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THE INVESTIGATION OF PROBLEM OF CLOSING OF CRACK IN THERMOELASTIC MEDIA

Martirosyan G.A., Bagdoev A.G., Martirosyan A.N., Dinunts A.S., Davtyan A.V.

Problems on building up of layer of inclusions, containing in fluid entering in crack, which infinite thermoelastic plane are considered. These problems were solved in [1] for geop environment without taking into account of elastic stresses. In present paper the coupled thermo and diffusion mixed boundary problem for crack with fluid is solved by Winner–Hopf method graphs of vertical displacements on crack surface are calculated numerically and the moment of he of crack in some point is determined. Experimental curves of healing of strips, filled by fluid inclusions in earth environment are done in [10]. In present paper mathematical solution is obtained biological cracks with conform oil.

1. **Statement of problem of crack in thermoelastic plane with fluid current.**

The problem of semi-infinite crack in thermoelastic plane, when there is current of fluid w inclusions, entering in crack at initial moment $t=0$, is considered. These problems are especi important for study of process of narrowing of crack due to action of crystallite inclusions in conform and in analogically by mathematical treatment problem of building up of cracks by inclusions conform oil in corresponding problems of technology [10]. In plane x, y equation of crack bounda $y=\pm b_1(x,t),\ b_1(x,t)=b_0+U_y(x,y,t), y\approx 0$, where thickness $2b_1$ is small and later is taken upper sign, and is solved problem for $y\geq 0$, due to symmetry. U_x, U_y are components displacements in elastic media.

Let the temperature T of elastic plane and fluid are approximately the same, denoting by constant initial value of T, by $q=\rho_f v b_0$ the current of entering fluid in crack, v–fluid velocity x axis of crack, ρ_f–density of fluid, i_0–constant diffusion current of inclusions along y axis, which supposed known, $\gamma=\dfrac{\partial c}{\partial T}$ which also supposed constant, approximately taken for moment $t=$ $\gamma=\dfrac{\partial \bar{c}_0}{\partial T_0}$, by K let us denote constant coefficient of building up of crack surface, than using equati of narrowing of crack surface [1], on account of stresses term, one obtains

$$\rho_s \frac{\partial b_1}{\partial t}=q\gamma\frac{\partial T}{\partial x}-K\sigma_{yy}-i_0 H(x)H(t) \qquad (1.1)$$

The crack equation $y=b_0+U_y(x,0,t), 0<x<\infty$.

One must put and solve corresponding problem of thermoelasticity. The equations of motion of thermoelastic media [4], where are excluding terms with T by dropping of small thermoconductivity, ar as follows

$$a^2\frac{\partial^2 U_x}{\partial x^2}+b^2\frac{\partial^2 U_x}{\partial y^2}+(a^2-b^2)\frac{\partial^2 U_y}{\partial x\partial y}+\bar{\delta}\frac{\partial}{\partial x}\left(\frac{\partial U_x}{\partial x}+\frac{\partial U_y}{\partial y}\right)=\frac{\partial^2 U_x}{\partial t^2},\quad a^2\frac{\partial^2 U_y}{\partial y^2}+b^2\frac{\partial^2 U_y}{\partial x^2}+(a^2-b^2)\frac{\partial^2 U_x}{\partial x\partial y}+\bar{\delta}\frac{\partial}{\partial y}\left(\frac{\partial U_x}{\partial x}+\frac{\partial U_y}{\partial y}\right)=\frac{\partial^2 U_y}{\partial t^2} \qquad (1.2)$$

where $\bar{\delta}=\dfrac{K_4}{\rho}\dfrac{C_\rho-C_v}{C_v}$, $K_4=\lambda+\dfrac{2}{3}\mu$ - volume elastic modulus, ρ density, C_ρ, C_v specifi thermal capacities.

The stresses components are [4]

$$\frac{\sigma_{xx}}{\rho} = (a^2 - 2b^2)\left(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}\right) + 2b^2\frac{\partial u_y}{\partial y} + \delta\left(\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y}\right), \quad \frac{\sigma_{xy}}{\rho} = b^2\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right) \tag{1.3}$$

In thermoelastic media one can write [4]

$$T - T_0 = -\frac{C_p - C_v}{\alpha C_v}\left(\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y}\right) \tag{1.4}$$

Which is obtained neglecting of thermoconductivity in equation

$$\frac{\partial T}{\partial t} + \frac{C_p - C_v}{\alpha C_v}\frac{\partial}{\partial t}\left(\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{1.5}$$

where α is coefficient of temperature–conductivity.

