АСТРОФИЗИКА

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# PHYSICAL ACCEPTABILITY OF THE RENYI HOLOGRAPHIC DARK ENERGY MODEL UNDER THE HUBBLE'S CUTOFF IN $f(T, B)$ GRAVITY

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The paper deals with the investigations of the behaviour and physical acceptability of the spatially homogeneous and isotropic FLRW space-time filled with pressureless matter and Rényi holographic dark energy under the Hubble's IR-cutoff in the framework of $f(T, B)$ gravity. We have calculated some cosmological parameters to study the astrophysical consequences of the constructed model. We discussed their behaviour during the cosmic evolution, in particular, the statefinder and EoS parameters. It is found that the constructed Rényi holographic dark energy model travels from Phantom, $\Lambda$CDM, and lastly enters & remains in Quintessence dark energy era with the increase in redshift.

Keywords: *$f(T,B)$ gravity: Renyi holographic dark energy: Hubble's cutoff: redshift*

1. *Introduction*. Cosmology aims to comprehend the universe on a large scale. Over recent years, one of the greatest challenges faced by cosmologists is to explain the nature and mechanism of cosmic acceleration [1-3], which has been confirmed by some observational data such as type Ia supernova [4-7], baryon acoustic oscillations (BAO) [8], weak lensing [9] and large scale structure (LSS) [10-12] etc. One of the key issues in modern cosmology and high-energy theoretical physics has been determining the phenomenological explanation of cosmic acceleration [13]. The dark energy (DE), which makes up 68.3% of the exotic component and possesses negative pressure, is what drives the expansion of the universe [14-17]. Modified theories of gravity offer an alternate way to study the cosmos and its accelerating expansion. A few suitable properties of modified theories of gravity are found in [18]. With modifications to the Einstein-Hilbert action, several researchers have constructed many cosmological models in modified theories of gravity, including $f(R)$ gravity [19-24], $f(T)$ gravity [25-30], $f(R,T)$ gravity [31-33], $f(T,B)$ gravity [34-36] etc. A comprehensive overview of modified theories of gravity was already given by Nojiri et al. [37]. Recently, Shankaranarayanan and Johnson [38] discussed modified theories of gravity: why, how and what. Also, Olmo et al. [39] provided the models of stellar structure in modified theories of gravity with their challenges and lessons.

The holographic dark energy (HDE), one of numerous dynamical DE models, has recently emerged as a viable tool for investigating the DE conundrum. The proposal was based on the quantum properties of black holes, which have been extensively researched in the literature [40,41] to study quantum gravity. The particle horizon [42], Hubble's horizon $H^{-1}$ [43,44], conformal-age-like [45], Granda-Oliveros [46,47], Ricci scalar radius [48] and event horizon [49] are the different kinds of *IR*-cutoffs that have been used in HDE models in explaining accelerating cosmic expansion which is compatible with the present astronomical data. Presently, to discuss various cosmological phenomena, the Rényi, Tsallis and Sharma-Mittal HDE models have been proposed [50,51]. These HDE models have been examined under different *IR*-cutoffs by many eminent researchers [52-55] etc. Recently, Nojiri et al. [56,57] showed that barrow entropic DE and different faces of DE like Tsallis entropic DE, the Rényi entropic DE, and the Sharma-Mittal entropic DE all can be regarded as different candidates for the generalized HDE family, with respective holographic cutoffs. Additionally, Nojiri & Odintsov [58,59] proposed the generalized HDE model where the *IR*-cutoff is identified with the combination of the FRW universe parameters like the Hubble rate, particle and future horizons, cosmological constant, the universe lifetime (if finite) and their derivatives.

In recent studies, many cosmologists have constructed Rényi HDE models in different modified gravity theories. Recently, Bharali and Das [60] constructed a modified Rényi HDE cosmological model in $f(R,T)$ theory of gravity. Also, Wankhade et al. [61] developed Rényi HDE cosmological model in $f(R)$ gravity with Hubble's *IR*-cutoff. Alam et al. [62] examined Rényi HDE and its behaviour in $f(G)$ gravity. Bhardwaj et al. [63] established Rényi HDE models in teleparallel gravity under Hubble's cutoff etc.

