Miguel LÓPEZ-ASTORGA

STOIC LOGIC FROM THE THEORY OF MENTAL MODELS

Abstract

An essential point about Stoic philosophy is why certain arguments and rules are basic in their logic. That is the case of the *indemonstrables* and the *themata*. It has been proposed that assuming the theory of mental models, one can think that the five *indemonstrables* and two of the *themata* are easy to understand for the human mind. This can explain why those arguments and rules are essential components in Stoic logic. In addition, it is relevant because, given that the theory of mental models tries to capture the real way people reason, it can show that Stoic logic is closer to the manner individuals naturally make inferences than modern propositional calculus. The present paper is intended to move forward in this direction. It has two aims: one of them is to give an account from the theory of mental models of all of the *themata*. The other one is to argue that a simple schema that is correct in modern propositional calculus, and which, however, is not deemed as a true syllogism in Stoic logic, is difficult for people according to the theory of mental models. Those are further pieces of evidence that Stoic logic describes the way human beings think to a greater extent than modern logic.

Keywords: indemonstrables, Stoic logic, syllogism, *themata*, theory of mental models.

Introduction

The reason why the *indemonstrables* (ἀναπόδεικτοι) in Stoic logic are deemed as indemonstrable is hard to understand from modern propositional calculus. This is because only one of them (and they are five) is actually indemonstrable following modern logic. However, this problem can be resolved if modern propositional calculus is ignored and another framework is taken into account (see, e.g., López-Astorga, 2017). Furthermore, it has already been said that modern logic is not the best approach to interpret the Stoic system (Bobzien, 1996).

In this way, it has been proposed that, while Stoic logic does not seem to be consistent with modern propositional calculus, it appears to be coherent with some contemporary cognitive theories, for example, the mental logic theory (e.g., López-Astorga, 2015) and the theory of mental models (e.g., López-Astorga, 2017). This paper will focus on the theory of mental models. Its

purpose will be to show that Stoic logic is even more compatible with that theory than what the literature has revealed. To do that, the paper will move forward in two senses. On the one hand, it will deal with the *themata* (θέματα). On the other hand, it will analyze an argument that, following Bobzien (1996), cannot be admitted as a real syllogism (συλλογισμός) in Stoic logic, although the argument is correct in modern logic.

The *themata* are important rules in Stoic logic. They help determine whether or not an argument is a syllogism. If the argument is an *indemonstrable* or can be transformed, employing an analysis (ἀνάλυσις) process, into an *indemonstrable*, it is a syllogism. The *themata* are essential because they are rules allowing making those analysis processes. It seems that the *themata* are four (De Lay, 1984, *Galeni De Placitis Hippocratis et Platonis* 114, 1-10) or five (there are two versions of the third one; see, e.g., Bobzien, 1996). However, only the first *thema* and the third one (its two versions) are preserved. Bob-

zien (1996) describes the *themata* kept and, in addition, presents a rebuilding of the missing ones. That reconstruction will be assumed here. Thus, the first point of this paper will be to review whether all of the *themata*, in the way Bobzien (1996) considers them, can be easily understood from the theory of mental models. The literature has already shown that with regard to the *indemonstrables* (López-Astorga, 2017) and the first *thema* and one of the versions of the third *thema* (López-Astorga, 2016), which is not the version Bobzien (1996) takes into account. So, Bobzien's reconstructions of the second and the fourth *themata*, as well as the version of the third one she accepts, will be addressed below.

The second point of this paper will have to do with an argument Bobzien (1996) also indicates. It is an argument sound in modern calculus. However, it is not admissible in Stoic logic. The reason for that is simple: it is not an *indemonstrable,* and it cannot be transformed into an *indemonstrable* by means of *themata*. Thereby, what the present paper will try to explain is that the argument is difficult for people within the theory of mental models. Accordingly, the accounts below can be further evidence in favour of the idea that Stoic logic is closer to the theory of mental models than to modern logic. This can mean that, if the theory of mental models explains how the human mind works, Stoic logic is also closer to the real way human beings reason than modern propositional calculus.

