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УЧЕНЫЕ ЗАПИСКИ АРМЯНСКОГО ГОСУДАРСТВЕННОГО ПЕДАГОГИЧЕСКОГО УНИВЕРСИТЕТА ИМ. Х. АБОВЯНА

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FREE INTERFACIAL AND EDGE VIBRATIONS OF MOMENTLESS CYLINDRICAL SHELL OF VARIABLE CURVATURE WITH FREE ENDS

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Free interfacial and edge vibrations of a closed cylindrical shell composed of finite orthotropic momentless cylindrical shells with variable curvature and different elastic properties are studied. It is assumed that the ends of shell are free.

Key words : *free vibrations, momentless cylindrical shell, ends of shell are free.*

Introduction. Investigation of free vibrations of composed shells plays an important role in the studies of dynamics of deformable solid bodies. Such studies are required by the needs of the theory itself and by practical needs of different branches of engineering industry, construction, instrument-engineering, seismic survey, etc. [1]. In many cases the objects of investigations are finite thin-walled composed cylindrical shells with variable curvature. For such shells much attention is attracted to the investigation of free vibrations localized near the ends of the shells, i.e. edge vibrations, and vibrations localized near the interface of material properties, i.e. interfacial vibrations. The investigation of elastic surface waves was initiated by the pioneering work of Lord Rayleigh [2], where the existence of the elastic waves propagating along the free boundary of semi-infinite space with amplitude strongly damping with the depth was shown. Such waves that appear in elastic bodies with different geometries are usually called Rayleigh type surface waves. The waves localized near the free edges of a semi-infinite plate and the waves in semi-infinite cylindrical shells damping from free edges along the generator, are also called Rayleigh type surface waves [3], [4]. The problems of the existence of free vibrations damping from free ends of momentless cylindrical shells along its generators are studied in [5,6-9].

The investigation of free interfacial vibrations was initiated by the Zilbergreit, et al. [10], and Getman, et. al. [11], where Stoneley wave analogues were investigated [12]. In paper [10] transverse vibrations running along the contact line of two semi-infinite plates and concentrated close to it are studied. In paper [11] the plane interfacial vibrations near the interface of two joined semi-strips with different elastic properties are investigated. Later, Kaplunov, et al. [13], [14], using the special asymptotic method studied the free interfacial vibrations of composed circular cylindrical shells [13] as well as the shells of revolution [14].

In the present paper free interfacial and edge vibrations of momentless close cylindrical shell, composed of finite orthotropic cylindrical shells with different elastic properties are studied. The dispersion equations to determine the appropriate frequencies of interfacial and edge vibrations of closed composed momentless cylindrical shell with variable curvature are obtained. An asymptotic link between dispersion equations of the considered problem and the analogous problem for plate-strip, composed of orthotropic plate-strips with different elastic properties are established. Also, an asymptotic link between dispersion equations of the considered problem and the problem of interfacial vibrations of semi-infinite and infinite composed cylindrical shells are established. The derived dispersion equations and related asymptotic formulae can be used for controlling the spectrum of frequencies of the stated problem by varying the geometry of the shell and mechanical properties of materials. In particular, one can control the spectrum by shifting either the origin of the spectrum or the points of condensation from the undesirable resonance region [15].

1. **Statement of the problem and some mathematical features.** Free

interfacial and edge vibrations of closed cylindrical shell composed of finite orthotropic momentless cylindrical shells with different elastic coefficients are considered. The choice of the coordinate system and the possible shell form are shown in Fig. 1.

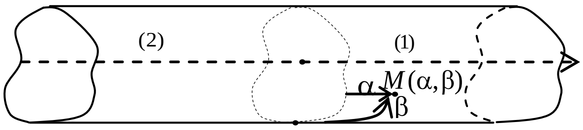

Fig. 1

Here α is the orientated length of the generatrix $-l^{(2)} \le \alpha \le l^{(1)}$, and $\alpha = 0$ corresponds to the interface of material properties separation. β is the length of the arc of the directing curve $0 \le \beta \le s$, where s is the complete length of the directing curve. It is supposed that the square of curvature of the composed cylindrical shell can be presented as Fourier series:

$$R^{-2} = k^2\left(\frac{r_0}{2} + \sum_{m=1}^{\infty} r_m \cos km\beta\right) \quad 0 \le \beta \le s,\ k = \frac{2\pi}{s},\ \sum_{m=1}^{\infty} | r_m | < +\infty \tag{1.1}$$

Note that depending on the curvature of the directing curve the values of k, may be multiple of the presented ones .

For $\alpha = 0$ full contact conditions are set. All the values corresponding to the right shell ($0 \le \alpha \le l^{(1)}$) on (Fig.1) are marked with superscript (1). Similarly, for the left shell ($-l^{(2)} \le \alpha \le 0$) superscript (2) is used. The equations corresponding to momentless classical theory of orthotropic cylindrical shells are used for describing vibrations of shells as given below [16]

$$\begin{gathered}
-B_{11}^{(r)}\frac{\partial^2 u_1^{(r)}}{\partial\alpha^2} - B_{66}^{(r)}\frac{\partial^2 u_1^{(r)}}{\partial\beta^2} - \left(B_{12}^{(r)} + B_{66}^{(r)}\right)\frac{\partial^2 u_2^{(r)}}{\partial\alpha\partial\beta} + \frac{B_{12}^{(r)}}{R}\frac{\partial u_3^{(r)}}{\partial\alpha} = \lambda^{(r)} u_1^{(r)} \\
-\left(B_{12}^{(r)} + B_{66}^{(r)}\right)\frac{\partial^2 u_1^{(r)}}{\partial\alpha\partial\beta} - B_{66}^{(r)}\frac{\partial^2 u_2^{(r)}}{\partial\alpha^2} - B_{22}^{(r)}\frac{\partial^2 u_2^{(r)}}{\partial\beta^2} + B_{22}^{(r)}\frac{\partial}{\partial\beta}\left(\frac{u_3^{(r)}}{R}\right) = \lambda^{(r)} u_2^{(r)} \\
-\frac{B_{12}^{(r)}}{R}\frac{\partial u_1^{(r)}}{\partial\alpha} - \frac{B_{22}^{(r)}}{R}\frac{\partial u_2^{(r)}}{\partial\beta} + \frac{B_{22}^{(r)}}{R^2} u_3^{(r)} = \lambda^{(r)} u_3^{(r)}\ ,\quad (r = 1,2)
\end{gathered} \tag{1.2}$$

Here $u_1^{(r)}, u_2^{(r)}, u_3^{(r)}$ $(r = 1,2)$ are the projections of the displacement vector to the directions of α, β and the normal to the shell surface, respectively. $R^{-1} = R^{-1}(\beta)$ is the radius of the curvature of the directing curve; $\lambda^{(r)} = \rho^{(r)}\omega^2$, where ω is the angular frequency of free vibrations, and $\rho^{(r)}(r=1,2)$ are the densities of the materials; $B_{ij}^{(r)}(r=1,2)$ are the coefficients of elasticity of the composed shell. The boundary conditions have the form [16]

$$T_1^{(1)}|_{\alpha=0}=T_1^{(2)}|_{\alpha=0},\quad S_{12}^{(1)}|_{\alpha=0}=S_{12}^{(2)}|_{\alpha=0},\quad u_1^{(1)}|_{\alpha=0}=u_1^{(2)}|_{\alpha=0},\quad u_2^{(1)}|_{\alpha=0}=u_2^{(2)}|_{\alpha=0} \quad (1.3)$$

$$\left.\frac{\partial u_1^{(r)}}{\partial\alpha}+\frac{B_{12}^{(r)}}{B_{11}^{(r)}}\left(\frac{\partial u_2^{(r)}}{\partial\beta}-\frac{u_3^{(r)}}{R}\right)\right|_{\alpha=(-1)^{r-1}l^{(r)}}=0,\quad \left.\frac{\partial u_1^{(r)}}{\partial\beta}+\frac{\partial u_2^{(r)}}{\partial\alpha}\right|_{\alpha=(-1)^{r-1}l^{(r)}}=0,\ r=1,2 \quad (1.4)$$

where $T_1^{(r)},\ S_{12}^{(r)}, r=1,2,$ are tangential normal and shear forces, respectively:

$$T_1^{(r)}=hB_{11}^{(r)}\left(\frac{\partial u_1^{(r)}}{\partial\alpha}+\frac{B_{12}^{(r)}}{B_{11}^{(r)}}\left(\frac{\partial u_2^{(r)}}{\partial\beta}-\frac{u_3^{(r)}}{R}\right)\right),\quad S_{12}^{(r)}=hB_{66}^{(r)}\left(\frac{\partial u_1^{(r)}}{\partial\beta}+\frac{\partial u_2^{(r)}}{\partial\alpha}\right) \quad (1.5)$$

where h is the thickness of shell. Relations (1.3) and (1.4) are full contact conditions at $\alpha=0$ and free ends conditions at $\alpha=l^{(1)},\alpha=-l^{(2)}$, respectively (Fig. 1).

It is known that any boundary-value problem originated from the system of equations (1.2) (with fixed index (r)) has a continuous spectrum band, coinciding with the segment $0\le\lambda^{(r)}\le\lambda_0^{(r)}$, which is the range of function [9]

$$\Omega^{(r)}(\beta,\theta^{(r)})=\frac{B_{66}^{(r)}\Delta^{(r)}R^{-2}(\beta)\sin^4\theta^{(r)}}{B_{66}^{(r)}(B_{11}^{(r)}\sin^4\theta^{(r)}+B_{22}^{(r)}\cos^4\theta^{(r)})+(\Delta^{(r)}-2B_{12}^{(r)}B_{66}^{(r)})\cos^2\theta^{(r)}\sin^2\theta^{(r)}} \quad (1.6)$$

$$0\le\beta\le s,\quad 0\le\theta^{(r)}\le 2\pi,\ \Delta^{(r)}=B_{11}^{(r)}B_{22}^{(r)}-(B_{12}^{(r)})^2\quad r=1,2$$

Note, that the appearance of the continuous spectrum band is the result of violation of ellipticity of system (1.2) by Douglas-Nierenberg and it is not related to boundary conditions ([17], p. 97).