In this paper, we have taken up our study of the cosmological model in the framework of $f(T,B)$ gravity. The $f(T,B)$ gravity has been established by Bahamonde et al. [64] as the precise relationship between very popular $f(R)$ and $f(T)$ gravity. In this new theory, the boundary term $B$ is taken into account, which is the difference between the Ricci scalar $R$ and torsion scalar $T$ given by $R=-T+B$. This relation between $R$, $T$ and $B$ is regarded as one of the basic equations of general theory of relativity and its teleparallel equivalent. Bahamonde et al. [65] explored the validity of laws of thermodynamics, and Zubair et al. [66] derived the energy constraints for de Sitter (exponential), power-law, $\Lambda$CDM and Phantom models, in the framework of $f(T,B)$ gravity. Bahamonde and Capozziello [67] adopted the Noether symmetry approach to study dynamical systems and explored cosmological solutions. Capozziello et al. [68] derived gravitational waves (GW's) for $f(T,B)$ gravity and obtained the different polarization states of GW's. Paliathanasis and Leon [69] investigated the dynamics of $f(T,B)$ gravity in a spatially flat Friedmann-Lemaitre-Robertson-Walker (FLRW) universe, by applying the approach which is more general than that of Hubble's normalization and they found that Minkowski space-time as an exact solution for the field equation described by a stationary point. Rivera et al. [70] explored the possibility of using cosmographic parameters in terms of the derivatives of scale factor as tools for investigating the behaviour of cosmological models in $f(T, B)$ gravity.

Motivated by the above discussion, in this paper, we investigate the physical acceptability of the Rényi HDE model in $f(T, B)$ gravity under Hubble's *IR*-cutoff by considering the scale factor obtained by Pawar et al. [71]. The paper has been organized as follows: In section 2, we present the general framework of $f(T, B)$ gravity in brief. The metric and field equations are given in section 3. In section 4, we obtain the solutions of field equations. We discuss the physical acceptability of the $f(T, B)$ Renyi HDE model under Hubble's *IR*-cutoff in section 5. At the end, conclusions are presented in section 6.

2. *The framework of f(T, B) gravity*. In this section, we discuss the basic notions of $f(T, B)$ gravity and its field equations as per the description given in [64,67].

The action for $f(T, B)$ gravity is given as

$$S = \int e\left[\frac{f(T, B)}{k^2} + L_m\right] d^4x , \tag{1}$$

where $f(T, B)$ is the function of the torsion scalar $T$ and of the boundary term $B = 2\partial_\alpha \left(eT^\alpha\right)/e$. $L_m$ is the matter Lagrangian, $k^2 = 8\pi G$, $G$ is the Newtonian gravitational constant, and the speed of light $c$ is taken as 1. Here $e$ represents the determinant of tetrad, $\left[e^i_\alpha\right]$ i.e., $e = \left|e^i_\alpha\right| = \sqrt{-g}$ ; $T_\beta$ is the torsion vector given by $T_\beta = T^\alpha_{\alpha\beta}$, where the torsion tensor $T^\alpha_{\alpha\beta}$ is the antisymmetric part of Weitzenbocks connection $W^i_{\alpha\beta} = \partial_\alpha e^i_\beta$ defined as

$$T^i_{\alpha\beta} = W^i_{\alpha\beta} - W^i_{\beta\alpha} = \partial_\alpha e^i_\beta - \partial_\beta e^i_\alpha \tag{2}$$

The contorsion tensor is the difference between the Levi-Civita and Weitzenbocks connection and is defined by

$$K^\alpha_{\beta\gamma} = \frac{1}{2}\left(T^\alpha_{\beta\gamma} - T^\alpha_{\gamma\beta} + T^\alpha_{\beta\gamma}\right). \tag{3}$$

A new tensor, $S^{\alpha\beta}_\gamma$, is constructed from the torsion and contorsion tensors for a better understanding of the definition of the scalar equivalent to the curvature scalar of Riemannian geometry as follows,

$$S^{\alpha\beta}_\gamma = \frac{1}{2}\left(K^{\alpha\beta}_\gamma - \delta^\alpha_\gamma T^\beta - \delta^\beta_\gamma T^\alpha\right). \tag{4}$$

The torsion scalar $T$ which is similar to the scalar curvature $R$ in GTR is defined by

$$T = S_{\gamma}^{\alpha\beta} T_{\alpha\beta}^{\gamma} . \tag{5}$$

The scalar curvature $R$ and the torsion scalar $T$ are connected by the relation,

$$R = -T + B . \tag{6}$$

By varying the action given in the equation (1) w.r.t. the tetrad field, the field equations are obtained as

$$\begin{gathered} 2e\delta_{\beta}^{\alpha}\nabla^{\mu}\nabla_{\mu} f_B - 2e\nabla^{\alpha}\nabla_{\beta} f_B + eBf_B\,\delta_{\beta}^{\alpha} + 4e\left(\partial_{\mu} f_B + \partial_{\mu} f_T\right)S_{\beta}^{\mu\alpha} + 4e_{\beta}^{i}\,\partial_{\mu}\left(eS_{i}^{\mu\alpha}\right)f_T \\ - 4ef_T T_{\mu\beta}^{\gamma} S_{\gamma}^{\alpha\mu} - ef\,\delta_{\beta}^{\alpha} = 16\pi e\Theta_{\beta}^{\alpha} , \end{gathered} \tag{7}$$

where $\Theta_{\beta}^{\alpha} = e_{\beta}^{i}\,\Theta_{i}^{\alpha}$ is the standard energy-momentum tensor.

