FINDING STRUCTURAL CHANGES IN FINANCIAL MARKETS WITH DISTANCE BASED PORTFOLIOS

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We propose analyzing structural changes in financial markets based on portfolios constructed with minimal distance to normality. There are several ways to check changes in one time series, but tracking structural changes in many times series simultaneously should be based on some functionality. Here we analyze possibilities of linear combinations of returns of underlying securities. Utilizing techniques of Hellinger's probability metric, we construct the portfolio closest to Gaussian in distribution with parameters found from efficient frontier of Markowitz's mean-variance portfolio. The Hellinger's distance though cannot address any change of the whole bunch of assets, such as momentary shift in return; it is sensitive to changes within market dependencies. We display the use of the method on an obvious example of exchange market, where we detect the obvious structural change. Thus this diagnostic tool may be a good invariant for structural change detection, or at least a good approximation of one.

Keywords: *Hellinger distance, market structural change, ForEx, distance portfolio.*

I. Introduction and motivation

Lately we have proposed an alternative to classical Markowitz's mean-variance model, based on minimal Hellinger's distance [1].

Distance based portfolios are vastly used in construction of tracking rival's portfolios when the risk measure or strategy they use are unknown [2]. We proposed tracking distance from normal distributed portfolios with minimal variance for all levels of desired expected return. Two reasons for this construction are less sensitivity (variability) of that portfolio to small changes and less corner solutions. (These are essential drawbacks of mean-variance portfolios [3] [4] [5].)

However, the main purpose is to track possible reactions (changes in portfolio weights) to extreme changes in market. For analyzing such extreme changes in one time series, bunch of mechanisms already exist (most known one being unit root tests). Yet, simultaneous changes in the whole market (like shifts due to inflation) will keep the optimal portfolios unchanged.

This is a very desirable feature as only relative changes have specific effects of portfolios managers' decisions (or changes in subclass of securities).

Measures of non-gaussianity are internal in signal processing, especially in disentangling many intertwined signals (time series) [6]. In independent component analysis framework one seeks to maximize distance, here we stick to an opposite direction. We try to linearly entangle the portfolio returns so as to maximize the normality.

Obviously, the larger the number of securities, the closer their linear combination will be normal in distribution, by central limit theorem. However, during the optimization phase, we seek portfolios closer to specific normal distribution. More exactly, we first solve the mean-variance optimization problem for all levels of expected return, and then for each level we minimize the Hellinger's distance to normal distribution with that level mean and minimal variance found.

Other metrics, the most famous ones based on Wasserstein metric and KL-divergence, used to get less sensitive solution were also proposed, [7] [8], .

The paper is organized as follows. In the 2nd part, the minimal Hellinger's distance portfolio construction process is described. In the 3rd part, the method is applied to ForEx market in Armenia (evaluating structural changes before and after war in Nagorno-Karabagh). The paper ends with a conclusion.

II. Problem statement changes in optimal portfolio

The classical mean variance problem seeks the portfolio, i.e. vector of weights by solving the following optimization problem

$$\begin{cases} Var(X) \rightarrow min \\ E(X) = \overline{e} \\ X = \sum_{i=1}^{n} w_i X_i \\ \sum_{i=1}^{n} w_i = 1 \end{cases} \quad (1)$$

where

X_i – represents return of i-th asset. Return can be calculated by either subtracting previous price (value) of given asset and dividing by it, or dividing by previous period price and taking natural logarithm (called log-return), which is exactly our case.

w_i – s are weighted sought. The optimization is exactly by weights. They represent the part of the original money spent for buying i-th asset.

X – represent the portfolios return. It is linear combination of returns of assets.

$\overline{e}$- is the desired level of expected return.

n – number of considered assets (or number of types of assets).

For our purposes, we add one additional condition on weights, prohibiting short selling: $w_i \geq 0$.

One originally deals with price processes $S_i(t)$. Note that calculated returns $\ln\left(\frac{S_i(t)}{S_i(t-1)}\right) = X_i(t)$, will generally depend on t. However, to make them usable in Markowitz framework, one always assumes that $X_i(t)$ – s have same distributions for each t [9].

While generally for short time horizon this can be regarded true, when structural changes take place, the change should be notable by the change in return distribution [10].

Thus, it is one of the problems of modeling in mean-variance framework. One simple thing to do is exponential (or other form of) smoothing. While it overall smooths out the differences between $X_i(t)$-s at different t-s, it cannot be used to account structural changes of jumps.

Obviously, changes in best portfolios will indicate the structural changes in a group of assets.

Generally, several drawbacks of Markowitz mean-variance portfolio are mentioned in literature. One of them is its extreme sensitivity to change of initial condition [5].

This sensitivity may lead to extreme changes in variance-minimizing portfolio from one period to another. While this is quite natural, it means we cannot take the minimum variance portfolio (portfolio with minimum variance among all portfolios) as something stable over time and represent it for the given market.

