond to the lower limit of this magnitude, then it must be confessed that for the present we know only a small part of all flare stars even in comparatively better studied systems like the Pleiades and the Orion.

Evolutionary Stage of Flare-Activity.

The physical similarity between the emission of T Tauri type stars and the emission appearing during stellar flares, revealed by Ambartsumian ([7,8]), gave him a reason to conclude, that the non-stable stars of these two classes are related.

Later on Haro and Chavira (9) on the basis of the results of flare stars photographic observations in associations and clusters stated an idea, that the stage of flare stars follows the stage of T Tau stars. A telling argument in favour of the evolutionary connection between these two stages was the discovery by Haro and Chavira and by Rosino et al ([10]) of some T Tau type stars showing classical flare-ups in the Orion and the Monocerotis associations.

This discovery has shown that the evolutionary stages of T Tau and UV Cet partly cover each other in time. During the period of coverage the star is of the T Tau type and flare star, simultaneously.

The statistical study of observational data concerning to the Orion association has shown ([11]) that the time of coexistence of these stages is equal to approximately one fourth of the duration of the T Tau stage. Recent estimations show that this time is somewhat longer about 40% of the T Tau stage duration ([12]). The duration of the flare activity stage itself varies in large limits: 10^6 - 10^9 years.

Thus, in the Orion association (age ~ 10^6 years ([13])) there are stars of high enough luminosities which don't already show a flare activity, at least available for photographic observations. In the older systems, as Pleiades (age ~ 7×10^7 years), Hyades and Praesepe (age higher than 10^8 years) there are still some flare stars as well as the stars of high luminosities which already lost their flare activity. At last, between stars of the Solar vicinity there are flare stars the ages of which exceed 10^9 years ([14,15]). A direct dependence has been found between the mean energy of flares and the age of flare stars, which gives possibility to obtain the ages of stellar aggregates and single flare stars ([14]).

It can be assumed that for a separate star the initial and ending phases of flare activity depend on its luminosity (mass): the higher the luminosity the earlier the flare activity begins and correspondingly it ends.

In the case of the ending of flare activity phase this regularity is confirmed by the data, related to the mean luminosities of flare stars in the systems of different ages ([16]). They show that the older the system (flare stars) the lower is the mean luminosity of flare stars in it. This regularity can explain the fact, that there are practically no flare stars of comparatively high luminosities in the general galactic field.

The reverse correlation existing between the mean luminosity offlare stars and the age of the system, to which they belong, can be considered as a direct observational evidence in favour of the idea, that the evolution rates of stars depend directly on their luminosities (masses): the stars possessing higher luminosities evolve more quickly compared with the stars of lower luminosities.

Thus, the observational data allow to outline the following, evolutionary sequence of dwarf stars: T Tau stars - T Tau stars, possessing flare activity - flare stars - stars of practically constant brightness.

Certainly, some questions connected with the presented evolutionary sequence haven't obtained yet their final decision. For example, the transition from the T Tau stage to the flare star stage is apparently connected with the difficulty connected with the problem of masses.

The important question whether all stars, at least dwarfs, pass through flare activity stage, has not yet a final solution. For example, the observations of flare stars in the Pleiades testify that the positive answer to this question needs to assume that flare activity of stars has a cyclic nature: the periods of high flare activity alternate with the periods of comparatively low activity ([17]).

However, it is not likely that new studies can bring to the essential changes of foregoing main evolutionary sequence.

Optical Manifestations of Flare Activity.

The photoelectric observations of flares of the UV Cet type stars carried out with high time resolution brought to the idea, that the stellar flare is a complex phenomenon and, in the general case, represents a multiple appearance of the additional energy released during the flare ([18]). Though the shape of the light-curve does not depend on power of the flare (very powerful flares,which have spike-like light-curves and small flares with complex shape of light-curves have been observed ([19]) however, there are some evidences showing that the powerful is a flare, in average, the higher is probability to observe the complex shape of light-curve ([20]).