To achieve these goals, first, the paper will present the Stoic *indemonstrables* and the four *themata* rebuilt by Bobzien (1996). Then, it will explain some important theses of the theory of mental models. The third section will briefly comment on the accounts based on the theory of mental models in the literature about the reasons why the *indemonstrables*, the first *thema*, and one of the versions of the third *thema* were deemed as basic inferences in Stoic logic. Fourth, the other version of the third *thema* and the reconstructions of the second and fourth ones by Bobzien (1996) will be considered in order to

show that they are coherent with the way people make inferences according to the theory of mental models. The last section will deal with the mentioned argument Bobzien identifies and argues that it is hard to accept following that very theory.

The Stoic *Indemonstrables* and Four *Themata*

It seems that Chrysippus of Soli was the Stoic philosopher that first presented the *indemonstrables* (e.g., Sextus Empiricus, *Adversus Mathematicos* 8, 223) (Mau, 2011; O'Toole & Jennings, 2004). As said, only one of them is indemonstrable in modern propositional calculus (see also, e.g., López-Astorga, 2017). It is MPP (Modus Ponendo Ponens) (Marcovich, 1999, Diogenes Laertius, *Vitae Philosophorum* 7, 80).

MPP:
First premise: If p, then q
Second premise: p
Conclusion: q

An important point about MPP is that its first premise (λῆμμα) is a conditional (συνημμένον). This is relevant because there is a difference from modern logic here. Modern logic follows the view of the conditional Philo of Megara gave (e.g., Sextus Empiricus, *Adversus Mathematicos* 8, 113) (Mau, 2011; Mates, 1953). That view is known as 'the material interpretation of the conditional'. It provides that a conditional such as the first premise of MPP is true "…whenever is not the case that the antecedent is true and the consequent false" (O'Toole & Jennings, 2004, p. 479). However, Chrysippus of Soli, and, hence, Stoic logic, claims one more requirement: to be true, a conditional needs "…a certain relationship between the clauses. In particular, the contrary (ἀντικείμενον) of the second clause (λῆγον) must be inconsistent with (μάχεται) the first clause (ἡγούμενον)" (López-Astorga, 2017, p. 311). Therefore, the first premise in MPP is true if and only if the contrary of q fights (μάχεται) p. Undoubtedly, this makes Stoic logic very different

from frameworks such as that of Gentzen (1934, 1935). Likewise, it also removes paradoxes such as those of the implication (for a description, see, e.g., Orenes & Johnson-Laird, 2012). As it is well known, the material interpretation of the conditional allows inferring a conditional with any antecedent from any formula if this last formula is taken as the consequent of the new conditional. If, as in Stoic logic, a relation between the two clauses is necessary, a paradox of that kind is not possible.

The second *indemonstrable* is MTT (Modus Tollendo Tollens) (see Diogenes Laertius, *Vitae Philosophorum* 7, 80) (Marcovich, 1999).

MTT:

First premise: If p, then q

Second premise: Not-q

Conclusion: Not-p

Of course, in MTT, as far as the first premise is concerned, not-q must also be inconsistent with p.

MPT I (Modus Ponendo Tollens I) is the third *indemonstrable* (see Diogenes Laertius, *Vitae Philosophorum* 7, 80) (Marcovich, 1999).

MPT I:

First premise: Not-(p and q)

Second premise: p

Conclusion: Not-q

Another *indemonstrable* is MPT II (Modus ponendo Tollens II) (see Diogenes Laertius, *Vitae Philosophorum* 7, 81) (Marcovich, 1999).

MPT II:

First premise: Either p or q, but not both of them

Second premise: p

Conclusion: Not-q

The first premise in MPT II includes 'but not both of them' because disjunction (διεζευγμένον) is exclusive in Stoic logic (e.g., Cicero, Topica 14, 56-57) (Reinhardt, 2003; Bocheński, 1963). This is another difference from modern logic, in which disjunction is inclusive.

And MTP (Modus Tollendo Ponens) is the last *indemonstrable* (see Diogenes Laertius, *Vitae Philosophorum* 7, 81) (Marcovich, 1999).

MTP:

First premise: Either p or q, but not both of them

Second premise: Not-p

Conclusion: q

Obviously, disjunction keeps being exclusive in MTP.