The main boundary condition (1.1) on crack on account of healing of crack due to diffusion current of inclusions and building up of its surface, as well as on account of friction, can be write down

$$\rho_s\frac{\partial U_y}{\partial t} = \alpha\gamma\left.\frac{\partial T}{\partial x}\right|_{y=0} + \lambda_1\frac{\partial c}{\partial y} - K\sigma_{yy}, \qquad \frac{\partial U_x}{\partial t} = \frac{K_1}{\rho_s}\sigma_{xy}, x > 0 \tag{1.8}$$

where $\lambda_1 = -\rho_s D$, D - diffusion coefficient, K tribological coefficient of crack due to building up, K_1 – coefficient of horizontal wearing of surface. Besides due to symmetry out of crack $y = 0,\ x < 0$, $U_y = 0, \sigma_{xy} = 0$. To simplify of solution one can approximately write $\frac{\partial T}{\partial x} = \frac{T - T_0}{l}$, where l is some mean length along crack, and diffusion current $\lambda_1\frac{\partial c}{\partial y}$ can be assumed known and equal to $-i_0 H(x)H(t), i_0 = const$. The diffusion equation in this treatment is not used, replacing by assumption on approximately constant transversal diffusion current of inclusions to crack surface. Then approximated boundary conditions yield

$$\frac{\partial U_y}{\partial t} = -\frac{\rho}{\rho_s}\frac{C_p - C_v}{\alpha C_v l}\left(\frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y}\right)v_0 b_0\gamma - i_0 H(x)H(t) - \frac{K\sigma_{yy}}{\rho_S}, \quad y = 0,\ x > 0, \quad \frac{\partial U_x}{\partial t} = \frac{K_1}{\rho_s}\sigma_{xy} \tag{1.9}$$

where $H(t) = 1,\ t > 0 \quad H(t) = 0,\ t < 0$ is Heaviside unit function.

Problem (1.2), (1.9) for U_x, U_y for $y > 0$ is closed.

2. Solution of problem of building up of crack in absence of friction.

Let us denote $U_x = U,\ U_y = V$.

One can consider further simplified boundary value problem for half–plane: $y = 0$,

$$\sigma_{xy} = 0, -\infty < x < \infty \qquad V = 0, x < 0$$

$$\frac{\partial V}{\partial t} = -\left\{\frac{K(\bar{a}^2 - b^2)\rho}{\rho_s} + \xi\right\}\frac{\partial U}{\partial x} - \left(\frac{K\bar{a}^2\rho}{\rho_s} + \xi\right)\frac{\partial V}{\partial y} - \frac{i_0}{\rho_s}H(x)H(t), x > 0 \tag{2.1}$$

where $\xi = \frac{C_p - C_v}{C_v l}\frac{v b_0\gamma\rho}{\alpha\rho_s}$.

The solution is looked for by integral transformations $\bar{U};\bar{V}$ of Laplace on t and $\bar{\bar{U}};\bar{\bar{V}}$ Fourier on x. In plane x, y one can write solution as

$$\bar{U};\bar{V} = \sum_{n=1}^{2}\int_{-\infty}^{\infty} e^{i\bar{\alpha}x + i\bar{\beta}_n y}\,\bar{\bar{U}}_n;\bar{\bar{V}}_n\, d\bar{\alpha}, \quad \bar{\beta}_n = \sqrt{\frac{\omega^2}{c_n^2} - \bar{\alpha}^2}, c_1 = \bar{a}, c_2 = b \tag{2.2}$$

where $s = -i\omega$ is parameter of Laplace transformation. Besides $\bar{\bar{V}}_1 = \frac{\beta_1}{\alpha}\bar{\bar{U}}_1$, $\quad \bar{\bar{V}}_2 = -\frac{\alpha}{\beta_2}\bar{\bar{U}}_2$