Very essential property of stellar flares has been discovered by Haro ([21]). He has divided all flares into two groups "fast"and "slow", according to the flare rise time, using his multiexposure photographic observations. For the majority of flares the rise time was very short ("fast"), while there are rare flares for which the flare rise time reaches 20-30 and more minutes ("slow"). Probably the "fast" and "slow" flares differ from each other apparently by colours too ("slow" flares are, in average, redder than the "fast" ones ([21]).

The difference between "fast" and "slow" flares can be successfully explained, if one following to Ambartsumian ([9,10]) assumes, that the flare rise time is determined by the depth of stellar atmospheric layers, where the flare takes place: the larger this depth the longer the flare rise time is. The observations of "slow" flares in the Orion and the Pleiades show, that the larger the energy of "slow" flare the shorter its rise-time in agreement of this idea on the flare nature is ([22]).

At present it can be said that Haro's classification which was very fruitful, actually is a conditional one and is determined by the method of his observations. In the reality the distribution of flare rise time durations is continuous: there is no sharp transition betweer- "fast" and "slow" flares. In favour of the idea that the flare rise time indeed is determined by the characteristics of those layers of stellar atmosphere where the flare occurs the important fact can be considered that the majority of stars which have shown the "slow" flares were also observed in "fast" flare.

The fuor-like variations of star brightness (FUOri phenomenon) can be considered as a remarkable manifestation of flare activity. After the brightnening, of V 1057 Cyg, having before the T Tau type spectrum, it has been revealed that such wonderful variations take place with some T Tau type stars (see, for example, ([23]). Ambartsumian ([24]) proceeded from the idea on the liberation of excess flare energy in the surface layers of stars having different depths, has shown that a definite parallel exists between the differences in radiation of a prefuor and a postfuor, on the one hand, and the differences between radiation of "fast" and "slow" flares, on the other hand.

As some confirmation of this point of view one can consider the results of observations obtained for the objects Chanal and Sugano Ab 24 (see, for example, ([25,26])). They show that fuor-like variations of star brightness in a smaller scale can occur in the flare activity stage. These observations give some reason to assume that the phenomena which occur during fuor-like variations of star brightness and during "slow" flares have the same physical nature.

It can be added that fuor-like variations of star brightness are connected apparently with the ejection of some noticeable quantity of matter by a star bringing to the formation of an envelope. Ambartsumian's ([24]) interpretation of the fuor phenomenon is based namely on this assumption. There are indications of the appearance of a gas envelope around the star V 1057 Cyg after its brightening ([27,28]).

Optical Observations of Flare Emission.

For determination of the nature of the emission, originated during stellar flares it is important to obtain spectral composition and its variations during the flare.

The optical spectrum of flare emission is unusual. The colour indecies U-B and B-V of the flare radiations correspond to the different temperatures.

In the flare maximum they are, in average, equal to ([29]):

$$U\text{-}B \sim 1.0, \quad B\text{-}V \sim +0.3.$$

These colour indecies varies for different flares and vary somewhat irregularly during given flare. The spectral observations of the UV Cet flare stars, have confirm also the dominate role of the continuous emission, mainly at short waves, in the sharp increase of brightness at the beginning of the flare, noted already in the pioneer paper by Joy and Humason ([30]).

This significant result has been confirmed with a special clearness by parallel spectral and photoelectric flare observations of the UV Cet stars, carried out by Moffett and Bopp ([19,31]), showing that the continuous emission is primary one compared with the line emission, at least.

It should be added that at present the essential observational data were obtained on the flare radiation and the radiation of flare stars, in general, in radio and X-ray spectral regions. Parallel observations of stellar flares in different spectral regions didn't shows any correlation between obtained results ([32]). This fact shows large diversity of flare emission spectrum.

Conclusion.

The results of the flare star study obtained during last decades turned completely unexpected for the existing theoretical stellar models.

This concerns, first of all, to the conclusion that flare stars represent an evolutionary stage, obtained on the basis of their observations in star clusters and associations. No stellar evolution theory doesn't suspect this.