Regarding the *themata*, only the descriptions of the first one (Pseudo-Apuleius, *De Interpretatione* 209, 12-14) (Thomas, 1970) and two versions of the third one (respectively, Alexander of Aphrodisias, *Aristotelis Analyticorum Priorum* 278, 12-14) (Wallies, 1883); Simplicius, *In Aristotelis De Caelo* 273, 2-4 (Heiberg, 1894)) are known. There are discussions about the problem of the *themata* (e.g., Mignucci, 1993). Nevertheless, the rebuilding Bobzien (1996) made will be assumed here. This is because that reconstruction can suffice to make a point of this paper. Bobzien's (1996, pp. 152-153) rebuilding is as follows ('T1', 'T2', 'T3', and T4' refer, respectively, to the first, second, third, and fourth *themata*; they are not axioms, and the reason for that is explained below):

T1: If [if (p and q) then r] then [if (p and not-r) then not-q]

T2: If {[if (p and q) then r] and [if (r and p) then s]} then [if (p and q) then s]

T3: If {[if (p and q) then r] and [if (r and s) then t]} then [if (p and q and s) then t]

T4: If {[if (p and q) then r] and [if (r and p and s) then t]} then [if (p and q and s) then t]

As said, Bobzien (1996) only assumes one of the two versions of the third *thema* (that in Simplicius, *In Aristotelis De Caelo* 273, 2-4). On the other hand, T1, T2, T3, and T4 are expressed above in a simplified way. For example, Bobzien (1996) also includes an 'expanded version' of T1. In that version, the first conditional, that is, (1), does not have two conjuncts in its first clause. It can have more conjuncts.

(1) If (p and q) then r

In addition, it is obvious that in T2, conditional (2) can have other forms such as (3) and (4).

(2) If (r and p), then s

(3) If (r and q) then s

(4) If (r and p and q) then s

Likewise, s can be replaced in T3 with a set of propositions such as (5).

(5) $s_1,...,s_n$

And these two circumstances can happen in T4 too. On the one hand, (6) could be substituted with (7) or (8).

(6) If (r and p and s) then t

(7) If (r and q and s) then t

(8) If (r and p and q and s) then t

On the other hand, s could be replaced with (5) in T4 as well.

Furthermore, although Bobzien (1996) does not express the *themata* as conditionals, but resorting to premises and conclusions, it is justified to express them as above. It seems that the Stoics admitted conditionalization, that is, the process by means of which the premises of an argument can be deemed as the first clause of a conditional, and the conclusion as the second clause of that very conditional (e.g., Sextus Empiricus, *Pyrrhoneae Hypotyposes* 2, 137) (Mau, 2011; O'Toole & Jennings, 2004). This does not make T1, T2, T3, and T4 axioms. The reason is that, as indicated, Stoic logic is not modern logic. So, the conditionals suppose relations between their antecedents and consequents which are not required in modern logic.

Thus, the way the *themata* worked was akin to this one:

Given an argument such as the following (which is taken from Bobzien, 1996, p. 153):

First premise: p

Second premise: Not-q

Conclusion: Not-(if p then q)

That argument can be transformed into MPP by virtue of T1.

The theory of mental models shows that all these components of Stoic logic appear to be natural for the human mind and easy to understand. This can explain the basic character of those components in Stoic philosophy. However, before describing the account of the *indemonstrables* and the *themata* that can be given from the theory of mental models, it is necessary to comment on some theses of this last theory.

The Theory of Mental Models as a Dual Process Theory

The theory of mental models proposes several explanations of human reasoning (e.g., Khemlani, Byrne, & Johnson-Laird, 2018). It is important to note that many of those theses move the theory away from modern logic (see also, e.g., Johnson-Laird, 2010). Nevertheless, the central thesis of the theory of mental models to make a point of this paper is that their proponents deem it a dual-process theory (see also, e.g., Johnson-Laird, Khemlani, & Goodwin, 2015). A dual-process theory (e.g., Evans, 2008) is a theory distinguishing two systems in the human mind. Those systems are usually named 'System 1' and 'System 2'. System 1 refers to intuitive processes. When using System 1, people do not spend much time drawing conclusions. On the other hand, when mental processes are more reflexive, individuals resort to System 2. In that case, they spend time and think in a more logical way. There are several dual-process theories (see also, e.g., Evans, 2009), but the manner this applies to the theory of mental models is explained below.

The theory of mental models claims that 'sentential connectives' lead people to consider the possibilities representing the situations that can be true for those connectives and their propositions (see also, e.g., Johnson-Laird & Ragni, 2019). If Stoic logic is addressed, the relevant sentential connectives are the conditional, exclusive disjunction, and conjunction. Given a conditional such as (9),

(9) If p, then q

Its possibilities are in (10).