Placing (2.2) in (2.1) and inverting of Fourier transformation, one obtains

$$\overline{V}_1 + \overline{V}_2 = V^-, \quad \overline{\beta}_1 \overline{U}_1 + \overline{\beta}_2 \overline{U}_2 + \overline{\alpha} V^- = 0 \tag{2.3}$$

$$sV^- = -K_2 i\overline{\alpha}\left(\overline{U}_1 + \overline{U}_2\right) - K_3\left(i\overline{\beta}_1 \overline{V}_1 + i\overline{\beta}_2 \overline{V}_2\right) - \frac{i_0}{2\pi i\alpha s\rho_s} + \Omega_2^+ \tag{2.4}$$

where $\Omega_2^+ = \frac{1}{2\pi}\int_{-\infty}^{0}\left(\overline{V}_1 + \overline{V}_2\right)\Big|_{y=0} e^{-i\alpha x}dx$, $V^- = \frac{1}{2\pi}\int_{0}^{\infty}\overline{V}\Big|_{y=0} e^{-i\overline{\alpha} x}dx$

index (+) gives analytic functions of $\overline{\alpha}$ in upper half–plane, (-) – analytic in lower half-plane $\overline{\alpha}$.

From (2.4), (2.3) one obtains Winner–Hopf equation:

$$K_2 = \frac{K\left(\overline{a}^2 - 2b^2\right)\rho}{\rho_s} + \xi, \quad K_3 = \frac{K\overline{a}^2\rho}{\rho_s} + \xi, \quad \Omega_2^+ - \frac{i_0}{2\pi i\alpha s\rho_s} = iR(\overline{\alpha})\beta_2 C_0 V^-, \quad C_0 = \frac{(K_2 + K_3)b^2}{\overline{a}^2} \tag{2.5}$$

$$R(\overline{\alpha}) = \frac{-\overline{\beta}_1\frac{\omega^3}{b^2} + \overline{\alpha}^2 K_2\left(\overline{\beta}_2^2 - \overline{\alpha}^2 - 2\overline{\beta}_1\overline{\beta}_2\right) + K_3\overline{\beta}_1\left(\overline{\beta}_1\left(\overline{\beta}_2^2 - \overline{\alpha}^2\right) + 2\overline{\alpha}^2\overline{\beta}_2\right)}{\frac{\omega^2}{b^2}\overline{\beta}_1\overline{\beta}_2 C_0}, \tag{2.6}$$

$R(\overline{\alpha})$ has two imaginary and two real roots: $\pm\overline{\alpha}_2 i, \pm\overline{\alpha}_1$, and $R(\overline{\alpha}) \to 1$ for $\overline{\alpha} \to \infty$. For factorization of $R(\overline{\alpha})$ one can introduce function

$$F(\overline{\alpha}) = R(\overline{\alpha})\frac{\left(\frac{\omega}{a} - \overline{\alpha}\right)\left(\frac{\omega}{a} + \overline{\alpha}\right)^2}{\left(\overline{\alpha}^2 - \overline{\alpha}_1^2\right)\left(\overline{\alpha}^2 + \overline{\alpha}_2^2\right)}, \quad R^+(\overline{\alpha}) = \frac{R(\overline{\alpha})}{R^-(\overline{\alpha})}, \tag{2.7}$$

and obtain

$$R^-(\overline{\alpha}) = \frac{G^-(\overline{\alpha})(\overline{\alpha} - \overline{\alpha}_1)(\overline{\alpha} - \overline{\alpha}_2 i)}{\left(\frac{\omega}{a} - \overline{\alpha}\right)^{1/2}\left(\frac{\omega}{b} - \overline{\alpha}\right)^{1/2}},$$

$$G^-(\overline{\alpha}) = Exp\left\{\frac{1}{\pi}\int_{\frac{\omega}{a}}^{\frac{\omega}{b}} arctg\frac{\frac{\omega^3}{b^2}\overline{\beta}_1^{\bullet} - 2(K_3 - K_2)\overline{\zeta}^2\overline{\beta}_1^{\bullet}\overline{\beta}_2}{\left(K_3\left(\overline{\beta}_1^{\bullet}\right)^2 - K_2\overline{\zeta}^2\right)\left(\overline{\zeta}^2 - \overline{\beta}_2^2\right)}\frac{d\overline{\zeta}}{\overline{\zeta} - \overline{\alpha}} + \right.$$