This concerns also the results of study of physical peculiarities of flare emission and, in general, of stellar flare phenomenon. The observational data in some cases contradict the theoretical calculations. The difficulties in this field increased essentially after the space observations of flare stars. Recent new CAII, UV and X-ray Observations have shown that-behaviour of "activity" on stars is substantially more complex than hitherto suspected ([33]).

Therefore, we have some grounds to hope, that the further studies in this actual branch of the astrophysics can bring to the esentially new consequences in physics and evolution of stars.

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Բռնկվող աստղերն աստղակույտերում և աստղասփյուռներում

Աշխատանքը հանդիսանում է համառոտ ակնարկ՝ նվիրված աստղակույտերում և աստղասփյուռներում աստղային բռնկումների ուղղությամբ ստացված դիտողական արդյունքներին: Առավել կարևոր եզրակացությունն այն է, որ բռնկումային ակտիվությունն աստղային ագրեգատների և արեգակի շրջապատի թզուկ աստղերի էվոլյուցիայի փուլ է հանդիսանում:

Академик НАН Армении **Л. В. МИРЗОЯН, Э. С. ПАРСАМЯН**

Звездные вспышки в звездных скоплениях и ассоциациях

Работа является кратким обзором наблюдений звездных вспышек в звездных скоплениях и ассоциациях. Наиболее важным является вывод о том, что вспышечная активность представляет собой эволюционный этап в жизни звезд-карликов в звездных агрегатах и в окрестности солнца.

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Line Formation in an Atmosphere with Arbitrary Velocity Gradient

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1. Introduction

In studying the spectra of various cosmic objects we find them very often being affected essentially by the macroscopic motions within a medium where they are formed. Gaseous nebulae, shells of the novae or supernovae, a wide range of the non-stationary phenomena in the solar atmosphere, stellar winds represent typical examples of such objects. The presented list is obviously far from being exhaustive. It should be emphasized that the theory of the spectral line formation with allowance for the velocity field in an atmosphere is much more involved compared to that for the static media. This primarily concerns the strong resonance lines formed in the optically thick atmosphere when the effects of the multiple scattering become dominating. This kind of problem is encountered, for instance, in treating the extended atmospheres or the dense winds of the LBV (Luminous Blue Variable) stars (see e.g.([1]) and list of references therein).

The main objective of this report is to demonstrate that Ambartsumian's invariance principle in conjunction with the previous results of the present authors ([2,3]) may be used successfully in tackling the problem of the spectral line formation in a scattering atmosphere characterized by the internal motions. For expository reasons, we shall limit the discussion to considering the model problem of non-coherent scattering in an one-dimensional semi-infinite atmosphere. No constraints will be imposed on the velocity gradient in the atmosphere. Note that a kind of such problem but for the velocity increasing linearly with depth was treated by Sobolev ([4]), who obtained approximate solutions of the problem under various assumptions concerning the velocity gradient.

The invariance technique employed below in deriving the basic equations is largely similar to that by Sobolev ([5]) elaborated for the non-homogeneous atmosphere. For simplicity, we shall adopt the probabilistic language and deal with quantities possessing transparent physical significance.

2. Description of the scattering process.

Let us start with specifying the elementary event of scattering. Throughout the paper the scattering process is assumed to be isotropic and following by the partial redistribution over frequencies. For the redistribution function (averaged over directions) we introduce the commonly used notation $r(x',x)$, where x and x' are dimensionless frequencies (measured from the center of the line ν_0 in the units of Doppler widths $\Delta\nu_D = (u/c)\nu_0$, where u is the thermal velocity) of the incident and scattered quanta, respectively. The function $r(x',x)$ obeys the condition

$$\int_{-\infty}^{\infty} r(x',x)dx' = \alpha(x), \tag{1}$$

where $\alpha(x)$ is the line absorption profile normalized as follows

$$\int_{-\infty}^{\infty} \alpha(x)dx = \sqrt{\pi}. \tag{2}$$

Note also that in some specific but important cases of the frequency redistribution the function $r(x',x)$ admits bilinear expansion over the set of certain functions $\{\alpha_k(x)\}$