(10) $\Diamond$(p & q) & $\Diamond$(not-p & q) & $\Diamond$(not-p & not-q)

(10) expresses a 'conjunction of possibilities' (see also, e.g., Espino, Byrne, & Johnson-Laird, 2020). '$\Diamond$' stands for possibility. Nonetheless, it

does not work as the operator of the possibility in normal modal logics. In normal modal logics, (11) does not follow from (9).

(11) ◊p

This is because normal modal logics understand the conditional as Philo of Megara does. Thereby, (9) is false only when, as indicated, 'the antecedent is true and the consequent false'. Accordingly, if p is false, (9) is true even if p is, in addition, impossible. This is different in the theory of mental models. As (10) shows, (12) can be deduced from (9) (see, e.g., Espino et al., 2020).

(12) ◊(p & q)

As far as a sentence such as (13) is concerned,

(13) Either p or q, but not both of them

It also has a conjunction of possibilities. It is (14) (see also, e.g., Khemlani, Hinterecker, & Johnson-Laird, 2017).

(14) ◊(p & not-q) & ◊(not-p & q)

The theory of mental models is different from logic in this case too. In normal modal logics, the inference of (11) from (13) is not allowed either. The reason is obvious. (13) can be true even if (11) is false, that is, even if p is not possible. If the other disjunct, that is, q, is true, that already makes (13) true. However, as (14) reveals, one of the possibilities that (13) enables to deduce in the theory of mental models is (15) (see, e.g., Khemlani et al., 2017).

(15) ◊(p & not-q)

Lastly, conjunction is easy to capture in the theory of mental models. Conjunction such as (16) only expresses one possibility: (12).

(16) p and q

In this case, (12) cannot be really denominated 'possibility'. Given that there is only one possibility, (12) is a fact (see, e.g., Khemlani et al., 2017).

However, the most interesting point of the theory for this paper is that, as said, it is a dual-process theory. This means that the possibilities indicated for the conditional and exclusive disjunction are not always identified. To note all of them, it is necessary to use System 2. If people only resort to System 1, they can consider just what is true, ignoring what is false in the conjunctions of possibilities (see also, e.g., Byrne & Johnson-Laird, 2020).

Thus, System 1 only allows being aware of (12) in the case of the conditional. The other two possibilities in (10) are not taken into account because they refer to situations with p being false. So, they cannot be represented with just System 1 (see also, e.g., Johnson-Laird, 2012).

Regarding exclusive disjunction, what System 1 does not consider is the false disjuncts. Thereby, the resulting conjunction of possibilities is (17) (see also, e.g., Quelhas, Rasga, & Johnson-Laird, 2019).

(17) ◊p & ◊q

In the first possibility in (17), q is missing. The reason is that it is false, and System 1 does not pay attention to falsity. The same can be said with regard to p in the second possibility.

This framework makes it possible to account for why the Stoics deemed the *indemonstrables* and the *themata* as essential components in their logic. In fact, that account has already been partly given in the literature. The next section shows this.

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The Theory of Mental Models,
the *Indemonstrables*, the First *Thema*,
and the Version of the Third *Thema* by
Alexander of Aphrodisias

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Actually, there are two versions of the theory of mental models. The description in the previous section corresponds to the second one, that is, the most updated version (for the first one, see, e.g., Oakhill & Garnham, 1996). This is important because, in the literature analyses of the *indemonstrables*, the first *thema*, and Alexander's version of the third *thema* based on the theory are to be found (for the *indemonstrables*, see López-Astorga, 2017; for the first *thema* and Alexander's version of the third one, see López-Astorga, 2016). Nevertheless, those analyses have been made from the initial version.

But, as far as the aims of the present paper are

concerned, this does not have significant relevance. The main difference between the two versions is just in the relation between the possibilities. The original version proposes that the possibilities are linked by means of disjunctions. In the second version, the links are by means of conjunctions since, as indicated, the proponents use the expression 'conjunction of possibilities' (e.g., Khemlani et al., 2018). This has no influence on the accounts of Stoic logic given from the theory of mental models. To remove any doubt in this way, this section will describe the explanations in the literature based on the initial version of the theory again, but using the machinery and terminology of the second version.