It is well known that the ellipticity of the system is not sufficient for correct formulation of Dirichlet problem even in the case of homogeneous systems [19-22]. In order to the problem (1.2)-(1.4) has nontrivial solution it is necessary to impose additional algebraic conditions along the boundary of the shell and the interface of materials. This condition is called condition of complementation or Shapiro-Lopatinsky condition [9,19-22]. In analogy with [20] one can show that the Shapiro-Lopatinsky condition on the interface of materials has the form

$$\Omega(\beta,\lambda^{(1)},\lambda^{(2)})=\frac{1}{S^{(1)}(\beta,\lambda^{(1)})S^{(2)}(\beta,\lambda^{(2)})}\left\{\frac{\Delta^{(1)}}{B_{66}^{(2)}}\lambda^{(1)}Q^{(2)}(\beta,\lambda^{(2)})+\frac{\Delta^{(2)}}{B_{66}^{(1)}}\lambda^{(2)}Q^{(1)}(\beta,\lambda^{(1)})\right\}+$$
$$(\gamma_1^{(1)}+\gamma_2^{(1)})(\gamma_1^{(2)}+\gamma_2^{(2)})(\gamma_1^{(1)}\gamma_2^{(1)}+\gamma_1^{(2)}\gamma_2^{(2)})- \quad (1.7)$$
$$-2\left(\gamma_1^{(1)}\gamma_2^{(1)}-\frac{\lambda^{(1)}B_{12}^{(1)}}{S^{(1)}(\beta,\lambda^{(1)})}\right)\left(\gamma_1^{(2)}\gamma_2^{(2)}-\frac{\lambda^{(2)}B_{12}^{(2)}}{S^{(2)}(\beta,\lambda^{(2)})}\right)\neq 0,\ 0\le\beta\le s;$$

$$Q^{(r)}(\beta,\lambda^{(r)})=B_{66}^{(r)}(\lambda^{(r)}-B_{22}^{(r)}R^{-2}(\beta))+\sqrt{B_{22}^{(r)}\lambda^{(r)}(B_{11}^{(r)}\lambda^{(r)}-\Delta^{(r)}R^{-2}(\beta))}$$
$$S^{(r)}(\beta,\lambda^{(r)})=B_{11}^{(r)}\lambda^{(r)}-\Delta^{(r)}R^{-2}(\beta). \quad (1.8)$$

Note, that for boundary conditions (1.4) the Shapiro-Lopatinsky condition is satisfied out of the range $[0,\lambda_0^{(1)}/\rho^{(1)}]\cup[0,\lambda_0^{(2)}/\rho^{(2)}]$. This fact can be proved analogously, as in [9]. Therefore, the Shapiro-Lopatinsky condition of problem (1.2)- (1.4) has the form (1.7).

Let us denote the range of ω^2 where $\Omega(\beta,\lambda^{(1)},\lambda^{(2)})=0$ as Ω_γ.

The following statement is true: *the spectrum of frequencies of problem (1.2)- (1.4) out of the range* $[0,\lambda_0^{(1)}/\rho^{(1)}]\cup[0,\lambda_0^{(2)}/\rho^{(2)}]\cup\Omega_\gamma$ *consists of isolated free frequencies of finite multiplicity* [17], [22].

2. Derivation and analysis of dispersion equations. For further calculations it is convenient to reduce the system of equations (1.2) to the system of equations

$$\Gamma^{(r)}u_1^{(r)}=\frac{B_{12}^{(r)}}{B_{11}^{(r)}}\frac{\partial^3 w^{(r)}}{\partial\alpha^3}-\frac{B_{22}^{(r)}}{B_{11}^{(r)}}\frac{\partial^3 w^{(r)}}{\partial\alpha\partial\beta^2}+\frac{B_{12}^{(r)}}{B_{11}^{(r)}}\frac{\lambda^{(r)}}{B_{66}^{(r)}}\frac{\partial w^{(r)}}{\partial\alpha}$$
$$\Gamma^{(r)}u_2^{(r)}=\frac{B_{22}^{(r)}}{B_{11}^{(r)}}\frac{\partial^3 w^{(r)}}{\partial\beta^3}+\frac{\Delta^{(r)}-B_{12}^{(r)}B_{66}^{(r)}}{B_{11}^{(r)}B_{66}^{(r)}}\frac{\partial^3 w^{(r)}}{\partial\alpha^2\partial\beta}+\frac{B_{22}^{(r)}}{B_{11}^{(r)}}\frac{\lambda^{(r)}}{B_{66}^{(r)}}\frac{\partial w^{(r)}}{\partial\beta} \quad (2.1)$$
$$-\frac{B_{12}^{(r)}}{B_{22}^{(r)}}\frac{1}{R^2}\frac{\partial u_1^{(r)}}{\partial\alpha}-\frac{1}{R^2}\frac{\partial u_2^{(r)}}{\partial\beta}+\frac{w^{(r)}}{R^2}=\frac{\lambda^{(r)}}{B_{22}^{(r)}}w^{(r)},\quad r=1,2$$

where $w^{(r)}=u_3^{(r)}/R$, and operators $\Gamma^{(r)}(r=1,2)$ have the form

$$\Gamma^{(r)}=\frac{\partial^4}{\partial\alpha^4}+\frac{\Delta^{(r)}-2B_{12}^{(r)}B_{66}^{(r)}}{B_{11}^{(r)}B_{66}^{(r)}}\frac{\partial^4}{\partial\alpha^2\partial\beta^2}+\frac{B_{22}^{(r)}}{B_{11}^{(r)}}\frac{\partial^4}{\partial\beta^4}+$$
$$+\frac{B_{11}^{(r)}+B_{66}^{(r)}}{B_{11}^{(r)}B_{66}^{(r)}}\lambda^{(r)}\frac{\partial^2}{\partial\alpha^2}+\frac{B_{22}^{(r)}+B_{66}^{(r)}}{B_{11}^{(r)}B_{66}^{(r)}}\lambda^{(r)}\frac{\partial^2}{\partial\beta^2}+\frac{(\lambda^{(r)})^2}{B_{11}^{(r)}B_{66}^{(r)}} \quad (2.2)$$

The solution of the system (2.1) is searched in the form

$$u_1^{(r)}=\exp((-1)^r\chi^{(r)}k\alpha)(\sum_{m=1}^{\infty}u_m^{(r)}\sin km\beta),\ u_2^{(r)}=\exp((-1)^r\chi^{(r)}k\alpha)(\sum_{m=1}^{\infty}v_m^{(r)}\cos km\beta)$$
$$w^{(r)}=k\exp((-1)^r\chi^{(r)}k\alpha)(\sum_{m=1}^{\infty}w_m^{(r)}\sin km\beta),\ r=1,2$$
(2.3)

Substitute expressions (2.3) into system (2.1). From the first two equations (2.1) by equalizing the corresponding coefficients of the obtained trigonometric series we get

$$C_m^{(r)}u_m^{(r)} = (-1)^r \chi^{(r)} a_m^{(r)} w_m^{(r)}, \ \ C_m^{(r)} v_m^{(r)} = m b_m^{(r)} w_m^{(r)}, \ r = 1,2$$

$$a_m^{(r)} = \frac{B_{12}^{(r)}}{B_{11}^{(r)}}(\chi^{(r)})^2 + \frac{B_{22}^{(r)}}{B_{11}^{(r)}} m^2 + \frac{B_{12}^{(r)}}{B_{11}^{(r)}}(\eta^{(r)})^2, \ (\eta^{(r)})^2 = \frac{\lambda^{(r)}}{k^2 B_{66}^{(r)}} \tag{2.4}$$

$$b_m^{(r)} = \frac{\Delta^{(r)} - B_{12}^{(r)} B_{66}^{(r)}}{B_{11}^{(r)} B_{66}^{(r)}}(\chi^{(r)})^2 - \frac{B_{22}^{(r)}}{B_{11}^{(r)}}(m^2 - (\eta^{(r)})^2)$$

$$C_m^{(r)} = (\chi^{(r)})^4 - \frac{\Delta^{(r)} - 2B_{12}^{(r)} B_{66}^{(r)}}{B_{11}^{(r)} B_{66}^{(r)}} m^2 (\chi^{(r)})^2 + \frac{B_{11}^{(r)} + B_{66}^{(r)}}{B_{11}^{(r)}}(\eta^{(r)})^2 (\chi^{(r)})^2 +$$

$$+ (m^2 - (\eta^{(r)})^2)\left(\frac{B_{22}^{(r)}}{B_{11}^{(r)}} m^2 - \frac{B_{66}^{(r)}}{B_{11}^{(r)}}(\eta^{(r)})^2\right), \quad m = \overline{1,+\infty}$$

From the third equation of system (2.1) by considering the relationship (2.4) and the rule for multiplying the trigonometric series ([23], p. 592) we come to an infinite systems of equations

$$\sum_{n=1}^{\infty}(r_{n-m} - r_{n+m}) A_n^{(r)} w_n^{(r)} - 2\frac{B_{66}^{(r)}}{B_{22}^{(r)}}(\eta^{(r)})^2 w_m^{(r)} = 0, \ m = \overline{1,+\infty}, \ r = 1,2 \tag{2.5}$$

$$A_n^{(r)} = P_n^{(r)} / C_n^{(r)}, \ P_n^{(r)} = C_n^{(r)} + n^2 b_n^{(r)} - B_{12}^{(r)} / B_{22}^{(r)} (\chi^{(r)})^2 a_n^{(r)}, \ n = \overline{1,+\infty} \tag{2.6}$$

From the rule of multiplication of trigonometric series for $h > 0$ we have $r_{-h} = r_h$. Since in the region of determination of $A_n^{(r)}$ we have $A_n^{(r)} = O(1/n^2)$ then $\sum_{n=1}^{\infty} | A_n^{(r)} | < +\infty$. Taking into account representation (1.1), we have

$$\sum_{n,m=1}^{\infty} | A_n^{(r)} | (| r_{n+m} | + | r_{n-m} |) \le 3(| r_0 | / 2 + \sum_{m=1}^{\infty} | r_m |)(\sum_{n=1}^{\infty} | A_n^{(r)} |) < +\infty \tag{2.7}$$

Hence, infinite determinants of systems (2.5) at $\lambda^{(r)} \notin [0, \lambda_0^{(r)}], r = 1,2$ in the range of definition of coefficients (2.6) belong to the class of concurrent determinants known as normal determinants [24]. In order to systems (2.5) have nontrivial solutions, it is necessary and sufficient, that their determinants be equal to zero

$$D^{(r)}((\chi^{(r)})^2, (\eta^{(r)})^2, B_{11}^{(r)}, B_{22}^{(r)}, B_{12}^{(r)}, B_{66}^{(r)}, r_0, r_1, \ldots, r_m, \ldots) = 0, \ \ r = 1,2 \tag{2.8}$$

Assume, that $\chi_1^{(r)}, \chi_2^{(r)}$ $(r = 1,2)$ are different roots of equations (2.8) with positive real parts, then $\chi_3^{(r)} = -\chi_1^{(r)}, \chi_4^{(r)} = -\chi_2^{(r)}$ are other roots of equations (2.8), as well. Then, let us consider the solution of problem (1.2)- (1.4) in the form

$$u_1^{(r)} = \sum_{j=1}^{4}\sum_{m=1}^{\infty}\exp((-1)^r \chi_j^{(r)} k\alpha) u_{mj}^{(r)} \sin km\beta,\ u_2^{(r)} = \sum_{j=1}^{4}\sum_{m=1}^{\infty}\exp((-1)^r \chi_j^{(r)} k\alpha) v_{mj}^{(r)} \cos km\beta$$

$$w^{(r)} = k\sum_{j=1}^{4}\sum_{m=1}^{\infty}\exp((-1)^r \chi_j^{(r)} k\alpha) w_{mj}^{(r)} \sin km\beta,\ r = 1,2 \tag{2.9}$$