$$r(x',x) = \sum_{k=0}^{\infty} A_k \alpha_k(x')\alpha_k(x), \tag{3}$$

where A_k are some constants. Referring the reader for the details of this point to the papers ([6,7]), we note that for pure Doppler redistribution in frequencies when

$$r(x',x) = \int_{\max(|x|,|x'|)}^{\infty} e^{-t^2} dt \tag{4}$$

and $\alpha(x) = e^{-x^2}$, we have $A_k = 1/(2k+1)$, and

$$\alpha_k(x) = \left(\pi^{1/4} 2^k \sqrt{(2k)!}\right)^{-1} e^{-x^2} H_{2k}(x), \tag{5}$$

where $H_k(x)$ is the Hermit polynomial of the kth order.

During multiple scattering in an atmosphere, the photon may be thermalized by absorbing in the continuous spectrum. This process is characterized by the ratio β of the absorption coefficient in the continuum to that in the center of the spectral line.

Before proceeding directly to our problem we introduce two important characteristics of the diffusion process in the source-free homogeneous and stationary atmosphere. One of them is the reflectance ρ of the semi-infinite atmosphere, also referred to as 'reflection function' or 'reflection coefficient'. The probabilistic meaning assigned to function ρ in case of non-coherent scattering (occurring in a one-dimensional medium) is given as follows: $\rho(x',x)dx$ is the probability that a photon of frequency x' incident over the semi-infinite atmosphere will be reflected in the frequency domain $(x, x+dx)$. Being applied to the reflectance of atmosphere, Ambartsumian's invariance principle leads to the functional equation

$$\begin{aligned}(2/\lambda)[v(x',) + v(x)]\rho(x',x) = r(x',x) + \int_{-\infty}^{\infty} r(x',x'')\rho(x'',x)dx'' + \\ + \int_{-\infty}^{\infty} \rho(x',x'')r(x'',x)dx'' + \int_{-\infty}^{\infty} \rho(x',x'')dx'' \int_{-\infty}^{\infty} r(x'',x''')\rho(x''',x)dx''',\end{aligned} \tag{6}$$

where $v(x) = \alpha(x) + \beta$, and λ is the single-scattering albedo. Eq. (6) in the specific case of $\beta = 0$ was written for the first time by Sobolev ([8]) (see also ([9,10])).

The second quantity needed in further discussion is the function P defined as to have the following probabilistic meaning: $P(\tau,x',x)dx$ is the probability that a photon of frequency x', *moving* initially at depth τ in either direction, will escape it as a result of multiple scattering as a photon with frequency from the interval $(x, x+dx)$. The principle of invariance, which requires that the addition (or removal) of an infinitesimal layer to (from) the surface of the semi-infinite atmosphere must not change the mentioned probability, yields ([2])

$$\frac{dP(\tau,x',x)}{d\tau} = -\nu(x)P(\tau,x',x) + \int_{-\infty}^{\infty} P(\tau,x',x'')\alpha(x'')p(0,x'',x)dx'', \qquad (7)$$

where $p(\tau,x',x)$ is the photon escape probability designed for the photon of frequency x' *absorbed* at the optical depth τ. It is obvious that $P(0,x',x) = \rho(x',x)$. Note also that the function $p(0,x',x)$ is also simply expressed in terms of the reflectance ρ and the redistribution function

$$\alpha(x')p(0,x',x) = r(x',x) + \int_{-\infty}^{\infty} r(x',x'')\rho(x'',x)dx''. \qquad (8)$$

In fact, the knowledge of the function P allows to find the outgoing intensity for any given distribution of energy sources in the static atmosphere.

3. Formulation of the problem and basic equations

Now let us consider a semi-infinite atmosphere, the various parts of which are moving relative to each other. Suppose $V(\tau)$ be the velocity field in the atmosphere. This function is assumed to be given and we do not impose any restrictions on its behaviour.

In discussing the scattering process in such an atmosphere one must distinguish from each other the frequency x of a photon in the laboratory or observer's frame and that (denoted hereafter by $x_1(\tau)$ or $x_2(\tau)$ depending on the direction of propagation) in the comoving frame at a given depth τ. These frequencies are related by means of Doppler formula

$$x_1(\tau) = x + \delta(\tau); \quad x_2(\tau) = x - \delta(\tau), \qquad (9)$$

where $\delta(\tau) = V(\tau)/u$.