3. *Metric and field equations in f(T, B) gravity*. We consider the spatially flat FLRW line element in the form:

$$ds^2 = dt^2 - a^2(t)\left[dx^2 + dy^2 + dz^2\right], \tag{8}$$

where $a(t)$ is the scale factor of the universe.

Then the set of diagonal tetrads related to the metric (8) is

$$\left[e_{\beta}^{i}\right] = \mathrm{diag}\left[1, a(t), a(t), a(t)\right], \tag{9}$$

The determinant of a matrix (9) is

$$e = a^3(t). \tag{10}$$

The components of field equation (7), the Ricci scalar $R$, the torsion scalar $T$ and boundary term $B$, for the line element (8) are calculated in [65-67] as

$$-3H^2\left(3f_B + 2f_T\right) + 3H\dot{f}_B - 3\dot{H}f_B + \frac{1}{2}f = k^2\rho , \tag{11}$$

$$-\left(3H^2 + \dot{H}\right)\left(3f_B + 2f_T\right) - 2H\dot{f}_T + \ddot{f}_B + \frac{1}{2}f = -k^2 p , \tag{12}$$

$$R = -T + B = 12H^2 + 6\dot{H} , \tag{13}$$

$$T = 6H^2 , \tag{14}$$

$$B = 6\left(\dot{H} + 3H^2\right), \tag{15}$$

where $H = \dot{a}/a$ is the Hubble's parameter and the overhead dot represents the differentiation w.r.t. the cosmic time $t$.

We consider the matter distribution as a combination of pressureless matter and isotropic DE in the form

$$\Theta_{\alpha\beta} = \Theta_{\alpha\beta}^{(m)} + \Theta_{\alpha\beta}^{(DE)} , \tag{16}$$

where $\Theta_{\alpha\beta}^{(m)}$ and $\Theta_{\alpha\beta}^{(DE)}$ are the energy-momentum tensors of pressureless matter and isotropic DE, respectively, given by

$$\Theta_{\alpha\beta}^{(m)} = \rho_m u_\alpha u_\beta , \tag{17}$$

$$\Theta_{\alpha\beta}^{(DE)} = \left(\rho_{DE} + p_{DE}\right) u_\alpha u_\beta - p_{DE} g_{\alpha\beta} , \tag{18}$$

where $\rho_m$ is the matter-energy density, $\rho_{DE}$ and $p_{DE}$ are respectively the energy density and pressure of HDE fluid, $u^\alpha = (0, 0, 0, 1)$, where $u^\alpha$ is the four-velocity vector of the fluid with $u^\alpha u_\alpha = 1$.

The EoS parameter of HDE is defined as

$$\omega_{DE} = \frac{p_{DE}}{\rho_{DE}} . \tag{19}$$

Parameterization of the energy-momentum tensor of dark energy $\Theta_{\alpha\beta}^{(DE)}$ leads to

$$\Theta_{\alpha\beta}^{(DE)} = \left[1, -(\omega_{DE})_x, -(\omega_{DE})_y, -(\omega_{DE})_z\right] \rho_{DE} , \tag{20}$$

where $(\omega_{DE})_x$, $(\omega_{DE})_y$, $(\omega_{DE})_z$ are the directional EoS parameters on *x*, *y* and $z$ axis respectively.

Then the field equations (11) and (12) with the energy-momentum tensor (16) (for $k^2 = 1$) become

$$-3H^2\left(3f_B + 2f_T\right) + 3H\dot{f}_B - 3\dot{H}f_B + \frac{1}{2}f = \left(\rho_m + \rho_{DE}\right), \tag{21}$$

$$-\left(3H^2 + \dot{H}\right)\left(3f_B + 2f_T\right) - 2H\dot{f}_T + \ddot{f}_B + \frac{1}{2}f = -\omega_{DE}\rho_{DE} , \tag{22}$$

We consider the $f(T, B)$ gravity model of the form [34,67] as

$$f(T, B) = \alpha B^m + \beta T^n , \tag{23}$$

where $\alpha$, $\beta$, *m* and *n* are constants.

For this model it was already shown in [72] that for $m < 0$, the Friedmann equations will be affected mostly in the accelerating late-time universe, whereas the same situation will be for $m > 0$ at early time, when boundary contribution is zero.