Here $u_{mj}^{(r)}, v_{mj}^{(r)}$ are the values of $u_{mj}^{(r)}, v_{mj}^{(r)}$, and $(w_{1j}^{(r)}, w_{2j}^{(r)}, ..., w_{mj}^{(r)}, ...)$ $j = 1,2$ are solutions of system (2.5) at $\chi^{(r)} = \chi_j^{(r)}$ $j = \overline{1,4},\ r = 1,2$, respectively. Taking into account boundary conditions (1.3)-(1.4) and relations (2.4) we come to the totality of systems of equations

$$\sum_{j=1}^{4}\frac{R_{1j}^{(1)}}{C_{mj}^{(1)}} w_{mj}^{(1)} - \frac{B_{11}^{(2)}}{B_{11}^{(1)}}\sum_{j=1}^{4}\frac{R_{1j}^{(2)}}{C_{mj}^{(2)}} w_{mj}^{(2)} = 0,\quad \sum_{j=1}^{4}\frac{R_{2j}^{(1)}}{C_{mj}^{(1)}} w_{mj}^{(1)} + \frac{B_{66}^{(2)}}{B_{66}^{(1)}}\sum_{j=1}^{4}\frac{R_{2j}^{(2)}}{C_{mj}^{(2)}} w_{mj}^{(2)} = 0$$

$$\sum_{j=1}^{4}\frac{R_{3j}^{(1)}}{C_{mj}^{(1)}} w_{mj}^{(1)} + \sum_{j=1}^{4}\frac{R_{3j}^{(2)}}{C_{mj}^{(2)}} w_{mj}^{(2)} = 0,\qquad \sum_{j=1}^{4}\frac{R_{4j}^{(1)}}{C_{mj}^{(1)}} w_{mj}^{(1)} - \sum_{j=1}^{4}\frac{R_{4j}^{(2)}}{C_{mj}^{(2)}} w_{mj}^{(2)} = 0,\quad m = \overline{1,+\infty}$$

$$\sum_{j=1}^{4}\frac{R_{1j}^{(r)}}{C_{mj}^{(r)}}\exp(z_j^{(r)}) w_{mj}^{(r)} = 0,\qquad \sum_{j=1}^{4}\frac{R_{2j}^{(r)}}{C_{mj}^{(r)}}\exp(z_j^{(r)}) w_{mj}^{(r)} = 0,\qquad r = 1,2$$

(2.10)

$$R_{1j}^{(r)} = (\chi_j^{(r)})^2 a_{mj}^{(r)} - \frac{B_{12}^{(r)}}{B_{11}^{(r)}} m^2 b_{mj}^{(r)} - \frac{B_{12}^{(r)}}{B_{11}^{(r)}} c_{mj}^{(r)}$$

$$R_{2j}^{(r)} = \chi_j^{(r)}(a_{mj}^{(r)} + b_{mj}^{(r)}),\ R_{3j}^{(r)} = \chi_j^{(r)} a_{mj}^{(r)},\ R_{4j}^{(r)} = b_{mj}^{(r)} \tag{2.11}$$

and $a_{mj}^{(r)}, b_{mj}^{(r)}, c_{mj}^{(r)}$ are the values of $a_m^{(r)}, b_m^{(r)}, c_m^{(r)}$ from (2.4) at $\chi^{(r)} = \chi_j^{(r)}$ respectively. In order to the union of systems of equations (2.10) has a solution, it is sufficient that the totality of equations

$$\Delta_{\mathrm{m}} = \exp(-z_1^{(1)} - z_2^{(1)} - z_1^{(2)} - z_2^{(2)}) d_m = 0,\ m = \overline{1,+\infty}$$

(2.12)

$$d_m = \begin{Vmatrix} R_{11}^{(1)} & R_{12}^{(1)} & R_{11}^{(1)}e^{z_1^{(1)}} & R_{12}^{(1)}e^{z_2^{(1)}} & -cR_{11}^{(2)} & -cR_{12}^{(2)} & -cR_{11}^{(2)}e^{z_1^{(2)}} & -cR_{12}^{(2)}e^{z_2^{(2)}} \\ R_{21}^{(1)} & R_{22}^{(1)} & -R_{21}^{(1)}e^{z_1^{(1)}} & -R_{22}^{(1)}e^{z_2^{(1)}} & dR_{21}^{(2)} & dR_{22}^{(2)} & -dR_{21}^{(2)}e^{z_1^{(2)}} & -dR_{22}^{(2)}e^{z_2^{(2)}} \\ R_{31}^{(1)} & R_{32}^{(1)} & -R_{31}^{(1)}e^{z_1^{(1)}} & -R_{32}^{(1)}e^{z_2^{(1)}} & R_{31}^{(2)} & R_{32}^{(2)} & -R_{31}^{(2)}e^{z_1^{(2)}} & -R_{32}^{(2)}e^{z_2^{(2)}} \\ R_{41}^{(1)} & R_{42}^{(1)} & R_{41}^{(1)}e^{z_1^{(1)}} & R_{42}^{(1)}e^{z_2^{(1)}} & -R_{41}^{(2)} & -R_{42}^{(2)} & -R_{41}^{(2)}e^{z_1^{(2)}} & -R_{42}^{(2)}e^{z_2^{(2)}} \\ R_{11}^{(1)}e^{z_1^{(1)}} & R_{12}^{(1)}e^{z_2^{(1)}} & R_{11}^{(1)} & R_{12}^{(1)} & 0 & 0 & 0 & 0 \\ -R_{21}^{(1)}e^{z_1^{(1)}} & -R_{22}^{(1)}e^{z_2^{(1)}} & R_{21}^{(1)} & R_{22}^{(1)} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -cR_{11}^{(2)}e^{z_1^{(2)}} & -cR_{12}^{(2)}e^{z_2^{(2)}} & -cR_{11}^{(2)} & -cR_{12}^{(2)} \\ 0 & 0 & 0 & 0 & dR_{21}^{(2)}e^{z_1^{(2)}} & dR_{22}^{(2)}e^{z_2^{(2)}} & -dR_{21}^{(2)} & -dR_{22}^{(2)} \end{Vmatrix}$$

(2.13)

where $c = B_{11}^{(2)} / B_{11}^{(1)},\ d = B_{66}^{(2)} / B_{66}^{(1)}$, has ω^2 solution out of the range $[0, \lambda^{(1)} / \rho^{(1)}] \bigcup [0, \lambda^{(2)} / \rho^{(2)}] \cup \Omega_\gamma$. Numerical analysis shows, that determinant (2.13) becomes small, when any two roots of equations (2.8) become close to each other. It makes the calculations very difficult and may bring to the appearance of false solutions of equations (2.12). It turns out that the multiplier in (2.13) that tends to zero when the roots approach each other may be isolated. Performing elementary actions over the columns of determinant (2.13) we get

$$\Delta_m = m^{26}(x_2^{(1)} - x_1^{(1)})^2 (x_2^{(2)} - x_1^{(2)})^2 \exp(-z_1^{(1)} - z_2^{(1)} - z_1^{(2)} - z_2^{(2)}) Det\left\|m_{ij}\right\|_{i,j=1}^{8} \qquad (2.14)$$

where the elements m_{ij} are given in Appendix 1. Equations (2.12) are equivalent to equations

$$Det\left\|m_{ij}\right\|_{i,j=1}^{8} = 0, \quad m = \overline{1,+\infty} \qquad (2.15)$$

Equations (2.15) are dispersion equations of the problem (1.2)-(1.4).

For $l^{(2)} \to \infty$ equations (2.15) have the form

$$Det\left\|m_{ij}\right\|_{i,j=1}^{8} = Det\left\|m_{ij}\right\|_{ij=1}^{6} Det\left\|m_{ij}\right\|_{i=7,8}^{j=7,8} + \sum_{j=1}^{2} O(\exp(z_j^{(2)})) = 0, \quad m = \overline{1,+\infty} \qquad (2.16)$$

Therefore, for $l^{(2)} \to \infty$ equations (2.15) split into equations

$$Det\left\|m_{ij}\right\|_{ij=1}^{6} = 0, \quad m = \overline{1,+\infty} \qquad (2.17)$$

$$Det\left\|m_{ij}\right\|_{i=7,8}^{j=7,8} = 0, \quad m = \overline{1,+\infty} \qquad (2.18)$$

Equations (2.17) are dispersion equations of interfacial and boundary vibrations of a closed semi-infinite composed cylindrical shell, with a free edge at $\alpha = l^{(1)}$.

Equations (2.18) are generalized Rayleigh's dispersion equations for semi-infinite closed cylindrical shell, made of material (2), and with a free edge at $\alpha = -l^{(2)}$ (p. [7-9]).

For $l^{(1)} \to \infty$ the equations (2.17) get the form

$$Det\left\|m_{ij}\right\|_{i,j=1}^{6} = Det\left\|m_{ij}\right\|_{i=5,6}^{j=3,4} Det\left\|m_{ij}\right\|_{i=1,2,3,4}^{j=1,2,5,6} + \sum_{j=1}^{2} O(\exp(z_j^{(1)})) = 0, \quad m = \overline{1,+\infty} \qquad (2.19)$$

Hence, for $l^{(1)} \to \infty$ equations (2.17) split into the equations

$$Det\left\|m_{ij}\right\|_{i=1,2,3,4}^{j=1,2,5,6} = 0, \quad m = \overline{1,+\infty} \qquad (2.20)$$

$$Det\left\|m_{ij}\right\|_{i=5,6}^{j=3,4} = 0, \quad m = \overline{1,+\infty} \qquad (2.21)$$

Equations (2.20) are dispersion equations of interfacial vibrations of a closed infinite composed cylindrical shell. Equations (2.21) are generalized Rayleigh's dispersion equations for a semi-infinite closed cylindrical shell, made of material (1), and with a free edge at $\alpha = l^{(1)}$ (p. [7-9]). Note, that

$$Det\|m_{ij}\|_{i=5,6}^{j=3,4} = K_2^{(1)}(\eta_m^{(1)}, x_1^{(1)}, x_2^{(1)}),\ Det\|m_{ij}\|_{i=7,8}^{j=7,8} = cdK_2^{(2)}(\eta_m^{(2)}, x_1^{(2)}, x_2^{(2)})$$

$$K_2^{(r)}(\eta_m^{(r)}, x_1^{(r)}, x_2^{(r)}) = \delta_1^{(r)}(x_1^{(r)})^2(x_2^{(r)})^2 + \delta_2^{(r)}x_1^{(r)}x_2^{(r)} + \delta_3^{(r)}((x_1^{(r)})^2 + (x_2^{(r)})^2) + \delta_4^{(r1)},\ r = 1,2$$

$$\delta_1^{(r)} = \frac{B_{11}^{(r)}B_{22}^{(r)} - (B_{12}^{(r)})^2}{(B_{11}^{(r)})^2}\left(\frac{\Delta^{(r)}}{B_{11}^{(r)}B_{66}^{(r)}} - \frac{B_{12}^{(r)}}{B_{11}^{(r)}}(\eta_m^{(r)})^2\right)$$

$$\delta_2^{(r)} = -(\eta_m^{(r)})^2\left(\frac{B_{22}^{(r)}\Delta^{(r)}}{(B_{11}^{(r)})^3} + \frac{B_{12}^{(r)}(\Delta^{(r)} - B_{12}^{(r)}B_{66}^{(r)} - B_{22}^{(r)}B_{66}^{(r)})}{(B_{11}^{(r)})^3}(\eta_m^{(r)})^2\right) \tag{2.22}$$

$$\delta_3^{(r)} = \frac{B_{12}^{(r)}\Delta^{(r)}}{(B_{11}^{(1)})^3}(\eta_m^{(r)})^2\left(1 - (\eta_m^{(r)})^2\right),\ \delta_3^{(r)} = \frac{B_{12}^{(r)}B_{66}^{(r)}(B_{12}^{(r)} + B_{66}^{(r)})}{(B_{11}^{(r)})^3}(\eta_m^{(r)})^4\left(1 - (\eta_m^{(r)})^2\right).$$

Taking into account (2.16), (2.19) and (2.22) equations (2.15) may by written in the form

$$Det\|m_{ij}\|_{i,j=1}^{8} = K_2^{(1)}(\eta_m^{(1)}, x_1^{(1)}, x_2^{(1)}).K_2^{(2)}(\eta_m^{(2)}, x_1^{(2)}, x_2^{(2)}).Det\|m_{ij}\|_{i=1,2,3,4}^{j=1,2,5,6} +$$

$$\sum_{j=1}^{2} O(\exp(z_j^{(1)})) + \sum_{j=1}^{2} O(\exp(z_j^{(2)})) = 0,\ \ m = \overline{1,+\infty} \tag{2.23}$$

For large $l^{(1)}$ and $l^{(2)}$ equations (2.15) split into the equations

$$K_2^{(1)}(\eta_m^{(1)}, x_1^{(1)}, x_2^{(1)}) = 0,\ K_2^{(2)}(\eta_m^{(2)}, x_1^{(2)}, x_2^{(2)}) = 0,\ Det\|m_{ij}\|_{i=1,2,3,4}^{j=1,2,5,6} = 0,\ \ m = \overline{1,+\infty}$$

Thus, for large $l^{(1)}$ and $l^{(2)}$ vibrations of the composite cylindrical shell may be divided into Rayleigh type boundary vibrations at the end-walls $\alpha = l^{(1)}$ and $\alpha = -l^{(2)}$ of the shell and interfacial vibrations at interface $\alpha = 0$ of the shell material properties.