Our purpose in this section is to evaluate the line intensity radiated by the atmosphere described, in the presence of the internal energy sources, the power of which we denote by $\varepsilon(\tau,x)$. Before turning immediately to this problem we shall discuss first an auxiliary problem of determining the reflectance of the non-stationary atmosphere.

In contrast to the stationary case, the problem of finding the reflectance must be embedded into family of problems that concern atmospheres, the top parts of which, referenced from some optical depth t, are removed. We shall call such atmospheres truncated, and t is the truncation parameter. Thus, we are interested in the task of evaluating the reflectance $\rho(t,x',x)$ for the family of atmospheres dependent on the parameter t. On setting $t = 0$ in equations derived below, we arrive obviously at the requisite results pertaining our particular problem.

The standard invariance procedure, being applied to the reflectance $\rho(t,x',x)$ of a truncated atmosphere, yields

$$(2/\lambda)[v(x_1') + v(x_2)]\rho(t,x',x) - \frac{\partial\rho(t,x',x)}{\partial t} = r(x_1',x_2) +$$
$$+\int_{-\infty}^{\infty} r(x_1',x_1'')\rho(t,x'',x)dx'' + \int_{-\infty}^{\infty} \rho(t,x',x'')r(x_2'',x_2)dx'' + \tag{10}$$
$$+\int_{-\infty}^{\infty} \rho(t,x',x'')dx'' \int_{-\infty}^{\infty} r(x_2'',x_1''')\rho(t,x''',x)dx''',$$

where $x_{1,2}(t) = x \pm \delta(t)$, $x_1'(t) = x' \pm \delta(t)$, and so on. For simplicity of exposition, the dependence of x and x', on t in Eq.(10) is not marked explicitly.

If the redistribution function $r(x',x)$ admits the bilinear expansion of the type given by Eq.(3), the formal solution of Eq.(10) may be written as

$$\rho(t,x',x) = (\lambda/2)\sum_{k=0}^{\infty} A_k \int_t^{\infty} \varphi_k(\tau,x_1')\overline{\varphi_k}(\tau,x_2)\exp\left(-\int_t^{\tau}\{v[x_1'(t')] + v[x_2(t')]\}dt'\right)d\tau, \tag{11}$$

where

$$\varphi_k[t,x_1'(t)] = \alpha_k[x_1'(t)] + \int_{-\infty}^{\infty} \rho(t,x',x'')\alpha_k[x_2''(t)]$$
$$\overline{\varphi}_k[t,x_2(t)] = \alpha_k[x_2(t)] + \int_{-\infty}^{\infty} \rho(t,x'',x)\alpha_k[x_1'(t)]dx''. \tag{12}$$

Similarly, by using the invariance technique one can find the following equation

$$\frac{\partial P(t,\tau,x',x)}{\partial \tau} - \frac{\partial P(t,\tau,x',x)}{\partial t} = -v(x_2)P(t,\tau,x',x) +$$
$$+\int_{-\infty}^{\infty} P(t,\tau,x',x'')\alpha(x_2'')p(t,0,x'',x)dx'' \tag{13}$$

where the functions $P(t,\tau,x',x)$ and $p(t,\tau,x',x)$ have the probabilistic meaning similar to that of functions $P(\tau,x',x)$ and $p(\tau,x',x)$ introduced in the preceding paragraph, with only difference that these new ones concern the truncated atmosphere.

On the other hand, taking into account the probabilistic meaning of $P(t,\tau,x',x)$ one can immediately write down the expression for the intensity of outgoing radiation

$$I(t,x) = \int_{-\infty}^{\infty} dx' \int_0^{\infty} \{\varepsilon[t+\tau, x_1'(t+\tau)] + \varepsilon[t+\tau, x_2'(t+\tau)]\} P(t,\tau,x',x)d\tau. \tag{14}$$

Then, by using Eqs.(13) and (14) one may obtain

$$v(x_2)I(t,x) - \frac{\partial I(t,x)}{\partial t} = \int_{-\infty}^{\infty} I(t,x')\alpha(x_2')p(t,0,x',x)dx' +$$
$$+\varepsilon(t,x_2) + \int_{-\infty}^{\infty} \varepsilon(t,x_1')\rho(t,x',x)dx' \tag{15}$$

It is easily seen that for $\delta(t) = 0$ Eqs. (10) and (13) transform into Eqs.(6) and (7), respectively.