Regarding the *indemonstrables*, it is easy to note why they were deemed as basic schemata in Stoicism. The explanation resorting to the initial version of the theory of mental models for the five *indemonstrables* can be found in López-Astorga (2017). The following accounts are based on that explanation. However, as said, the present paper uses the updated version of the theory.

MPP is not hard because it only requires System 1. If the only possibility for (9) is (12), it is evident that in a scenario with p, q needs to be as well.

One might think that MTT is different. Given that its second premise is not-q, one might suppose that all of the possibilities corresponding to the first premise, that is, to a sentence such as (9), have to be taken into account. As pointed out, those possibilities are (10). Nevertheless, in particular, the necessary possibility is the third one in (10), that is, (18).

(18) $\Diamond$(not-p & not-q)

Possibility (18) is the necessary one because not-q appears in it. Thus, it reveals that not-q can be the case only if not-p is also the case. In fact, this is the usual argumentation from the theory of mental models to explain why MTT is often more difficult than MPP for people: individuals need to use System 2 and detect (18) (see, e.g., Byrne & Johnson-Laird, 2009).

Nonetheless, it is enough to remember Chrysippus' thesis with regard to the case in which a conditional can be true: the denial of the consequent has to fight the antecedent. This means that when it is known that (9) is true, it is known that (19) is true too.

(19) If not-q then not-p

Accordingly, given (19) and only resorting to System 1, the possibility identified is, again, (18). And, as indicated, (18) shows that the only circumstance in which not-q can happen is when not-p occurs as well.

The account for MPT I is also easy. The first premise presents a negated conjunction. The only possibility for conjunction is (12). However, in MPT I, the conjunction is negated. So, if the second premise is p, q needs to be false. This can be underlain to a greater extent with the possibilities that the theory of mental models assigns to sentential connectives when negated. When a proposition is negated, its possibilities are those missing in the set of that very proposition when affirmed. In other words, when a sentential connective is negated, its conjunction of possibilities matches the complement of the possibilities of that connective when it is not negated (e.g., Khemlani, Orenes, & Johnson-Laird, 2014). Therefore, the possibilities of a sentence such as (20) are (21).

(20) Not-(p & q)

(21) $\Diamond$(p & not-q) & $\Diamond$(not-p & q) & $\Diamond$(not-p & not-q)

In (21), p, the second premise, is true just in the first possibility, that is, in (15). Furthermore, in that possibility, not-q is the case.

The first premise of MPT II is an exclusive disjunction. Hence, if only System 1 is used, the conjunction of possibilities is (17). Given that the second premise is p, the possibility of q is eliminated. Thus, because the disjunction is exclusive, the second premise also reveals that q cannot happen. This leads to not-q, that is, to what corresponds to the first possibility of the conjunction of possibilities of exclusive disjunction when System 2 works. In other words, that leads to

(15).

Finally, MTP does not require a lot of effort either. It needs to use System 2, as the first premise is again a sentence such as (13). As explained, in the case of exclusive disjunction, System 1 only allows identifying (17). Nevertheless, the second premise of MTP is the negation of one of the disjuncts, which implies considering the possibilities System 2 can deploy, that is, (14). In this way, from the first possibility in (14), that is, (15), it is possible to conclude that, if not-q is true, p must happen.

However, although System 2 leads this process, it is not necessarily hard. The first premise is an exclusive disjunction. Accordingly, it is evident that one of its two disjuncts has to be true. If the information the second premise provides is that one of the disjuncts is false, the only scenario that can be thought is that the other disjunct is the case. Thereby, the possibilities of System 2 are displayed without a great cognitive analysis.

As far as the *themata* are concerned, as mentioned, the first one and a version of the third one (that of Alexander of Aphrodisias, *Aristotelis Analyticorum Priorum* 278, 12-14) (Wallies, 1883) have already been dealt with from the theory of mental models. That has been done in López-Astorga (2016). As in the case of the *indemonstrables*, the analysis was based on the old version of the theory. That analysis is reproduced again then. Nonetheless, as also in the previous case, the accounts here follow the updated version of the theory of mental models.

The analysis in the literature also resorted to the Stoic idea of conditionalization. Therefore, it presented the first *thema* and Alexander's version of the third *thema* as conditionals too (López-Astorga, 2016). Starting with T1, its first conditional is (1). Compound conditionals of this kind, with a conjunction in the antecedent, were not a problem in Stoic logic (e.g., Bobzien, 1996). Hence, they can be deemed as any other conditional. That allows making two points. First, System 1 leads to the possibility in which both clauses (the if-clause and the then-clause)

are true, that is, to (22).