In general, the solution of equations (2.8) is a difficult problem. That is why, for the establishment of asymptotic formulas for dispersion equations (2.15) we consider the following particular cases.

3. Particular cases. *Case a):* $R^{-2}(\beta) \equiv 0\ (r_m = 0,\ m = \overline{0,+\infty})$. In the expressions (1.2)-(1.5) we formally put $R^{-1}(\beta) \equiv 0$ everywhere. As a result, we obtain a system of equations of small planar vibrations of orthotropic plates [25]

$$\begin{aligned} &-B_{11}^{(r)}\frac{\partial^2 u_1^{(r)}}{\partial\alpha^2} - B_{66}^{(r)}\frac{\partial^2 u_1^{(r)}}{\partial\beta^2} - (B_{12}^{(r)} + B_{66}^{(r)})\frac{\partial^2 u_2^{(r)}}{\partial\alpha\partial\beta} = \lambda^{(r)}u_1^{(r)} \\ &-(B_{12}^{(r)} + B_{66}^{(r)})\frac{\partial^2 u_1^{(r)}}{\partial\alpha\partial\beta} - B_{66}^{(r)}\frac{\partial^2 u_2^{(r)}}{\partial\alpha^2} - B_{22}^{(r)}\frac{\partial^2 u_2^{(r)}}{\partial\beta^2} = \lambda^{(r)}u_2^{(r)} \end{aligned} \tag{3.1}$$

where $u_1^{(r)}, u_2^{(r)}\ (r = 1,2)$ are tangential displacement components of the middle plane point. $B_{ik}^{(r)}(r=1,2)$ are the coefficients of elasticity of plates. $\lambda^{(r)}=\rho^{(r)}\omega^2$, where ω is the angular frequency of free vibrations. $\rho^{(r)}(r=1,2)$ are the densities of the materials. All the values for the right plate ($0\le\alpha<l^{(1)}$) are marked with superscript (1), and the values for the left plate $(-l^{(2)}<\alpha\le 0)$ are marked with superscript (2).

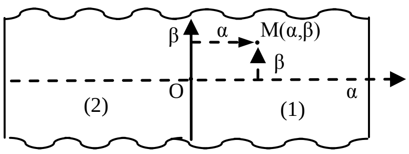

Fig. 2

The question of the existence of planar interfacial and edge vibrations of the composed plate-layer (Fig. 2) is investigated. It is assumed, that $\alpha(-l^{(2)}<\alpha<l^{(1)})$ and $\beta(-\infty<\beta<\infty)$ are rectilinear orthogonal coordinates of the middle surface point of the plate-layer (Fig. 2). At the interface of material property ($\alpha=0$) a full contact takes place. The boundary conditions have the form

$$T_1^{(1)}|_{\alpha=0}=T_1^{(2)}|_{\alpha=0},\ S_{12}^{(1)}|_{\alpha=0}=S_{12}^{(2)}|_{\alpha=0},\ u_1^{(1)}|_{\alpha=0}=u_1^{(2)}|_{\alpha=0},\ u_2^{(1)}|_{\alpha=0}=u_2^{(2)}|_{\alpha=0} \quad (3.2)$$

$$\left.\frac{\partial u_1^{(r)}}{\partial\alpha}+\frac{B_{12}^{(r)}}{B_{11}^{(r)}}\frac{\partial u_2^{(r)}}{\partial\beta}\right|_{\alpha=(-1)^{r-1}l^{(r)}}=0,\ \left.\frac{\partial u_1^{(r)}}{\partial\beta}+\frac{\partial u_2^{(r)}}{\partial\alpha}\right|_{\alpha=(-1)^{r-1}l^{(r)}}=0,\ r=1,2 \quad (3.3)$$

$$T_1^{(r)}=hB_{11}^{(r)}\left(\frac{\partial u_1^{(r)}}{\partial\alpha}+\frac{B_{12}^{(r)}}{B_{11}^{(r)}}\frac{\partial u_2^{(r)}}{\partial\beta}\right),\quad S_{12}^{(r)}=hB_{66}^{(r)}\left(\frac{\partial u_1^{(r)}}{\partial\beta}+\frac{\partial u_2^{(r)}}{\partial\alpha}\right) \quad (3.4)$$

where h is the thickness of plate. The relations (3.2) express the conditions of full contact at $\alpha=0$, (3.3) are conditions of free edges at $\alpha=-l^{(2)}$ and $\alpha=l^{(1)}$. The solution of system (3.1), with wave number m is searched in the form

$$u_1^{(r)}=u_m^{(r)}\sin km\beta\exp((-1)^r y^{(r)}km\alpha),\ u_2^{(r)}=v_m^{(r)}\cos km\beta\exp((-1)^r y^{(r)}km\alpha),\ r=1,2 \quad (3.5)$$

where $k=n_0 2\pi/s$, $n_0\in N$ and s is any positive number. Substituting expressions (3.5) in system (3.1), we obtain the system of equations

$$\begin{aligned}&\left(B_{11}^{(r)}(y^{(r)})^2-B_{66}^{(r)}+\frac{\lambda^{(r)}}{m^2k^2}\right)u_m^{(r)}-(B_{12}^{(r)}+B_{66}^{(r)})(-1)^r y^{(r)}v_m^{(r)}=0\\&(B_{12}^{(r)}+B_{66}^{(r)})(-1)^r y^{(r)}u_m^{(r)}+\left(B_{66}^{(r)}(y^{(r)})^2-B_{22}^{(r)}+\frac{\lambda^{(r)}}{m^2k^2}\right)v_m^{(r)}=0\end{aligned} \quad (3.6)$$

Equalizing the determinant of system (3.6) to zero, we get characteristic equations

$$c_m^{(r)} = (y^{(r)})^4 - \frac{\Delta^{(r)} - 2B_{12}^{(r)}B_{66}^{(r)}}{B_{11}^{(r)}B_{66}^{(r)}}(y^{(r)})^2 + \frac{B_{11}^{(r)} + B_{66}^{(r)}}{B_{11}^{(r)}}(\eta_m^{(r)})^2(y^{(r)})^2 + \\ + (1-(\eta_m^{(r)})^2)\left(\frac{B_{22}^{(r)}}{B_{11}^{(r)}} - \frac{B_{66}^{(r)}}{B_{11}^{(r)}}(\eta_m^{(r)})^2\right) = 0 \quad (3.7)$$

$$(\eta^{(r)})^2 = \frac{\lambda^{(r)}}{k^2 B_{66}^{(r)}};\ \eta_m^{(r)} = \frac{\eta^{(r)}}{m};\ r = 1,2;\ m = \overline{1,+\infty} \quad (3.8)$$

Let $y_j^{(r)},\ j = 1,2$ be different roots of equation (3.7) with positive real parts, then $y_3^{(r)} = -y_1^{(r)}, y_4^{(r)} = -y_2^{(r)},\ r = 1,2$ are other roots of equation (3.7). As a solution of equations (3.6) one can take

$$u_{mj}^{(r)} = (-1)^r y_j^{(r)}\frac{B_{12}^{(r)} + B_{66}^{(r)}}{B_{11}^{(r)}},\quad v_{mj}^{(r)} = (y_j^{(r)})^2 - \frac{B_{66}^{(r)}}{B_{11}^{(r)}}(1-(\eta_m^{(r)})^2),\ j = \overline{1,4} \quad (3.9)$$

The solution of problem (3.1)-(3.4) is searched in the form

$$u_1^{(r)} = \sum_{j=1}^{4} u_{mj}^{(r)} w_j^{(r)} \exp((-1)^r y_j^{(r)} km\alpha)\sin km\beta, \\ u_2^{(r)} = \sum_{j=1}^{4} v_{mj}^{(r)} w_j^{(r)} \exp((-1)^r y_j^{(r)} km\alpha)\cos km\beta,\ r = 1,2 \quad (3.10)$$

Taking into account boundary conditions (3.2), (3.3) we obtain the following system of equations

$$\sum_{j=1}^{4} P_{1j}^{(1)} w_j^{(1)} - \frac{B_{11}^{(2)}}{B_{11}^{(1)}}\sum_{j=1}^{4} P_{1j}^{(2)} w_j^{(2)} = 0,\ \sum_{j=1}^{4} P_{2j}^{(1)} w_j^{(1)} + \frac{B_{66}^{(2)}}{B_{66}^{(1)}}\sum_{j=1}^{4} P_{2j}^{(2)} w_j^{(2)} = 0 \\ \sum_{j=1}^{4} P_{3j}^{(1)} w_j^{(1)} + \sum_{j=1}^{4} P_{3j}^{(2)} w_j^{(2)} = 0,\ \sum_{j=1}^{4} P_{4j}^{(1)} w_j^{(1)} - \sum_{j=1}^{4} P_{4j}^{(2)} w_j^{(2)} = 0 \\ \sum_{j=1}^{4} P_{1j}^{(1)} \exp(z_j^{(1)}) w_j^{(1)} = 0,\ \sum_{j=1}^{4} P_{2j}^{(1)} \exp(z_j^{(1)}) w_j^{(1)} = 0 \\ -\frac{B_{11}^{(2)}}{B_{11}^{(1)}}\sum_{j=1}^{4} P_{1j}^{(2)} \exp(z_j^{(2)}) w_j^{(2)} = 0,\ \frac{B_{66}^{(2)}}{B_{66}^{(1)}}\sum_{j=1}^{4} P_{2j}^{(2)} \exp(z_j^{(2)}) w_j^{(2)} = 0. \quad (3.11)$$

$$P_{1j}^{(r)} = \frac{B_{66}^{(r)}}{B_{11}^{(r)}}((y_j^{(r)})^2 + \frac{B_{12}^{(r)}}{B_{11}^{(r)}}(1-(\eta_m^{(r)})^2)),\quad P_{2j}^{(r)} = y_j^{(r)}((y_j^{(r)})^2 + \frac{B_{12}^{(r)}}{B_{11}^{(r)}} + \frac{B_{66}^{(r)}}{B_{11}^{(r)}}(\eta_m^{(r)})^2) \\ P_{3j}^{(r)} = y_j^{(r)}\frac{B_{12}^{(r)} + B_{66}^{(r)}}{B_{11}^{(r)}},\quad P_{4j}^{(r)} = (y_j^{(r)})^2 - \frac{B_{66}^{(r)}}{B_{11}^{(r)}}(1-(\eta_m^{(r)})^2) \quad (3.12)$$

Producing elementary actions over the columns of the determinant of system (3.11) and setting it to zero, we obtain dispersion equations

$$\Delta_m^* = (y_2^{(1)} - y_1^{(1)})^2 (y_2^{(2)} - y_1^{(2)})^2 \exp(-z_1^{(1)} - z_2^{(1)} - z_1^{(2)} - z_2^{(2)}) Det\left\| m_{ij}^* \right\|_{ij=1}^{8} = 0, \; m = \overline{1,+\infty}$$
(3.13)

where the elements m_{ij}^* are given in Appendix 2.