By the use of (23), the field equations (21) and (22) becomes

$$\begin{aligned} -3H^2\left\{3m\alpha B^{m-1} + 2n\beta T^{n-1}\right\} + 3m(m-1)\alpha H B^{m-2}\dot{B} - 3m\alpha\dot{H}B^{m-1} \\ + \frac{1}{2}\left(\alpha B^m + \beta T^n\right) = \rho_m + \rho_{DE} \end{aligned} \tag{24}$$

$$\begin{aligned} -\left(3H^2 + \dot{H}\right)\left[3m\alpha B^{m-1} + 2n\beta T^{n-1}\right] - 2n(n-1)\beta H T^{n-2}\dot{T} + m(m-1)(m-2)\alpha B^{m-3}\dot{B}^2 \\ + m(m-1)\alpha B^{m-2}\ddot{B} + \frac{1}{2}\left(\alpha B^m + \beta T^n\right) = -\omega_{DE}\rho_{DE} . \end{aligned} \tag{25}$$

Using (14) and (15) in (24) and (25) we obtain

$$\rho_m + \rho_{DE} = -3\alpha(m-1)(6)^{m-1}\left(\dot{H}+3H^2\right)^{m-2}\left\{\left(\dot{H}+3H^2\right)^2 - mH\left(\ddot{H}+6H\dot{H}\right)\right\} + 3\beta(1-2n)(6)^{n-1}H^{2n} \tag{26}$$

$$-\omega_{DE}\rho_{DE} = \alpha(6)^{m-1}(m-1)\left(\dot{H}+3H^2\right)^{m-3}\left\{\begin{array}{l} m\left(\dot{H}+3H^2\right)\left(\dddot{H}+6H\ddot{H}+6\dot{H}^2\right) \\ +m(m-2)\left(\ddot{H}+6H\dot{H}\right)^2 - 3\left(\dot{H}+3H^2\right)^3 \end{array}\right\} - \beta(6)^{n-1}(2n-1)H^{2n-2}\left(2n\dot{H}+3H^2\right). \tag{27}$$

4. *Solutions of the field equations*. In order to solve the field equations completely, we consider the power law relation of an average scale factor $a$ as described by Pawar et al. [71] as

$$a = \left(t^2 + \frac{\lambda}{\mu}\right)^{1/2\mu} \tag{28}$$

where $\lambda$ and $\mu$ are constants.

Using (28), the metric (8) becomes

$$ds^2 = dt^2 - \left(t^2 + \frac{\lambda}{\mu}\right)^{1/\mu}\left[dx^2 + dy^2 + dz^2\right]. \tag{29}$$

The metric potential of this model assumes a constant value at $t = 0$ and do not vanish for any $t$ and $\mu > 0$, $\lambda > 0$. Hence the model is free from any type of singularities for finite values of $t$.

Now, we define and calculate some cosmologically important physical and kinematical parameters.

The spatial volume $V$ is

$$V = a^3 = \left(t^2 + \frac{\lambda}{\mu}\right)^{3/2\mu}. \tag{30}$$

The average Hubble's parameter $H$ is

$$H = \frac{1}{3}\left(H_1 + H_2 + H_3\right) = \frac{\dot{a}}{a} = \frac{t}{\mu t^2 + \lambda}, \tag{31}$$

where $H_1$, $H_2$, $H_3$ are the directional Hubble's parameters.

The mean anisotropy parameter $A_m$ is

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i}{H} - 1\right)^2 = 0, \tag{32}$$

because $H_i = H = a$, for $i = 1, 2, 3$.

The expansion scalar $\theta$ and the shear scalar $\sigma^2$ are respectively, obtained as

$$\theta = u^{\alpha}_{;\alpha} = 3H = \frac{3t}{\mu t^2 + \lambda}, \tag{33}$$

$$\sigma^2 = \frac{3}{2} A_m H^2 = 0. \tag{34}$$

The deceleration parameter $q$ is obtained as

$$q = -1 + \frac{d}{dt}\frac{1}{H} = -\frac{\ddot{a}}{aH^2} = -1 + \mu - \frac{\lambda}{t^2}. \tag{35}$$

The expressions (28), (30), (31), (33) and (35) show that the $a$, $V$, $H$, $\theta$ and $q$ are all time-dependent. The scale factor, spatial volume and deceleration parameter have non-zero constant values, whereas Hubble's parameter and expansion scalar have zero values, at $t=0$. Thus the universe starts to expand with a very small constant volume which increases with time, which is very clear from Fig.1. Furthermore, from expression (35) it is observed that the decelerating or accelerating phase of cosmic expansion depends upon the values of $\lambda$ and $\mu$. We obtained the accelerating expansion of the universe for $(\mu - 1)t^2 < \lambda$. The graph of the deceleration parameter versus cosmic time is depicted in Fig.2. It is observed from the figure that $q \approx -1$ for $t=0$, and it increases with time and becomes constant at nearly -0.5 (approx.), which shows the accelerating expansion of the universe throughout the evolution.