When the frequency redistribution law $r(x',x)$ admits the bilinear expansion (3), the formal solution of the Eq.(15) may be written as follows

$$I(t,x) = (\lambda/2)\sum_{k=0}^{\infty} A_k \int_t^{\infty} \varphi_k[\tau,x_2(\tau)]I_k(\tau)\exp\left(-\int_t^{\tau} v[x_2(t')]dt'\right)d\tau +$$

$$+\int_{t}^{\infty}\left\{\varepsilon\left[\tau,x_2(\tau)\right]+\int_{-\infty}^{\infty}\varepsilon\left[\tau,x_1'(\tau)\right]\rho(\tau,x',x)dx'\right\}\exp\left(-\int_{t}^{\tau}v\left[x_2(t')\right]dt'\right)d\tau, \tag{16}$$

where

$$I_k(\tau)=\int_{-\infty}^{\infty}I(\tau,x')\alpha_k\left[x_2'(\tau)\right]dx'. \tag{17}$$

Thus we obtained the analytical solution of the problem. For the practical use of the results it is necessary to elaborate the facilitate methods of numerical calculations which is the subject of a separate study to be performed in the following papers.

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Հ. Ա. ՀԱՐՈՒԹՅՈՒՆՅԱՆ, Ա. Գ. ՆԻԿՈՂՈՍՅԱՆ

Սպեկտրալ գծերի առաջացումն արագությունների կամայական գրադիենտով մթնոլորտում

Համբարձումյանի ինվարիանտության սկզբունքը կիրառվում է այն դեպքի համար, երբ կիսանվերջ մթնոլորտում գոյություն ունի արագությունների գրադիենտ: Ստացվել են անդրադարձման ֆունկցիայի հատված մթնոլորտից քվանտի դուրս գալու հավասարումները: Նույնանման հավասարում է ստացվել նաև միջավայրից դուրս եկող ճառագայթման ինտենսիվության համար: Ստացված բոլոր հավասարումներն արագությունների գրադիենտի բացակայության դեպքում վերափոխվում են արդեն հայտնի ավելի պարզ հավասարումների:

Г. А. АРУТЮНЯН, А. Г. НИКОГОСЯН

Образование спектральных линий в атмосфере с произвольным градиентом скоростей

Применяется модифицированный принцип инвариантности Амбарцумяна в случае, когда в полубесконечной атмосфере существует градиент скоростей. Получены соответствующие уравнения для функции отражения и вероятности выхода кванта из усеченной атмосферы. Аналогичное уравнение получено также и для интенсивности выходящего из среды излучения. Все полученные уравнения в случае отсутствия градиента скоростей переходят в известные, более простые уравнения.

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ДОКЛАДЫ НАЦИОНАЛЬНОЙ АКАДЕМИИ НАУК АРМЕНИИ

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ASTROPHYSICS

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A. L. Gyulbudaghian

On Trapezium (in Orion) - Type Systems

(Submitted by Academician of NAS RA E. Ye. Khachikian 13/ VII 1998)

1.STARS. One of the basic statements of the Ambartsumian's conception of stellar systems formation is the existence in these systems of the Trapezium-type configurations. The distances between the components in such systems are the values of the same order ([1]). The Trapezium in Orion consists of OB stars and belongs to the association Ori OB 1.