Therefore, dispersion equations (3.13) are equivalent to equations

$$Det\left\| m_{ij}^* \right\|_{ij=1}^{8} = 0, \; m = \overline{1,+\infty} \tag{3.14}$$

Equations (3.14) are dispersion equations of problem (3.1)-(3.3).

For $l^{(2)} \to \infty$ we have asymptotic formulae

$$Det\left\| m_{ij}^* \right\|_{ij=1}^{8} = Det\left\| m_{ij}^* \right\|_{i=7,8}^{j=7,8} Det\left\| m_{ij}^* \right\|_{ij=1}^{6} + \sum_{j=1}^{2} O(\exp z_j^{(2)}), \quad m = \overline{1,+\infty} \tag{3.15}$$

Therefore, for $l^{(2)} \to \infty$ equations (3.14) split into the totality of equations

$$Det\left\| m_{ij}^* \right\|_{i=7,8}^{j=7,8} = 0, \; m = \overline{1,+\infty}; \; Det\left\| m_{ij}^* \right\|_{ij=1}^{6} = 0, \; m = \overline{1,+\infty}, \tag{3.16}$$

The first equations of (3.16) are Rayleigh's equations for a semi-infinite plate made of material (2) and with a free edge at $\alpha = -l^{(2)}$. The second equations of (3.16) are dispersion equations of semi-infinite composed plate with a free edge at $\alpha = -l^{(1)}$.

For $l^{(2)} \to \infty$, the second equations from (3.16) have asymptotic representation

$$Det\left\| m_{ij}^* \right\|_{ij=1}^{6} = Det\left\| m_{ij}^* \right\|_{i=5,6}^{j=3,4} Det\left\| m_{ij}^* \right\|_{i=1,2,3,4}^{j=1,2,5,6} + \sum_{j=1}^{2} O(\exp z_j^{(1)}), \quad m = \overline{1,+\infty} \tag{3.17}$$

Therefore, at $l^{(2)} \to \infty$ and $l^{(1)} \to \infty$ equations (3.14) have asymptotic representation

$$Det\left\| m_{ij}^* \right\|_{ij=1}^{8} = Det\left\| m_{ij}^* \right\|_{i=5,6}^{j=3,4} Det\left\| m_{ij}^* \right\|_{i=7,8}^{j=7,8} Det\left\| m_{ij}^* \right\|_{i=1,2,3,4}^{j=1,2,5,6} + \sum_{j=1}^{2} O(\exp(z_j^{(1)})) + \sum_{j=1}^{2} O(\exp(z_j^{(2)})), \quad m = \overline{1,+\infty} \tag{3.18}$$

$$\begin{aligned}
Det\left\| m_{ij}^* \right\|_{i=1,2,3,4}^{j=1,2,5,6} &= -\frac{B_{66}^{(1)}}{B_{11}^{(1)}} \frac{B_{12}^{(1)} + B_{66}^{(1)}}{B_{11}^{(1)}} \frac{B_{12}^{(2)} + B_{66}^{(2)}}{B_{11}^{(2)}} L(\eta_m^{(1)}, \eta_m^{(2)}), \; m = \overline{1,+\infty} \\
Det\left\| m_{ij}^* \right\|_{i=5,6}^{j=3,4} &= -\frac{B_{66}^{(1)}}{B_{11}^{(1)}} \frac{B_{12}^{(1)} + B_{66}^{(1)}}{B_{11}^{(1)}} K_2^{(1)}(\eta_m^{(1)}) \\
Det\left\| m_{ij}^* \right\|_{i=7,8}^{j=7,8} &= -\frac{\left(B_{66}^{(2)}\right)^2}{B_{11}^{(1)} B_{66}^{(1)}} \frac{B_{12}^{(2)} + B_{66}^{(2)}}{B_{11}^{(2)}} K_2^{(2)}(\eta_m^{(2)})
\end{aligned} \tag{3.19}$$

$$L(\eta_m^{(1)},\eta_m^{(2)})=K_2^{(1)}(\eta_m^{(1)})Q^{(2)}(\eta_m^{(2)})+\left(\frac{B_{66}^{(2)}}{B_{66}^{(1)}}\right)^2 K_2^{(2)}(\eta_m^{(2)})Q^{(1)}(\eta_m^{(1)})+$$

$$+\frac{B_{66}^{(2)}}{B_{66}^{(1)}}\left[2\left(y_1^{(1)}y_2^{(1)}-\frac{B_{12}^{(1)}}{B_{11}^{(1)}}(1-(\eta_m^{(1)})^2)\right)\left(y_1^{(2)}y_2^{(2)}-\frac{B_{12}^{(2)}}{B_{11}^{(2)}}(1-(\eta_m^{(2)})^2)\right)+\right.$$

$$\left.+(y_2^{(1)}+y_1^{(1)})(y_2^{(2)}+y_1^{(2)})((1-(\eta_m^{(2)})^2)y_1^{(1)}y_2^{(1)}+(1-(\eta_m^{(1)})^2)y_1^{(2)}y_2^{(2)})\right]$$

$$Q^{(r)}(\eta_m^{(r)})=y_1^{(r)}y_2^{(r)}+\frac{B_{66}^{(r)}}{B_{11}^{(r)}}(1-(\eta_m^{(r)})^2),\ r=1,2$$

$$K_2^{(r)}(\eta_m^{(r)})=(1-(\eta_m^{(r)})^2)\left(\frac{\Delta^{(r)}}{B_{11}^{(r)}B_{66}^{(r)}}-(\eta_m^{(r)})^2\right)-(\eta_m^{(r)})^2 y_1^{(r)}y_2^{(r)},\ r=1,2$$

Dispersion equations (3.14) have the form

$$Det\left\|m_{ij}^*\right\|_{ij=1}^{8}=-\left(\frac{B_{66}^{(2)}}{B_{11}^{(1)}}\right)^2\frac{B_{66}^{(1)}}{B_{11}^{(1)}}\left(\frac{B_{12}^{(1)}+B_{66}^{(1)}}{B_{11}^{(1)}}\frac{B_{12}^{(2)}+B_{66}^{(2)}}{B_{11}^{(2)}}\right)^2 K_2^{(1)}(\eta_m^{(1)})K_2^{(2)}(\eta_m^{(2)})L(\eta_m^{(1)},\eta_m^{(2)})+$$

$$\sum_{j=1}^{2}O(\exp z_j^{(1)})+\sum_{j=1}^{2}O(\exp z_j^{(2)})=0,\ m=\overline{1,+\infty}$$

(3.20)

Therefore, for $l^{(1)}\to\infty$ and $l^{(2)}\to\infty$ dispersion equations of problem (3.1)-(3.3) split into the totality of equations

$$K_2^{(1)}(\eta_m^{(1)})=0,\quad K_2^{(2)}(\eta_m^{(2)})=0,\quad L(\eta_m^{(1)},\eta_m^{(2)})=0,\quad m=\overline{1,+\infty} \qquad (3.21)$$

The first two equations of (3.21) are Rayleigh's equations for semi-infinite orthotropic plates, made of materials (1) and (2) on free edges at $\alpha=l^{(1)}$ and $\alpha=-l^{(2)}$, respectively [7-9]. The third equation of (3.21) is the analogous of Stoneley's dispersion equation for a composed infinite plate [13],[3],[4].

Thus, planar vibrations of a composed plate-strip with free edges may be separated on the boundary vibrations of Rayleigh types and interfacial vibrations of Stoneley type.

Note, that in dispersion equations $L(\eta_m^{(1)},\eta_m^{(2)})=0,\ m=\overline{1,+\infty}$ the elasticity coefficients of the left and right plates and the corresponding roots of characteristic equations (3.7) enter in a symmetric way. Thus, for example, if the left plate (superscript (2)) is softer (i.e. $\rho^{(2)}/\rho^{(1)}<<1,\ B_{ij}^{(2)}/B_{ij}^{(1)}<<1,\ i,j=1,2,6$), than the right one, then we can write

$$L(\eta_m^{(1)},\eta_m^{(2)})=Q^{(2)}(\eta_m^{(2)})\{K_2(\eta_m^{(1)})+O(B_{66}^{(2)}/B_{66}^{(1)})+O(\rho^{(2)}/\rho^{(1)})\}=0 \qquad (3.22)$$

Therefore, the existence of interfacial vibrations of a composed plate depends on the existence of the boundary vibrations of the right semi-infinite plate with a free edge [7-8]: i.e. it is obvious that there are interfacial vibrations. If $B_{ij}^{(2)}/B_{ij}^{(1)} \approx 1,\ ij=1,2,6;\ \rho^{(2)}/\rho^{(1)} \approx 1$, then there is a small chance for the existence of interfacial vibrations.

Case b): $R^{-2}=k^2 r_0/2\ (r_m=0, m=\overline{1,+\infty})$, i.e. we have momentless elastic circular closed composed cylindrical shell. In this case the systems (2.5) have the form

$$(r_0 A_m^{(r)} - 2\frac{B_{66}^{(r)}}{B_{22}^{(r)}}(\eta^{(r)})^2) w_m^{(r)} = 0,\ m=\overline{1,+\infty},\ r=1,2 \tag{3.23}$$

Therefore, equations (2.8) split into two totalitis of equations

$$r_0 P_m^{(r)} - 2\frac{B_{66}^{(r)}}{B_{22}^{(r)}} C_m^{(r)} (\eta^{(r)})^2 = 0,\ m=\overline{1,+\infty},\ r=1,2 \tag{3.24}$$

or equations

$$\begin{aligned}&\left((\eta^{(r)})^2 - \frac{\Delta^{(r)}}{B_{11}^{(r)}B_{66}^{(r)}}\frac{r_0}{2}\right)(\chi^{(r)})^4 - (\eta^{(r)})^2\left(\frac{\Delta^{(r)}-2B_{12}^{(r)}B_{66}^{(r)}}{B_{11}^{(r)}B_{66}^{(r)}}m^2 -\right.\\&\left.-\frac{B_{11}^{(r)}+B_{66}^{(r)}}{B_{11}^{(r)}}(\eta^{(r)})^2 + \frac{\Delta^{(r)}+B_{22}^{(r)}B_{66}^{(r)}}{B_{11}^{(r)}B_{66}^{(r)}}\frac{r_0}{2}\right)(\chi^{(r)})^2 +\\&+(\eta^{(r)})^2(m^2-(\eta^{(r)})^2)\left(\frac{B_{22}^{(r)}}{B_{11}^{(r)}}m^2 - \frac{B_{66}^{(r)}}{B_{11}^{(r)}}(\eta^{(r)})^2 + \frac{B_{22}^{(r)}}{B_{11}^{(r)}}\frac{r_0}{2}\right)=0,\ m=\overline{1,+\infty},\ r=1,2\end{aligned} \tag{3.25}$$

In this case, for finding dimensionless characteristics of free frequencies $\eta^{(r)}\ (r=1,2)$ in dispersion equations (2.15) expressions $x_1^{(r)}=\chi_1^{(r)}/m,\ x_2^{(r)}=\chi_2^{(r)}/m$ are used, where $\chi_1^{(r)}$ and $\chi_2^{(r)}$ are the roots of equations (3.25) with positive real parts. For a circular cylindrical shell we set $k=n_0 2\pi/s,\ n_0\in N$, where s is the full length of the directing circle.