From the above respective expressions the Hubble's parameter and the expansion scalar seem to be decreasing functions of cosmic time. Additionally, the mean anisotropy parameter and the shear scalar are zero throughout the evolution of the universe, which describes that the universe is isotropic and shear-free.

On solving (26) and (27) with the use of (31), we obtain the matter-energy density and the EoS parameter of DE in terms of the energy density of DE in the form.

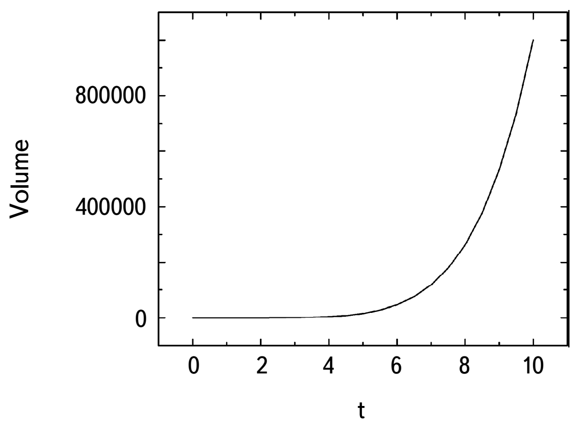


Fig.1. Variation of a spatial volume $V$ vs cosmic time $t$ for $\mu = 0.5$ and $\lambda = 0.005$.

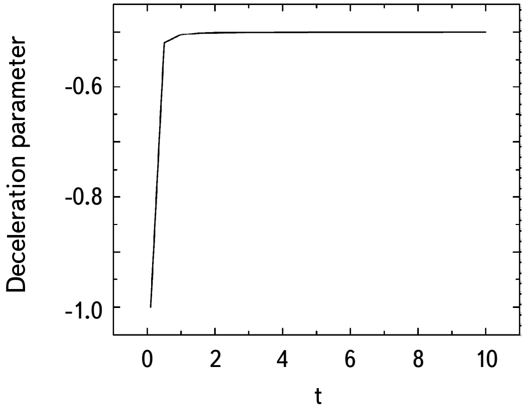


Fig.2. Variation of a deceleration parameter $q$ vs cosmic time $t$ for $\mu = 0.5$ and $\lambda = 0.005$.

$$\rho_m = -\rho_{DE} - 3\alpha(6)^{m-1}\left(t^2+\frac{\lambda}{\mu}\right)^{-2m}\left[\frac{\lambda}{\mu^2}+t^2\left(\frac{3}{\mu^2}-\frac{1}{\mu}\right)\right]^{m-2}\left\{(m+1)\left[\frac{\lambda}{\mu^2}+t^2\left(\frac{3}{\mu^2}-\frac{1}{\mu}\right)\right]^2\right.$$
$$\left. - m(m-1)\frac{2t}{\mu^2}\left[\left(1-\frac{3}{\mu}\right)t^3-\frac{3\lambda}{\mu}\left(1-\frac{1}{\mu}\right)t\right]\right\}+\left(n+\frac{1}{2}\right)\frac{\beta(-6)^n}{\mu^{2n}}t^{2n}\left(t^2+\frac{\lambda}{\mu}\right)^{-2n} \tag{36}$$

$$\omega_{DE} = -\frac{1}{\rho_{DE}}\left\{\alpha(6)^{m-1}\left[\frac{\lambda}{\mu^2}+t^2\left(\frac{3}{\mu^2}-\frac{1}{\mu}\right)\right]^{m-3}\left(t^2+\frac{\lambda}{\mu}\right)^{-2m}\left\{m(m-1)(m-2)\frac{4}{\mu^2}\right.\right.$$
$$\times\left[\left(1-\frac{3}{\mu}\right)t^3-\frac{3\lambda}{\mu}\left(1-\frac{1}{\mu}\right)t\right]^2+m(m-1)\frac{6}{\mu^4}\left[\frac{\lambda}{\mu^2}+t^2\left(\frac{3}{\mu^2}-\frac{1}{\mu}\right)\right]$$
$$\left.\times\left[(3-\mu)\mu^2t^4+2\lambda\mu t^2(3\mu-4)+(1-\mu)\lambda^2\right]-3(m+1)\left[\frac{\lambda}{\mu^2}+t^2\left(\frac{3}{\mu^2}-\frac{1}{\mu}\right)\right]\right\}$$
$$\left.-\beta(6)^{n-1}\frac{t^{2n-2}}{\mu^{2n-2}}\left(t^2+\frac{\lambda}{\mu}\right)^{-2n}\left[2n(2n-1)\left(\frac{\lambda}{\mu^2}-\frac{t^2}{\mu}\right)+3(2n+1)\frac{t^2}{\mu^2}\right]\right\}. \tag{37}$$