(22) $\Diamond[(p \& q) \& r]$

Second, (1) is a Stoic conditional. So, it also enables to detect the third possibility for conditionals in System 2 without difficulties. In this case, that possibility would be (23).

(23) $\Diamond[\text{not-}(p \& q) \& \text{not-}r]$

In this way, individuals know that not-r can be the case only if (20) is. Nonetheless, not-r is not the only component of the antecedent of the second conditional in T1, which is (24).

(24) If (p and not-r), then not-q

The antecedent of (24) includes p too. Accordingly, as explained for MPT I, (20), which is in (23), and p allow drawing not-q (remember that conjunction is truth-functional in Stoic logic; see, e.g., Bobzien, 1996). Therefore, as indicated in T1, given p and not-r, the conclusion is not-q.

As also mentioned, the version of the third *thema* that has been reviewed under the theory of mental models is not that Bobzien (1996) assumed, but the one coming from Alexander of Aphrodisias. If it is conditionalized, that version can be expressed as T3A.

T3A: If {[if (p and q) then r] and (if s then q)} then [if (p and s) then r]

Only using System 1, the possibility of the first conditional in T3A is (22). On the other hand, one of the second conditional, that is, (25), is (26).

(25) If s, then q

(26) $\Diamond(s \& q)$

Possibility (26) reveals that, if s happens, q needs to occur as well. So, if the situation is that both p and s happen, which is what the antecedent of the last conditional, that is, (27), provides,

(27) If (p and s) then r

There is no doubt that q is also present. But, as (22) points out, if both p and q occur, r is the case too, which leads to the consequent of (27).

Nonetheless, the theory of mental models not only can explain why T1 and T3A were essential rules in the Stoic system. The theory can also show that T2, T3, and T4 have structures that are easy to capture. The following section is devoted

T2, T3, and T4 from the Theory of Mental Models

System 1 also allows understanding T2 without difficulties. The possibility of its first conditional, which matches (1), is again (22). Likewise, following System 1 as well, the possibility of the second conditional, that is, (2), is (28).

(28) ◊[(r & p) & s]

This last possibility reveals that whenever r and p are true, s is also true. Thus, by virtue of (22), if p and q are the case, r is the case too. However, by virtue of (28), if p and r happen, s happens too. Hence, what the last conditional in T2 expresses, that is, (29), is correct.

(29) If (p and q) then s

The case of T3 is very similar: it can also be understood by considering only System 1. The first conditional is once again (1), which leads to possibility (22). The second conditional, that is, (30), is linked, if only System 1 is taken into account, to (31).

(30) If (r and s) then t

(31) ◊[(r & s) & t]

This means that it is not possible the conjunction of r and s without t. Accordingly, the result is evident. Given a scenario in which, as in the antecedent of the third conditional, that is, (32), propositions p, q, and s occur,

(32) If (p and q and s) then t

If, as (22) shows, p and q are not possible without r, then p, q, and s necessarily imply t. This is because t must happen when r and s occur, and p and q cause r to be true.

Lastly, the first conditional in T4 continues to be (1), which means that, following System 1, its possibility is in this case (22) as well. On the other hand, if the only system working is System 1, the second conditional, that is, (6), is linked to possibility (33).

(33) ◊[(r & p & s) & t]

Possibility (33) expresses that when r, p, and s occur, t must also happen. Hence, when, as in the antecedent of the last conditional, that is, (32), p, q and s are true, t is true too. As said in previous cases, p and q lead to r. If, in addition, s happens, t happens as well. The reason is that p along with r and s lead to t.

Therefore, T2, T3, and T4 are also, according to the theory of mental models, easy rules for the human mind. That can explain their role in Stoic philosophy. Nevertheless, the links between Stoic logic and the theory of mental models can be seen even clearer if another argument is reviewed. It is an argument that is sound in modern propositional calculus. However, it is not a syllogism in Stoic logic.