For $r_0\to 0$ equations (3.25) become

$$\begin{aligned}&(\chi^{(r)})^4 - \left(\frac{\Delta^{(r)}-2B_{12}^{(r)}B_{66}^{(r)}}{B_{11}^{(r)}B_{66}^{(r)}}m^2 - \frac{B_{11}^{(r)}+B_{66}^{(r)}}{B_{11}^{(r)}}(\eta^{(r)})^2\right)(\chi^{(r)})^2 +\\&+(m^2-(\eta^{(r)})^2)\left(\frac{B_{22}^{(r)}}{B_{11}^{(r)}}m^2 - \frac{B_{66}^{(r)}}{B_{11}^{(r)}}(\eta^{(r)})^2\right)=0,\ m=\overline{1,+\infty},\ r=1,2\end{aligned} \tag{3.26}$$

These equations are characteristic equations of systems describing planar vibrations of a composed plate-strip (at $k=n_0 2\pi/s,\ n_0\in N$, where s is an arbitrary positive number). The roots $\chi^{(r)}/m$ of equations (3.26) with positive real parts are denoted by $y_1^{(r)}, y_2^{(r)}$. Then, in this case, for dispersion equations (2.15) the following asymptotic formulae are valid

$$Det\left\|m_{ij}\right\|_{i,j=1}^{8}=\left(\frac{B_{11}^{(1)}}{B_{66}^{(1)}}\frac{B_{11}^{(2)}}{B_{66}^{(2)}}\frac{B_{11}^{(1)}}{(B_{12}^{(1)}+B_{66}^{(1)})}\frac{B_{11}^{(2)}}{(B_{12}^{(2)}+B_{66}^{(2)})}\right)^{2}\left(N^{(1)}(\eta_m^{(1)}).N^{(2)}(\eta_m^{(2)})\right)^{2}Det\left\|m_{ij}^{*}\right\|_{ij=1}^{8}+$$
$$O(r_0/(2m^2))=0,\quad m=\overline{1,+\infty}$$

(3.27)

$$N^{(r)}(\eta_m^{(r)})=\frac{B_{22}^{(r)}\Delta^{(r)}}{(B_{11}^{(r)})^3}+\frac{B_{12}^{(r)}(\Delta^{(r)}-B_{66}^{(r)}B_{12}^{(r)}-B_{66}^{(r)}B_{22}^{(r)})}{(B_{11}^{(r)})^3}(\eta_m^{(r)})^2 \tag{3.28}$$

From equation (3.27) it follows, that for $r_0/m^2\to 0$ the equations (2.15) transform into equations (3.14).

Thus, equations (3.27) establish the asymptotic connection between the dispersion equations of the considered problem and analogous problem for a composed plate-strip.

For $l^{(1)}\to\infty$ and $l^{(2)}\to\infty$ and by taking into account (3.18)-(3.20),(3.27), dispersion equations (2.15) in this case may be written in the form

$$\begin{gathered}Det\left\|m_{ij}\right\|_{i,j=1}^{8}=\frac{\left(B_{11}^{(2)}\right)^2}{B_{11}^{(1)}B_{66}^{(1)}}\left(N^{(1)}(\eta_m^{(1)})N^{(2)}(\eta_m^{(2)})\right)^2K_2^{(1)}(\eta_m^{(1)})K_2^{(2)}(\eta_m^{(2)})L(\eta_m^{(1)},\eta_m^{(2)})+\\ O\left(\frac{r_0}{2m^2}\right)+\sum_{j=1}^{2}O(\exp(z_j^{(1)}))+\sum_{j=1}^{2}O(\exp(z_j^{(2)}))=0,\ m=\overline{1,+\infty}\end{gathered} \tag{3.29}$$

From (3.29) it follows, that for $r_0/m^2\to 0$, $l^{(1)}\to\infty$ and $l^{(2)}\to\infty$ dispersion equations (2.15) split into equations (3.21). Therefore, for small r_0/m^2 and large $l^{(1)},l^{(2)}$ the roots of equation (3.21) are the approximate values of roots of equations (2.14) (see Tables 1,2).

Case c): $R^{-2}=k^2(r_0/2+r_1\cos k\beta)$, $r_m=0$, $m=\overline{2,+\infty}$, i.e. we have a noncircular composed closed cylindrical shell (with $k=2\pi/s$, where s is the full length of the directing curve). In this case the systems of equations (2.5) have the form

$$r_1P_{m-1}^{(r)}\omega_{m-1}^{(r)}+r_{mm}^{(r)}\omega_m^{(r)}+r_1P_{m+1}^{(r)}\omega_{m+1}^{(r)}=0,\ m=\overline{1,+\infty},\ r=1,2 \tag{3.30}$$

$$\omega_m=w_m^{(r)}/c_m^{(r)},\ r_{mm}^{(r)}=r_0P_m^{(r)}-2B_{66}^{(r)}(\eta^{(r)})^2C_m^{(r)}/B_{22}^{(r)} \tag{3.31}$$

Since the determinants of systems (3.30) are of the normal type, then in order to find a non-trivial solution, we equalize them to zero

$$D^{(r)}((\chi^{(r)})^2,(\eta^{(r)})^2,B_{11}^{(r)},B_{22}^{(r)},B_{12}^{(r)},B_{66}^{(r)},r_0,r_1)=0,\ r=1,2 \tag{3.32}$$

Solutions $(\chi^{(r)})^2$ of equations (3.32) can be obtained in analogous way as in [6-9].

The following statement is valid: for fixed $m\geq 2$ and at $\lambda^{(r)}\notin[0,\lambda_0^{(r)}]$, equations (3.32) have formal solutions of the form

$$(\chi_j^{(r)})^2=(\chi_{mj}^{(r)})^2+\alpha_{mj}^{(r)}r_1^2+\beta_{mj}^{(r)}r_1^4+..., j=1,2,\ r=1,2 \tag{3.33}$$

where $\chi_{mj}^{(r)}$ $(j=1,2)$ are the roots of the equation $r_{mm}^{(r)}=0$ (i.e. the equations (3.25) with positive real parts) and

$$\alpha_{mj}^{(r)}=\left.\frac{P_m^{(r)}(P_{m-1}^{(r)}r_{m+1m+1}^{(r)}+P_{m+1}^{(r)}r_{m-1m-1}^{(r)})}{r_{m-1m-1}^{(r)}r_{m+1m+1}^{(r)}r_{mm}^{(r)\prime}}\right|_{\chi^{(r)}=\chi_{mj}^{(r)}} \quad j=1,2,\ r=1,2 \tag{3.34}$$

where $r_{mm}^{(r)\prime}$ is the derivative of $r_{mm}^{(r)}$ with respect to $(\chi^{(r)})^2$.

Thus, in this case, for finding the coefficients of damping $k\chi_j^{(r)}/m$ $(j=1,2)$ the approximate formulae may be used

$$\chi_j^{(r)}/m=((\chi_{mj}^{(r)}/m)^2+\alpha_{mj}^{(r)}r_1^2/m^2)^{1/2},\ (j=1,2) \tag{3.44}$$

Equations (2.15) are used for finding the corresponding characteristics of free frequencies $\eta^{(r)}/m$.

4. Numerical investigation. In Tables 1,2 using dispersion equations (2.15), (3.14) dimensionless characteristics of free frequencies $\eta^{(1)}/m$ and characteristics of damping coefficients of the corresponding forms $k\chi_j^{(r)}/m$ $(j=1,2)$ depending on m, a,b are given for closed cylindrical shells with directing curves

$$x=a\cos t,\ y=b\sin t,\ a=2,b=1.5;\ a=2,b=1. \tag{4.1}$$

In Tables 1, 2 the results of calculations for three cases (1, 2, 3) are given for $R^{-2}=k^2(r_0/2+r_1\cos k\beta);$ $R^{-2}=k^2r_0/2;$ $R^{-2}=0$, respectively, applied to composed cylindrical shells with directions (4.1), and made of boroplate and paper with mechanical parameters [16], [26]:

Boroplate: $\rho^{(1)}=2\cdot10^3 кг/м^3, E_1^{(1)}=2.646\cdot10^{11}H/м^2, E_2^{(1)}=1.323\cdot10^{10}H/м^2,$

$$G^{(1)}=9.604\cdot10^9 H/м^2,\quad \nu_1^{(1)}=0.2, \nu_2^{(1)}=0.01, \tag{4.2}$$

Paper: $\rho^{(2)}=0.16 кг/м^3, E_1^{(2)}=2.95281\cdot10^9 H/м^2, E_2^{(2)}=2.2106\cdot10^9 H/м^2,$

$$G^{(2)}=9.77076\cdot10^8 H/м^2,\ \nu_1^{(2)}=\nu_2^{(2)}E_1^{(2)}/E_2^{(2)}, \nu_2^{(2)}=0.23, \tag{4.3}$$

and with geometrical parameters: in Table 1: $a=2, b=1.5, s=5.52587$ (half of the length of ellipse), $k=4\pi/s,\ r_0=0.273895,\ r_1=0.033796,\ l^{(1)}=l^{(2)}=15, l^{(1)}=l^{(2)}=2.5$; in Table 2: $a=2, b=1,\ s=4.8442$ (half of the length of ellipse), $k=4\pi/s,\ r_0=0.407139,\ r_1=0.229356,\quad l^{(1)}=l^{(2)}=15,\ l^{(1)}=l^{(2)}=2.5$.