Diagnostic statefinder parameters:

The pair of state finder parameters $\{r, s\}$ is defined in [73] and their values are obtained as follows:

$$r = \frac{\dddot{a}}{aH^3} = (1-\mu)(1-2\mu)+\frac{3\lambda(1-2\mu)}{t^2}, \tag{38}$$

$$s = \frac{r-1}{3(q-1/2)} = \frac{2}{3}\left[\frac{(2\mu-3)\mu t^2+3\lambda(1-2\mu)}{(2\mu-3)t^2-2\lambda}\right]. \tag{39}$$

For different DE models, the different sets of values of the pair are mentioned below:

- For $\Lambda$CDM model: $(r=1,\ s=0)$,
- For SCDM model: $(r=1,\ s=1)$,
- For HDE model: $(r=1,\ s=2/3)$,
- For CG model: $(r>1,\ s<0)$,
- For Quintessence model: $(r<1,\ s>0)$.

Fig.3 depicts the variation of state finder parameter $s$ versus cosmic time $t$ for $\mu=0.5$ and $\lambda=0.005$. It is observed that the parameter $s$ lie between 0.1 and 0.35 throughout the evolution of the universe. However, for the above mentioned values of $\lambda$ and $\mu$ we get the value of a parameter $r=0$ for all $t$. Thus the model so derived here is the Quintessence model.

In the next section, we consider Rényi holographic DE as a candidate of DE's and discuss the physical acceptability of the corresponding model under Hubble's *IR*-cutoff.

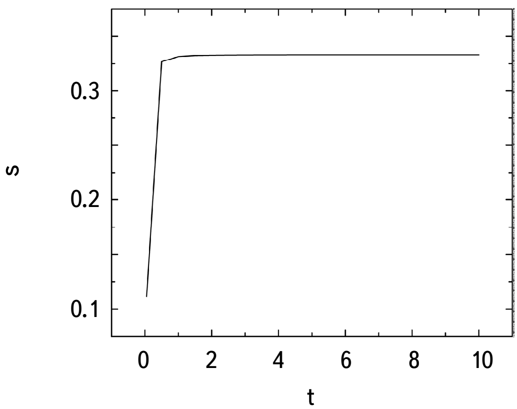


Fig.3. Variation of statefinder parameter $s$ vs cosmic time $t$ for $\mu=0.5$ and $\lambda=0.005$.

5. *Physical acceptability of Rényi HDE model with Hubble's IR-cutoff*. The energy density of Rényi HDE formulated in [74] is as follows:

$$\rho_{DE}=\frac{3d^2}{8\pi L^2}\left(1+\pi\delta L^2\right)^{-1} \tag{40}$$

with the constants $d$ and $\delta$.

Here, we consider the candidate for the *IR*-cutoff as Hubbles horizon i.e. $L=H^{-1}$.

So from (40), the Rényi HDE density under the Hubble horizon cutoff is obtained as

$$\rho_{DE}=\left(\frac{3d^2H^2}{8\pi}\right)\left(1+\frac{\pi\delta}{H^2}\right)^{-1}. \tag{41}$$

Now we use the relation between the average scale factor and the redshift $z$, which is given by

$$a = (1+z)^{-1}. \tag{42}$$

The equations (28) and (42) yield the time-redshift relation as

$$t = \mu^{-1/2}\left[\mu(1+z)^{-2\mu} - \lambda\right]^{1/2}. \tag{43}$$

Thus, with the use of (43), we obtain the Hubble's parameter in terms of $z$ as

$$H = \mu^{-3/2}(1+z)^{2\mu}\left[\mu(1+z)^{-2\mu} - \lambda\right]^{1/2}. \tag{44}$$

Using (44) in (41), we get the energy density of Rényi HDE under Hubble's cutoff as

$$\rho_{DE} = \frac{3d^2\mu^{-3}}{8\pi}\left\{\frac{(1+z)^{8\mu}\left[\mu(1+z)^{-2\mu} - \lambda\right]^2}{\pi\delta\mu^3 + (1+z)^{4\mu}\left[\mu(1+z)^{-2\mu} - \lambda\right]}\right\}. \tag{45}$$

The graphical behaviour of the energy density of Rényi HDE under Hubble's *IR*-cutoff versus redshift for the appropriate choice of constants is depicted in Fig.4, in which it is observed that the energy density of Rényi HDE increases with an increase in redshift throughout the evolution.