During last years the new type Trapezium-type systems were discovered. These systems consist of sources invisible in optics, but detectable in IR and/or radio. These sources are embedded in molecular clouds, their bolometric luminosities correspond to the luminosities of OB or WR type stars. Such systems were fount by us in ([2-7]). It is rather probable that these systems, after a definite period of time, when because of influence of their radiation and/or stellar wind the part of molecular cloud, in which they were embedded, is transformed into HII region, and as a consecuence the system will appear as bright in optics Trapezium-type system, consisting of OB or WR type stars. The distances between the components of these systems are systematically less than in the classical Trapezium-type systems (e.g. in Orion). Such systems are tight systems.

In ([8]), as well as in the catalogues ([9,10]) we have found Trapezium-type systems consisting of B stars. The distances between the components in these systems are several times more than in the classical Trapezium-type systems. These are broad Trapezium-type systems.

Hence we can make a conclusion about the existence of an evolutionary connection between the three preceiding types of Trapezium-type systems of OB stars: 1.the systems, consisting of IR and/or radio sources in molecular clouds (0.01-0.1pc), 2.the classic systems (like Trapezium in Orion) (~0.1pc), 3.the broad systems (~1pc).

There are also Trapezium-type systems, all components of which are T Tauri type stars ([11]) or some of the components are T Tauri type stars ([12]). The existence of such systems is an argument in favour of an origin of low mass stars by groups, like the stars of large masses (O, B, WR). We have to mention here that there is not a question about the stability of such systems, consisting of low mass stars. Such systems are unstable because of low values of gravitational contraction between the components.

2.GALAXIES. Ambartsumian mentioned in ([13]), that "The components of each multiple system of galaxies have common origin. The majority of multiple systems of galaxies has a Trapezium-type configuration. The Trapezium-type multiple systems of galaxies in the cases, when the masses of components are comparable, have to be unstable". As a confirmation of that statement we can mention the paper ([14]), where several examples of groups of galaxies are given, which are expanding, that is the groups are destroying.

3.GROUP OF CONDENSATIONS IN NGC5128. It is well known that Ambartsumian was the first who took attention on the activity of galactic nuclei. Especially interesting are the ejections from the nucleus of NGC5128 (Cen A). The comparable proximity of this galaxy (5Mpc) enables us to observe in optics the condensations, situated in the jet, ejected from the nucleus. As mentioned Ambartsumian ([13]): "it is possible to observe in the nature the ejection from the nucleus of galaxies of relatively small masses. These ejected masses can in a short period of time transform into conglomerates, consisting of young unstable stars, interstellar gas and the clouds of particles with high energy". It is probable that we have such a phenomena in the intermediate ejection from the nucleus of NGC5128. This ejection consists of diffuse matter, compact emission objects and the chains of blue objects, the spectra of last objects are identical with the spectra of B type supergiants ([14,15]). On the distance of 36kpc from the nucleus a group of diffuse emission regions is situated. This group looks like a Trapezium-type system. The difference between the velocities of separate objects in this group is about 800km/s ([16]). Such a high difference of velocities, with the small dimensions of the group, about 6kpc, coud not be originate at the moment of exit of these objects from the nucleus. The origin of the system had to take place near the modern coordinates of this group. For the holding of this group from the dissipation, from the virial theorem a mutual mass of about 5 orders more, than is obtained for this group, is needed.

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Տրապեցիայի տիպի համակարգերի մասին

Հոդվածում դիտարկվում են տրապեցիայի տիպի (սեղանակերպ) համակարգեր, որոնք բաղկացած են տարբեր տիպի օբյեկտներից՝ OB աստղասփյուռների մասը կազմող OB աստղերից, Տ Ցուլի տիպի աստղերից, NGC 5128 գալակտիկայում գտնվող խիտ միգամա-

ծությունների: Այդ օբյեկտներն ունեն մեկ ընդհանուր հատկություն՝ անկայուն են և քայքայվում են, ինչը համապատասխանում է Վ.Համբարձումյանի կանխատեսմանը:

А. Л. ГЮЛЬБУДАГЯН

О системах типа трапеции Ориона

В статье приводятся примеры систем типа Трапеции, которые представляют разные объекты - звезды типа OB в ассоциациях, звезды типа T Тельца, компактные сгущения в галактике NGC5128. Все эти системы объединяет их нестабильность, о которой в свое время было сказано Амбарцумяном.

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