The following quantities are taken as the characteristics of the attenuation factors

$$k\chi_0^{(1)}/m = \pm\min\{k\operatorname{Re}\chi_1^{(1)}/m, k\operatorname{Re}\chi_2^{(1)}/m\},\ k\chi_0^{(2)}/m = \pm\min\{k\operatorname{Re}\chi_1^{(2)}/m,\ k\operatorname{Re}\chi_2^{(2)}/m\} \quad (4.4)$$

In equalities (4.4) the sign plus corresponds to interfacial vibrations near the interface material properties $\alpha = 0$, and the sign minus corresponds to Rayleigh type near the end-wall of the shell $\alpha = l^{(1)}$ and $\alpha = -l^{(2)}$. Note, that $\eta^{(1)}$ and $\eta^{(2)}$ are related as follows

$$\eta^{(2)} = \frac{\rho^{(2)}}{\rho^{(1)}}\frac{B_{66}^{(1)}}{B_{66}^{(2)}}\eta^{(1)} \quad (4.5)$$

In Tables 1,2 after the characteristics of free frequencies the type of surface waves is given: $e(r),\ r = 1,2$ denotes Rayleigh waves near the end-wall of the component of cylindrical shell with index ("r"), "in" denotes interfacial vibrations, "iq" denotes those coefficients of damping which are pure image. The data for the case 3 is given for those wave numbers, which are given in tables. For small wave numbers they may differ from the presented numbers. Here $s = 4$.

Table 1

$l^{(1)}/l^{(2)}$	m	Case 1			Case 2		
		$k\chi_0^{(1)}/m$	$k\chi_0^{(2)}/m$	$\eta^{(1)}/m$	$k\chi_0^{(1)}/m$	$k\chi_0^{(2)}/m$	$\eta^{(1)}/m$
$15/15$	20	-0.1223 - iq	Iq - -0.8585	0.96470 - 32.5583	-0.1231 0.0322 Iq	iq 1.4657 -0.8585	0.96424 e(1) 0.99765 in 32.5583 e(2)
	21	-0.1231 0.0554 iq	1.4721 1.4818 - 0.8587	0.96423 e(1) 0.99301 in 32.5571 e(2)	-0.1231 0.0609 Iq	1.4707 1.4799 - 0.8587	0.96423 e(1) 0.99153 in 32.5577 e(2)
	100	-0.1232 0.0838 iq	1.7839 1.7854 - 0.8598	0.96420 e(1) 0.98380 in 32.5548 e(2)	-0.1232 0.0838 Iq	1.7838 1.7853 - 0.8597	0.96420 e(1) 0.98380 in 32.5548 e(2)
	250	-0.1232 0.0841 iq	1.8184 1.8187 - 0.8598	0.96420 e(1) 0.98369 in 32.5548 e(2)	-0.1232 0.0841 Iq	1.8184 1.8187 -0.8598	0.96420 e(1) 0.98369 in 32.5548 e(2)

	m	Case 3	$k\chi_0^{(1)}/m$=-0.1702 $k\chi_0^{(1)}/m$=0.1163 $k\chi_0^{(1)}/m$=iq	$k\chi_0^{(2)}/m$=2.5295 $k\chi_0^{(2)}/m$=2.5225 $k\chi_0^{(2)}/m$=-1.1877	$\eta^{(1)}/m$=0.96420 e(1) $\eta^{(1)}/m$=0.98367 in $\eta^{(1)}/m$=32.5548 e(2)		
	20	-0.1231 - Iq	Iq - -0.8585	0.96424 e(1) - 32.5580 e(2)	-0.1230 0.0280 Iq	iq 1.4659 -0.8585	0.96428 e(1) 0.99823 in 32.5580 e(2)
$2.5/2$	21	-0.1231 0.0546 Iq	1.4722 1.4819 -0.8586	0.96423 e(1) 0.99320 in 32.5577 e(2)	-0.1231 0.0664 Iq	1.4707 1.4800 -0.8586	0.96423 e(1) 0.99168 in 32.5577 e(2)
	100	-0.1232 0.0838 Iq	1.7839 1.7854 -0.8597	0.96420 e(1) 0.98380 in 32.5549 e(2)	-0.1232 0.0832 Iq	1.7838 1.7853 -0.8597	0.96420 e(1) 0.98380 in 32.5549 e(2)
	250	-0.1232 0.0841 Iq	1.8184 1.8187 -0.8598	0.96420 e(1) 0.98369 in 32.5548 (2)	-0.1232 0.0841 Iq	1.8184 1.8187 -0.8598	0.96420 e(1) 0.98369 in 32.5548 e(2)
	Case 3		$k\chi_0^{(1)}/m$=-0.1702 $k\chi_0^{(1)}/m$=0.1163 $k\chi_0^{(1)}/m$=iq	$k\chi_0^{(2)}/m$=2.5295 $k\chi_0^{(2)}/m$=2.5225 $k\chi_0^{(2)}/m$=-0.8598	$\eta^{(1)}/m$=0.96420 e(1) $\eta^{(1)}/m$=0.98367 in $\eta^{(1)}/m$=32.5548 e(2)		

Conclusion: In this paper it is shown that at the interface of materials properties of a composed momentless finite cylindrical shell with arbitrary plane direction, vibrations damping from the line of the material section along its generatrices may exist. It is shown, that near the free end-wall of the cylindrical shell waves of Rayleigh type may appear. It is shown that the frequencies of free interfacial and boundary vibrations of a composed cylindrical shell consisted of finite orthotropic momentless cylindrical shells with different elastic coefficients are determined from equations (2.15). Also, for a circular cylindrical shell the coefficients of damping χ are determined from equations (3.25), and for a plate they are determined from equations (3.26). The frequencies of free interfacial vibrations of a composed plate-strip are determined from equations (3.14). The existence of interfacial and boundary vibrations depends on the curvature of the directing curve, elasticity coefficients, materials density and the length of the component shells. For large wave numbers m or for a small curvature of the directing curve all the characteristics of free interfacial and boundary vibrations of momentless closed cylindrical shell tend to the characteristics of a planar interfacial vibrations of a plate-strip. Numerical analysis shows that with increasing the square of curvature of the directing curve of a cylindrical shell the first frequencies of interfacial and boundary vibrations appear for larger m and higher corresponding frequencies. The damping of vibrations depends on materials properties and geometrical parameters.

Table 2

$l^{(1)}/l^{(2)}$	m	Case 1			Case 2		
		$k\chi_0^{(1)}/m$	$k\chi_0^{(2)}/m$	$\eta^{(1)}/m$	$k\chi_0^{(1)}/m$	$k\chi_0^{(2)}/m$	$\eta^{(1)}/m$
$15/15$	24	-0.1404 - iq	iq - -0.8568	0.96424 e(1) - 32.5730 e(2)	-0.1405 - Iq	1.7270 - 0.9735	0.96423 e(1) - 32.5707 e(2)
	25	-0.1404 0.0213 iq	iq 1.7161 - 0.8576	0.96423 e(1) 0.99922 in 32.5571 e(2)	-0.1405 0.0613 Iq	1.7415 1.6799 - 0.9781	0.96423 e(1) 0.99342 in 32.5669 e(2)
	100	-0.1405 0.0955 iq	2.0195 2.0216 - 0.8597	0.96420 e(1) 0.98385 in 32.5548 e(2)	-0.1405 0.0954 Iq	2.0153 2.0175 - 0.9807	0.96420 e(1) 0.98380 in 32.5548 e(2)
	250	-0.1405 0.0959 iq	2.0705 2.0709 - 0.8598	0.96420 e(1) 0.98370 in 32.5548 e(2)	-0.1405 0.0959 Iq	2.0703 2.0707 - 0.9808	0.96420 e(1) 0.98370 in 32.5548 e(2)
	m	Case 3	$k\chi_0^{(1)}/m$ =-0.1702 $k\chi_0^{(1)}/m$ =0.1163 $k\chi_0^{(1)}/m$ =iq		$k\chi_0^{(2)}/m$ =2.5225 $k\chi_0^{(2)}/m$ =2.5225 $k\chi_0^{(2)}/m$ =-1.1877		$\eta^{(1)}/m$ =0.96420 e(1) $\eta^{(1)}/m$ =0.98367 in $\eta^{(1)}/m$ =32.5548 e(2)
$2.5/2.5$	24	-0.1404 - iq	iq - -0.9774	0.96424 e(1) - 32.5581 e(2)	-0.1407 - Iq	iq - -0.9793	0.96412 e(1) - 32.5707 e(2)
	25	-0.1404 0.0144 iq	iq 1.7162 -0.9794	0.96423 e(1) 0.99964 in 32.5580 e(2)	-0.1404 0.0612 Iq	iq 1.6799 -0.9794	0.96423 e(1) 0.99345 in 32.5578 e(2)
	100	-0.1405 0.0955 iq	2.0195 2.0216 -0.9807	0.96420 e(1) 0.98385 in 32.5550 e(2)	-0.1405 0.0955 Iq	2.0153 2.0175 - 0.9807	0.96420 e(1) 0.98386 in 32.5550 e(2)

	250	-0.1405 0.0959 iq	2.0705 2.0709 -0.9807	0.96420 e(1) 0.98370 in 32.5548 e(2)	-0.1405 0.0959 Iq	2.0702 2.0707 - 0.9807	0.96420 e(1) 0.98370 in 32.55480 e(2)
	Case 3		$k\chi_0^{(1)}/m$=-0.1702 $k\chi_0^{(1)}/m$=0.1163 $k\chi_0^{(1)}/m$=iq		$k\chi_0^{(2)}/m$=2.5225 $k\chi_0^{(2)}/m$=2.5225 $k\chi_0^{(2)}/m$=-0.9807		$\eta^{(1)}/m$=0.96420 e(1) $\eta^{(1)}/m$=0.98367 in $\eta^{(1)}/m$=32.55480 e(2)

Appendix 1. Analytical expressions for elements of matrix m_{ij} of (2.14) are as follows

$$m_{11}=R_{11}^{(1)},\ m_{12}=\left(\frac{\Delta^{(1)}}{(B_{11}^{(1)})^2}-\frac{B_{12}^{(1)}B_{66}^{(1)}}{(B_{11}^{(1)})^2}(\eta_m^{(1)})^2\right)(x_1^{(1)}+x_2^{(1)}),\ m_{13}=m_{11}\exp\left(z_1^{(1)}\right)$$

$$m_{14}=m_{12}\exp\left(z_2^{(1)}\right)+m_{11}[z_1^{(1)}z_2^{(1)}],\ m_{15}=-B_{11}^{(2)}/B_{11}^{(1)}R_{11}^{(2)},\ z_j^{(r)}=-kmx_j^{(r)}l^{(r)},\ j=1,2;r=1,2$$

$$m_{16}=-\frac{B_{11}^{(2)}}{B_{11}^{(1)}}\left(\frac{\Delta^{(2)}}{(B_{11}^{(2)})^2}-\frac{B_{12}^{(2)}B_{66}^{(2)}}{(B_{11}^{(2)})^2}(\eta_m^{(2)})^2\right)(x_1^{(2)}+x_2^{(2)})$$

$$m_{17}=-m_{15}\exp(z_1^{(2)}),\ m_{18}=-m_{16}\exp(z_2^{(2)})-m_{15}[z_1^{(2)}z_2^{(2)}];$$

$$m_{21}=R_{21}^{(1)},\ m_{22}=\frac{\Delta^{(1)}}{B_{11}^{(1)}B_{66}^{(1)}}((x_1^{(1)})^2+(x_2^{(1)})^2+x_1^{(1)}x_2^{(1)})+\frac{B_{12}^{(1)}+B_{22}^{(1)}}{B_{11}^{(1)}}(\eta_m^{(1)})^2$$

$$m_{23}=-m_{21}\exp(z_1^{(1)}),\ m_{24}=-m_{22}\exp(z_2^{(1)})-m_{21}[z_1^{(1)}z_2^{(1)}],\ m_{25}=B_{66}^{(2)}/B_{66}^{(1)}R_{21}^{(2)}$$