From (36) and (37), with the use of (43) and (45), we obtain the energy density of pressureless matter and the EoS parameter of Rényi HDE under the Hubble's cutoff respectively as

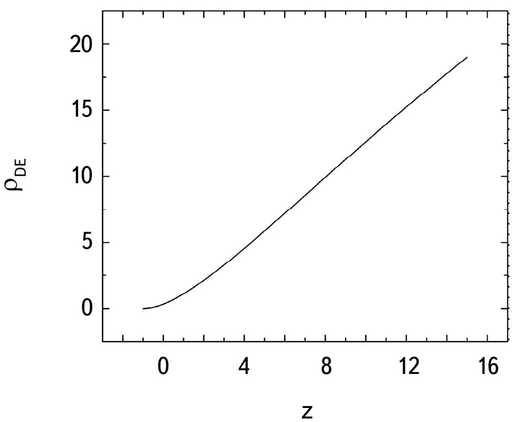


Fig.4. Variation of Rényi HDE density with Hubble's *IR*-cutoff vs redshift for $d = 2$, $\mu = 0.5$, $\lambda = 0.005$ and $\delta = 6$.

$$\rho_m = -3\alpha(6)^{m-1}\mu^{-3m}(1+z)^{4m\mu}\left[\lambda\mu+(3-\mu)\left(\mu(1+z)^{-2\mu}-\lambda\right)\right]^{m-2}$$
$$\times\left\{\begin{array}{l}(m+1)\left[\lambda\mu+(3-\mu)\left(\mu(1+z)^{-2\mu}-\lambda\right)\right]^2\\ -2m(m-1)\mu\left(\mu(1+z)^{-2\mu}-\lambda\right)\\ \times\left[(\mu-3)\left(\mu(1+z)^{-2\mu}-\lambda\right)-3\lambda(\mu-1)\right]\end{array}\right\}+\beta(6)^n\left(n+\frac{1}{2}\right)\mu^{-3n}(1+z)^{4n\mu} \tag{46}$$
$$\times\left(\mu(1+z)^{-2\mu}-\lambda\right)^n-\frac{3d^2\mu^{-3}}{8\pi}\frac{(1+z)^{8\mu}\left[\mu(1+z)^{-2\mu}-\lambda\right]^2}{\pi\delta\mu^3+(1+z)^{4\mu}\left[\mu(1+z)^{-2\mu}-\lambda\right]}.$$

$$\omega_{DE}=\frac{-8\pi}{3d^2}\left\{\frac{\pi\delta\mu^3+(1+z)^{4\mu}\left[\mu(1+z)^{-2\mu}-\lambda\right]}{(1+z)^{8\mu}\left[\mu(1+z)^{-2\mu}-\lambda\right]^2}\right\}$$
$$\times\left\{\begin{array}{l}\alpha(6)^{m-1}\mu^{-3(m-1)}(1+z)^{4m\mu}\left[\mu(3-\mu)(1+z)^{-2\mu}+\lambda(2\mu-3)\right]^{m-3}\\ \times\left[\begin{array}{l}4m(m-1)(m-2)\mu^2\left(\mu(1+z)^{-2\mu}-\lambda\right)\left(\mu(\mu-3)(1+z)^{-2\mu}+2\lambda(3-2\mu)\right)^2\\ +6m(m-1)\mu^2\left(\mu(3-\mu)(1+z)^{-2\mu}+\lambda(2\mu-3)\right)\end{array}\right]\\ \times\left[\left(\mu(1+z)^{-2\mu}-\lambda\right)\left(\begin{array}{l}\mu(3-\mu)(1+z)^{-2\mu}\\ +7(\mu-1)\lambda\end{array}\right)+(1-\mu)\lambda^2\right]-3(m+1)\left[\mu(3-\mu)+\lambda(2\mu-3)\right]^3\\ -\beta(6)^{n-1}\mu^{-3(n-1)}(1+z)^{4n\mu}\left(\mu(1+z)^{-2\mu}-\lambda\right)^{n-1}\left[\begin{array}{l}2n(2n-1)\mu\left(2\lambda-\mu(1+z)^{-2\mu}\right)\\ -3(2n+1)\left(\lambda-\mu(1+z)^{-2\mu}\right)\end{array}\right]\end{array}\right\}. \tag{47}$$