$$\left. + \frac{1}{\pi}\int_{\frac{\omega}{b}}^{\infty} arctg\frac{\frac{\omega^3}{b^2}\overline{\beta}_1^{\bullet}}{\left(K_3\left(\overline{\beta}_1^{\bullet}\right)^2 - K_2\overline{\zeta}^2\right)\left(\overline{\zeta}^2 - \overline{\beta}_2^2\right) - 2\overline{\beta}_1^{\bullet}\overline{\beta}_2\overline{\zeta}^2(K_3 - K_2)}\frac{d\overline{\zeta}}{\overline{\zeta} - \alpha}\right\} \tag{2.8}$$

Solution of (2.5) is by [6]

$$\Omega_2^+ = \frac{i_0}{2\pi i\alpha s\rho_s}\left(1 - \frac{\beta_2^+(\overline{\alpha})R^+(\overline{\alpha})}{\beta_2^+(0)R^+(0)}\right), \qquad V^- = -\frac{i_0}{2\pi\alpha s\rho_s\beta_2^+(0)R^+(0)\beta_2^-(\overline{\alpha})R^-(\overline{\alpha})C_0}$$

where $\overline{\beta}_2^{\pm} = \sqrt{\frac{\omega}{b} \pm \overline{\alpha}}$, $\overline{\beta}_* = \sqrt{\overline{\alpha}^2 - \frac{\omega^2}{c_*^2}}$, $c_1 = \overline{a}$, $c_2 = b$

Introducing of dimensionless variables $\bar{\alpha} = \frac{a}{\omega}\alpha$, $\bar{\zeta} = \frac{\zeta}{a}\omega$, $\bar{\beta}_n = \sqrt{\bar{\alpha}^2 - \frac{\omega^2}{c_n^2}} = \frac{\omega}{a}\sqrt{\alpha^2 - \frac{\bar{a}^2}{c_n^2}}$, $\bar{\alpha}_1 = \alpha_1\frac{\omega}{a}$, $\bar{\alpha}_2 = \alpha_2\frac{\omega}{a}$ one obtains solution (2.8) as

$$G(\alpha)=Exp\left[\frac{1}{\pi}\int_1^{*} arctg\frac{\frac{1}{b^2 a}\sqrt{\zeta^2-1}+\frac{2}{a^4}(K_2-K_3)\zeta^2\sqrt{\zeta^2-1}\sqrt{\frac{\bar{a}^{-2}}{b^2}-\zeta^2}}{\frac{1}{a^{-4}}\left(K_3(\zeta^4-1)-K_2\zeta^2\left(2\zeta^2-\frac{\bar{a}^{-2}}{b^2}\right)\right)}\frac{d\zeta}{\zeta-\alpha}+\frac{1}{\pi}\int^{*} arctg\frac{\frac{\bar{a}^{-2}}{b^2}\sqrt{\zeta^2-1}}{K_3(\zeta^4-1)-K_2\zeta^2\left(2\zeta^2-\frac{\bar{a}^{-2}}{b^2}\right)+2\sqrt{\zeta^2-1}\sqrt{\zeta^2-\frac{\bar{a}^{-2}}{b^2}}\zeta^2(K_2-K_3)}\frac{d\zeta}{\zeta-\alpha}\right]$$

The inverse transformations Laplace and Fourier from (3.5), gives solution in Smirnov–Sobolev form [3], and for $y = 0$ one obtains

$$\frac{1}{a}\frac{\partial^2}{\partial t^2}V = \mathrm{Re}\frac{\frac{i_0}{\rho_s a}G^-(0)\bar{\alpha}_1\bar{\alpha}_2\sqrt{1-\frac{\bar{a}t}{x}}}{\left(\frac{K_3}{a}-1\right)\left(\frac{\bar{a}t}{x}-\alpha_1\right)\left(\frac{\bar{a}t}{x}-\alpha_2 i\right)G^-\left(\frac{\bar{a}t}{x}\right)}, \quad V = x\mathrm{Re}\frac{\frac{i_0}{\rho_s a}G^-(0)\alpha_1\alpha_2}{\left(\frac{K_3}{a}-1\right)}\int_0^{\frac{\bar{a}t}{x}}\frac{\sqrt{1-\alpha'}\left(\frac{\bar{a}t}{x}-\alpha'\right)d\alpha'}{(\alpha'-\alpha_1)(\alpha'-\alpha_2 i)\alpha' G^-(\alpha')} \qquad (2.9)$$