$$m_{26}=\frac{B_{66}^{(2)}}{B_{66}^{(1)}}\left(\frac{\Delta^{(2)}}{B_{11}^{(2)}B_{66}^{(2)}}((x_1^{(2)})^2+(x_2^{(2)})^2+x_1^{(2)}x_2^{(2)})+\frac{B_{12}^{(2)}+B_{22}^{(2)}}{B_{11}^{(2)}}(\eta_m^{(2)})^2\right)\qquad(2.15)$$

$$m_{27}=-m_{25}\exp(z_1^{(2)}),\ m_{28}=-m_{26}\exp(z_2^{(2)})-m_{25}[z_1^{(2)}z_2^{(2)}];$$

$$m_{31}=R_{31}^{(1)},\ m_{32}=\frac{B_{12}^{(1)}}{B_{11}^{(1)}}((x_1^{(1)})^2+(x_2^{(1)})^2+x_1^{(1)}x_2^{(1)})+\frac{B_{22}^{(1)}}{B_{11}^{(1)}}+\frac{B_{12}^{(1)}}{B_{11}^{(1)}}(\eta_m^{(1)})^2$$

$$m_{33}=-m_{31}\exp(z_1^{(1)}),\ m_{34}=-m_{32}\exp(z_2^{(1)})-m_{31}[z_1^{(1)}z_2^{(1)}],\ m_{35}=R_{31}^{(2)}$$

$$m_{36}=\frac{B_{12}^{(2)}}{B_{11}^{(2)}}((x_1^{(2)})^2+(x_2^{(2)})^2+x_1^{(2)}x_2^{(2)})+\frac{B_{22}^{(2)}}{B_{11}^{(2)}}+\frac{B_{12}^{(2)}}{B_{11}^{(2)}}(\eta_m^{(2)})^2$$

$$m_{37}=-m_{35}\exp(z_1^{(2)}),\ m_{38}=-m_{36}\exp(z_2^{(2)})-m_{35}[z_1^{(2)}z_2^{(2)}];$$

$$m_{41}=R_{41}^{(1)},\ m_{42}=\frac{\Delta^{(1)}-B_{12}^{(1)}B_{66}^{(1)}}{B_{11}^{(1)}B_{66}^{(1)}}(x_1^{(1)}+x_2^{(1)})$$

$$m_{43}=m_{41}\exp(z_1^{(1)}),\ m_{44}=m_{42}\exp(z_2^{(1)})+m_{41}[z_1^{(1)}z_2^{(1)}],\ m_{45}=-R_{41}^{(2)}$$

$$m_{46}=-\frac{\Delta^{(2)}-B_{12}^{(2)}B_{66}^{(2)}}{B_{11}^{(2)}B_{66}^{(2)}}(x_1^{(2)}+x_2^{(2)})$$

$$m_{47}=-m_{45}\exp(z_1^{(2)}),\ m_{48}=-m_{46}\exp(z_2^{(2)})-m_{45}[z_1^{(2)}z_2^{(2)}];$$

$m_{51} = m_{13},\quad m_{52} = m_{14},\quad m_{53} = m_{11},\quad m_{54} = m_{12},\quad m_{5j} = 0,\quad j = 5,6,7,8;$

$m_{61} = m_{23},\quad m_{62} = m_{24},\quad m_{63} = m_{21},\quad m_{64} = m_{22},\quad m_{6j} = 0,\ j = 5,6,7,8;$

$m_{7j} = 0,\ j = 1,2,3,4\ ,\ m_{75} = m_{17},\quad m_{76} = m_{18},\quad m_{77} = m_{15},\quad m_{78} = m_{16};$

$m_{8j} = 0,\ j = 1,2,3,4\ ,\ m_{85} = m_{27},\quad m_{86} = m_{28},\quad m_{87} = m_{25},\quad m_{88} = m_{26};$

$[z_1^{(r)} z_2^{(r)}] = -kml^{(r)}(\exp(z_1^{(r)}) - \exp(z_2^{(r)}))/(z_1^{(r)} - z_2^{(r)});$

$\eta_m^{(r)} = \eta^{(r)} / m;\ x_i^{(r)} = \chi_i^{(r)} / m;\ i, r = 1,2;$

Appendix 2. Analytical expressions for elements of matrix m_{ij}^* of (3.13) are as follows

$$m_{11}^* = P_{11}^{(1)},\ m_{12}^* = \frac{B_{66}^{(1)}}{B_{11}^{(1)}}\left(y_1^{(1)} + y_2^{(1)}\right),\ m_{13}^* = m_{11}^* \exp(z_1^{(1)}),\ m_{14}^* = m_{12}^* \exp(z_2^{(1)}) + m_{11}^*[z_1^{(1)} z_2^{(1)}]$$

$$m_{15}^* = -\frac{B_{11}^{(2)}}{B_{11}^{(1)}} P_{11}^{(2)},\ m_{16}^* = -\frac{B_{66}^{(2)}}{B_{11}^{(1)}}\left(y_1^{(2)} + y_2^{(2)}\right),\ m_{17}^* = m_{15}^* \exp(z_1^{(2)}),\ m_{18}^* = m_{16}^* \exp(z_2^{(2)}) + m_{15}^*[z_1^{(2)} z_2^{(2)}]$$

$$m_{21}^* = P_{21}^{(1)},\ m_{22}^* = \frac{\Delta^{(1)} - B_{12}^{(1)} B_{66}^{(1)}}{B_{11}^{(1)} B_{66}^{(1)}} - (\eta_m^{(1)})^2 + y_1^{(1)} y_2^{(1)},\ m_{23}^* = -m_{21}^* \exp(z_1^{(1)})$$

$$m_{24}^* = -m_{22}^* \exp(z_2^{(1)}) - m_{21}^*[z_1^{(1)} z_2^{(1)}],\ m_{25}^* = \frac{B_{66}^{(2)}}{B_{66}^{(1)}} P_{21}^{(2)}$$

$$m_{26}^* = \frac{B_{66}^{(2)}}{B_{66}^{(1)}}\left(\frac{\Delta^{(2)} - B_{12}^{(2)} B_{66}^{(2)}}{B_{11}^{(2)} B_{66}^{(2)}} - (\eta_m^{(2)})^2 + y_1^{(2)} y_2^{(2)}\right)$$

$$m_{27}^* = -m_{25}^* \exp(z_1^{(2)}),\ m_{28}^* = -m_{26}^* \exp(z_2^{(2)}) - m_{25}^*[z_1^{(2)} z_2^{(2)}];$$

$$m_{31}^* = P_{31}^{(1)},\ m_{32}^* = \frac{B_{12}^{(1)} + B_{66}^{(1)}}{B_{11}^{(1)}},\ m_{33}^* = -m_{31}^* \exp(z_1^{(1)}),\ m_{34}^* = -m_{32}^* \exp(z_2^{(1)}) - m_{31}^*[z_1^{(1)} z_2^{(1)}] \tag{3.17}$$

$$m_{35}^* = P_{31}^{(2)},\ m_{36}^* = \frac{B_{12}^{(2)} + B_{66}^{(2)}}{B_{11}^{(2)}},\ m_{37}^* = -m_{35}^* \exp(z_1^{(2)}),\ m_{38}^* = -m_{36}^* \exp(z_2^{(2)}) - m_{35}^*[z_1^{(2)} z_2^{(2)}];$$

$$m_{41}^* = P_{41}^{(1)},\ m_{42}^* = \left(y_1^{(1)} + y_2^{(1)}\right),\ m_{43}^* = m_{41}^* \exp(z_1^{(1)}),\ m_{44}^* = m_{42}^* \exp(z_2^{(1)}) + m_{41}^*[z_1^{(1)} z_2^{(1)}]$$

$$m_{45}^* = -P_{41}^{(2)},\ m_{46}^* = -\left(y_1^{(2)} + y_2^{(2)}\right),\ m_{47}^* = m_{45}^* \exp(z_1^{(2)}),\ m_{38}^* = m_{46}^* \exp(z_2^{(2)}) + m_{45}^*[z_1^{(2)} z_2^{(2)}];$$

$$m_{51}^* = m_{13}^*,\ m_{52}^* = m_{14}^*,\ m_{53}^* = m_{11}^*,\ m_{54}^* = m_{12}^*,\ m_{5j}^* = 0,\ j = 5,6,7,8;$$

$$m_{61}^* = m_{23}^*,\ m_{62}^* = m_{24}^*,\ m_{63}^* = m_{21}^*,\ m_{64}^* = m_{22}^*,\ m_{6j}^* = 0,\ j = 5,6,7,8;$$

$$m_{7j}^* = 0,\ j = 1,2,3,4,\ m_{75}^* = m_{17}^*,\ m_{76}^* = m_{18}^*,\ m_{77}^* = m_{15}^*,\ m_{78}^* = m_{16}^*;$$

$$m_{8j}^* = 0,\ j = 1,2,3,4,\ m_{85}^* = m_{27}^*,\ m_{86}^* = m_{28}^*,\ m_{87}^* = m_{25}^*,\ m_{88}^* = m_{26}^*;$$

$$[z_1^{(r)} z_2^{(r)}] = -kml^{(r)}(\exp(z_1^{(r)}) - \exp(z_2^{(r)}))/(z_1^{(r)} - z_2^{(r)});\ z_j^{(r)} = -y_j^{(r)} mkl^{(r)},\ j = 1,2; r = 1,2\ .$$

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ԱՄՓՈՓՈՒՄ

ՓՈՓՈԽԱԿԱՆ ԿՈՐՈՒԹՅԱՆ ԵՎ ԱԶԱՏ ԾԱՅՐԵՐՈՎ ԱՆՄՈՄԵՆՏ ԳԼԱՆԱՅԻՆ ԹԱՂԱՆԹԻ ՍԵՓԱԿԱՆ ԻՆՏԵՐՖԵՅՍ ԵՎ ԵԶՐԱՅԻՆ ՏԱՏԱՆՈՒՄՆԵՐԸ

Գ.Ռ. ՂՈՒԼՂԱԶԱՐՅԱՆ, Լ.Գ. ՂՈՒԼՂԱԶԱՐՅԱՆ

Հետազոտվում է կամայական կորության փակ անմոմենտ գլանային թաղանթի սեփական ինտերֆեյս և եզրային տատանումները, որը բաղկացած է երկու վերջավոր, տարբեր առաձգականության հատկություններ ունեցող օրթոտրոպ գլանային թաղանթներից: Ենթադրվում է, որ թաղանթի ծայրերն ազատ են:

РЕЗЮМЕ

СОБСТВЕННЫЕ ИНТЕРФЕЙСНЫЕ И КРАЕВЫЕ КОЛЕБАНИЯ БЕЗМОМЕНТНОЙ ЦИЛИНДРИЧЕСКОЙ ОБОЛОЧКИ ПЕРЕМЕННОЙ КРИВИЗНЫ СО СВОБОДНЫМИ ТОРЦАМИ

Г.Р. ГУЛГАЗАРЯН, Л.Г. ГУЛГАЗАРЯН

Исследуются собственные интерфейсные и краевые колебания безмоментной замкнутой цилиндрической оболочки переменной кривизны, составленной из двух конечных ортотропных цилиндрических оболочек с разными упругими свойствами. Предполагается, что торцы оболочки свободны.