The graphical behaviour of the EoS parameter of Rényi HDE density with Hubble's *IR*-cutoff versus redshift for the appropriate choice of constants is shown in Fig.5. From the figure it is observed that we live in a phantom-dominated

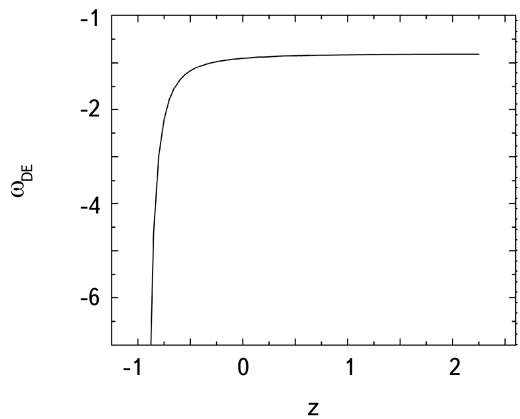


Fig.5. Variation of EoS parameter of Rényi HDE density with Hubble's *IR*-cutoff vs redshift for $d = 2$, $\mu = 0.5$, $\lambda = 0.005$, $\delta = 6$, $\alpha = 0.1$, $\beta = 0.01$, $m = 0.001$ and $n = 1$.

universe since the constructed model corresponds to $\omega_{DE} < -1$ for $-1 < z < -0.25$. Later on, it is also observed that $\omega_{DE} = -1$ for $z = -0.25$ which demonstrates that the universe passes through $\Lambda$CDM epoch and lastly for all $-0.25 < z$ the universe enters in Quintessence era i.e. $\omega_{DE} > -1$ and remains in the Quintessence DE region, since the EoS parameter lies in $-0.90 < \omega_{DE} < -0.82$ which is relatively close to $\Lambda$CDM region. These observations are fairly supported by [60,62,75,76]. For a late epoch the statefinder diagnostic parameters validated the observation.

6. *Conclusions*. In this work, authors have investigated the behaviour of the Rényi HDE model in $f(T, B)$ gravity under the Hubble's *IR*-cutoff by considering the power law form of an average scale factor obtained by Pawar et al. [71]. We have considered the spatially flat FLRW cosmological model and the $f(T, B) = \alpha B^m + \beta T^n$ gravity formalism. The physical acceptability of the model has been checked with the help of statefinder diagnostic and the EoS parameter of the model. The values of some physical and geometrical parameters and their graphical behaviour with time and redshift are obtained.

From the expressions of cosmological parameters and their graphical behaviour at $\mu = 0.5$ and $\lambda = 0.005$, it is observed that the constructed model starts to expand with a very small constant volume which increases with the increasing cosmic time. The model experiences an accelerating expansion throughout its evolution. It is observed that the model is isotropic and shear-free. The values of diagnostic statefinder parameters ($r < 1$, $s > 0$) confirms the constructed model is in Quintessence region.

The energy density of Rényi HDE model under Hubble's *IR*-cutoff is found to be increasing with an increase in redshift throughout its evolution. Furthermore, from the observations of the EoS parameter it is been found that initially, we live in a phantom-dominated universe, later on for a short period the universe passes through $\Lambda$CDM epoch and lastly, it enters and remains in the Quintessence DE era in which the values of EoS parameter are relatively close to $\Lambda$CDM region, which is as expected from the statefinder diagnostics parameter. The results so obtained are fairly supported by [60,62,75,76]. Thus the derived Rényi HDE model of the universe under Hubble's *IR*-cutoff in $f(T, B)$ gravity is found physically acceptable.

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# ФИЗИЧЕСКАЯ ПРИЕМЛЕМОСТЬ ГОЛОГРАФИЧЕСКОЙ МОДЕЛИ ТЕМНОЙ ЭНЕРГИИ РЕНЬИ ПРИ ОБРЕЗКЕ ХАББЛА В ГРАВИТАЦИИ $f(T, B)$

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Статья посвящена исследованию поведения и физической приемлемости пространственно однородного и изотропного пространства-времени FLRW, заполненного материей без давления и голографической темной энергией Реньи при ИК-пороге Хаббла в рамках гравитации $f(T, B)$. Рассчитаны некоторые космологические параметры для изучения астрофизических следствий построенной модели. Обсуждается их поведение в ходе эволюции, в том числе, параметры определителя состояния и параметры EoS. Обнаружено, что построенная голографическая модель темной энергии Реньи "путешествует" из Фантома, $\Lambda$CDM и, наконец, входит и остается в эре темной энергии Квинтэссенции с увеличением красного смещения.

Ключевые слова: *гравитация* $f(T, B)$*, голографическая темная энергия Реньи, обрезание Хаббла, красное смещение*

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