A Sound Argument in Modern Logic that is not a Syllogism

The argument is the following:
First premise: If not-(p and q), then r
Second premise: Not-p
Conclusion: r

This argument (presented in Bobzien, 1996, p. 174) is correct in modern propositional logic: the second premise makes the antecedent of the first premise true. If p is not true, (20) is true. Therefore, r also has to be true. Otherwise, the situation would match the only case in which, following the material view, that is, Philo's view, the conditional does not hold. As indicated, that is the case in which 'the antecedent is true and the consequent false'.

Nevertheless, according to Bobzien (1996), the argument is not a syllogism in Stoic logic. It cannot be transformed into an *indemonstrable* by means of the *themata*. Nevertheless, this point brings Stoic logic even closer to the theory of mental models. This is because the argument is difficult under this last theory. If only System 1 is considered, the possibility of the first premise is (34).

(34) ◊[not-(p & q) & r]

Possibility (34) establishes that whenever (20) is true, r happens. Nonetheless, the problem with the argument is that unlike MPT I, its second

premise denies one of the conjuncts of the negated conjunction. Hence, the second premise does not make the truth value of the other conjunct evident. In this way, to note that the argument is correct, it is necessary to deploy all of the possibilities the theory of mental models attributes to negated conjunctions; that is, it is necessary to use System 2 and transform (34) into (35).

(35) $\lozenge$[(not-p & q) & r] & $\lozenge$[(p & not-q) & r] & $\lozenge$ [(not-p & not-q) & r]

Furthermore, the additional difficulty is that even (35) is not the complete result of processing the first premise of the argument by means of System 2. The combinations corresponding to the situation in which the antecedent does not occur would have to be displayed too. Thus, all the possibilities related to the first premise would be those in (36).

(36) $\lozenge$[(not-p & q) & r] & $\lozenge$[(p & not-q) & r] & $\lozenge$ [(not-p & not-q) & r] & $\lozenge$[(p & q) & r] & $\lozenge$[(p & q) & not-r]

Actually, (35) would suffice to note that, if not-p is the case, r is the case as well, whether or not q is the case. In (35), q is the case in the first possibility, that is, in (37).

(37) $\lozenge$[(not-p & q) & r]

And it is not in the third one, that is, in (38).

(38) $\lozenge$ [(not-p & not-q) & r]

However, both in (37) and in (38) r also happens. In spite of this, to come to (35) already requires a detailed and extensive analysis, that is, already requires to resort to system 2. So, it is difficult to infer the conclusion.

Conclusion

The theory of mental models not only can, as the literature shows, offer an account of why the *indemonstrables*, T1, and T3A were considered as basic components in Stoic philosophy. The theory can also give similar explanations for T3, T2, and T4. Likewise, its framework allows understanding why certain arguments, as in the previous section, were not accepted as syllogisms in Stoicism.

The fact that the theory of mental models is a dual process theory, and that, therefore, distinguishes the processes linked to System 1 from the processes corresponding to System 2, seems to be the key in this way. To assume the *indemonstrables* and the *themata*, only System 1 is necessary in many cases. In the cases in which System 2 should be used, individuals can come to this last system in a direct and easy way. Furthermore, regarding arguments such as the last one addressed in the present paper, which are not syllogisms, the circumstances are different: the analysis of possibilities needs to be exhaustive, and System 2 is absolutely required. Thus, the inferences are deemed as difficult arguments.

All of this leads to several conclusions compatible with those of other works relating to Stoic logic and the theory of mental models (e.g., López-Astorga, 2016, 2017). First, the Stoic framework seems to be more similar to the theory of mental models than to modern propositional calculus. This is important since the theory of mental models is a theory about the manner people reason. So, the conclusion appears to be obvious: if the theory of mental models is a correct proposal, Stoic logic is closer to the way individuals reason than modern propositional logic.

This last point is interesting, as it leads to research to what extent Stoic logic is also able to explain experimental results that the theory of mental models accounts for. An example in this regard can be the results in different reasoning tasks that are compatible with the predictions of the theory of mental models (see, e.g., any of the works supporting the theory cited here). There are already studies in this direction (e.g., López-Astorga, 2021). However, given that the results published in cognitive science and psychology are significant, maybe there is much work to do.

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Researchers, code FCSEN2102, University of Talca (PIA Ciencias Cognitivas, Centro de Investigación en Ciencias Cognitivas, Instituto de Estudios Humanísticos, Universidad de Talca. Fondo Fondecyt de Continuidad para Investigadores Senior, código FCSEN2102, Universidad de Talca).

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