The calculations of (2.9) for $i_0(\rho_s\bar{a})^{-1} = 5\cdot10^{-6}$, $\alpha_1 = 0{,}99999987$, $\alpha_2 = 13333{,}3326$, $\bar{a} = 1000\frac{см}{сек}$, $\frac{V_0}{a} = \frac{1}{2\cdot10^4}$, $\frac{\bar{a}}{b} = 4$, $\frac{K_2}{a} = 0{,}00058$, $\frac{K_3}{a} = 0{,}00062$, , give graphs of fig. 1 and fig. 2

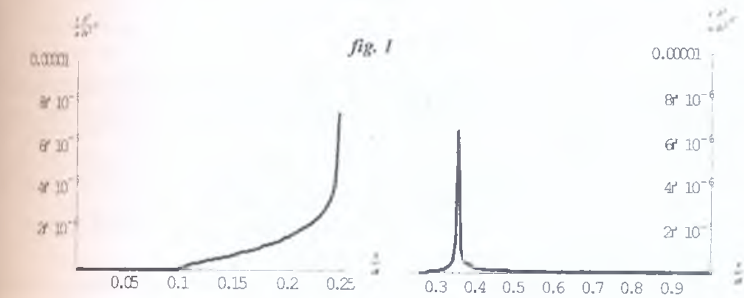

fig. 1

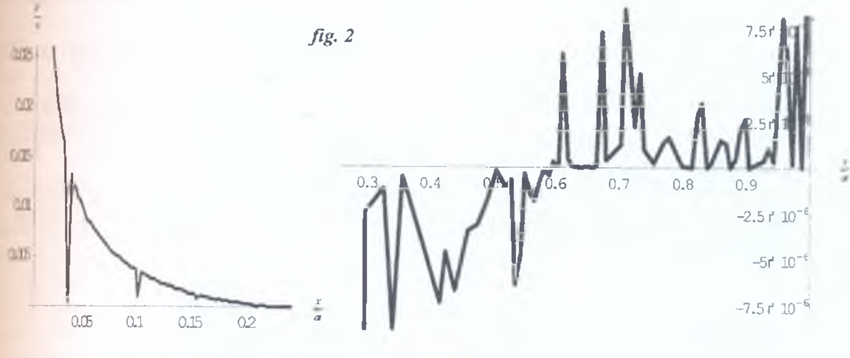

fig. 2

As it is seen $\frac{\partial^2}{\partial t^2}V$ out of crack is positive; fig.2. shows that for $0 \le \frac{x}{at} \le 0.25$, $\frac{V}{x} > 0$, a

$0.25 \le \frac{x}{at} \le 0.6$, $\frac{V}{x} < 0$, and width of crack $b_1 = b_0 + V$ is decreased. For $b_0 = 10^{-5}$ cm, one o

$\frac{x}{at} \approx 0.5, \frac{V}{x} = -7 \cdot 10^{-6}$, and healing condition $b_1 = 0$ yields $10^{-5} - 7 \cdot t \cdot 10^{-3} = 0$ an

$t = \frac{1}{700}$ sec crack is healed in point $x = \frac{5}{7}$ cm.

Резюме

Рассматриваются задачи о наращивании слоя примесей, содержащихся в жидк (флюиде), поступающей в трещину в бесконечной термоупругой плоскости. При некото упрощающих предположениях на характер влияния диффузии примесей на залечива трещины удается решить нестационарную задачу термоупругости при смешанных гранич условиях на границе трещине с учетом наращивания среды на границе за счет осажде примесей. Задачи решаются методом интегральных преобразований Лапласа и Фурь решением уравнений Винера – Хопфа с приведением решение к форме Смирнова-Соболева.

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