Hence, a less sensitive portfolio should be considered. We proposed the following portfolio based on Hellinger's distance optimization.

$$\begin{cases} H^2\left(X, N\left(\overline{e}, \sigma^2_{min}(\overline{e})\right)\right) \to min \\ E(X) = \overline{e} \\ X = \sum_{i=1}^{n} w_i X_i \\ \sum_{i=1}^{n} w_i = 1 \\ w_i \geq 0 \, ; i = \overline{1,n} \end{cases} \tag{2}$$

Where

$H^2(X,Y)$ – is squared Hellinger's distance from random variable X to random variable Y. We use random variables and their respective distributions interchangeably, when used as argument of Hellinger's distance.

More specifically, when we know respective densities we can rewrite $H^2(X,Y) = H^2(f_X, f_Y)$, with f standing for density functions.

$\sigma^2_{min}(\overline{e})$ – is the minimal variance found by solving (1), for given level of the desired expected return.

And

$N(a, \sigma^2)$ – for normal distribution with mean a and variance σ^2.

In case of absolutely continuous distributions

$$H^2(f_X, f_Y) = 1 - \int \sqrt{f_X(z) f_Y(z)} dz$$

We use exact Hellinger's distance for two main reasons. First, for its simple form, and second, for it being less sensitive than other statistical metrics. One additional reason is that it is indeed matric, while for example negentropy (see [6]) uses KL-divergence which is not symmetric, and thus is pseudo-metric.

The problem (2) is generally a non-smooth problem for any given data. While kernel methods of approximation can be used, they subtract useful distortion presented in data.

We use the traditional binning mechanism to make continuous histogram distribution out of discrete data, to be able to calculate squared Hellinger's distance.

III. Exposition of method with foreign exchange market

We took the data of Central Bank of Armenia on currency exchange rates for 5 currencies: Chinese yuan (CNY), EURO (EUR), Japanese yen (JPY), Russian ruble (RUB) and US dollar (USD), for a period from 26/07/2019 to 24/02/2022[1].

This period was chosen as the start day and end day are of the same distance as to November 9th of 2020[2].

We divided the data into two periods (26/07/2019-06/11/2020 and 09/11/2020-24/02/2022), and solved problems (1) and (2) for each period.

For details of construction solving portfolios, see original paper [1].

First, we solve mean-variance portfolio problem, determining minimal variance and/or standard deviation (the square-root of variance). Obtaining necessary parameters, we calculate squared Hellinger's distance to Gaussian distribution with those parameters and solve the minimization problem [2] for each level of the desired expected return. To do so, we constructed histogram-densities, taking the number of bins to be 10 (according to Sturges' formula). We determine the number of portfolio's returns in each bin. Then, to determine ten bin coefficients, we divide the count of each bin by the number of data points. Then we determine Hellinger's distance by using the averages found above but changing weights and, accordingly, deviations.

We aim to calculate minimal squared Hellinger's distances and find the according portfolios.

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[1] The end day was chosen the day of Russian intrusion into Ukraine.

[2] This day was proclaimed as the end of Nagorno-Karabagh war, when the truce was established.

Ultimately, we want to check, whether the structural changes can be reported according to the portfolio providing the overall minimal squared Hellinger.

More specifically, the portfolio weights with minimal Hellinger change substantially from one period to another.

Two ways to conduct the process, is by first solving the (2) for the first period (and this step is identical for both methods)

And next is either

To solve (2) for the second period and compare the optimal portfolio weights.

Or to find Hellinger's distance for optimal portfolio from the first period with the second period data.

We got the following results.

Table 1. Minimum squared Hellinger's distance, and respective portfolios for 26/07/2019-06/11/2020 data.

H^2	mean	St. dev.	w_1 (CNY)	w_2 (EUR)	w_3 (JPY)	w_4 (RUB)	w_5 (USD)
0.026639	-0.000504382	0.010407	0	0	0	1	0
0.023976	-0.000423109	0.009291	0.051205	0.035139	0.019605	0.894052	0
0.023857	-0.000341836	0.008112	0.118461	0.039154	0.056908	0.785477	0
0.023154	-0.000260564	0.007055	0.009026	0.178194	0.009053	0.665278	0.138449
0.02229	-0.000179291	0.005802	0.24039	0.078994	0.010624	0.548156	0.121837
0.020428	$-9.80176*10^{-5}$	0.004463	0.094266	0.076995	0.242433	0.440276	0.146029
0.021867	$-1.67447*10^{-5}$	0.002737	0.015627	0.037572	0.080547	0.243433	0.622821
0.018182	$6.45282*10^{-5}$	0.001992	0.148056	0.020171	0.157278	0.149977	0.524518
0.016739	0.000145801	0.001761	0.27683	0.102833	0.110871	0.059215	0.450251
0.012382	0.000227074	0.002172	0.576477	0.130773	0.145174	0.003343	0.144233
0.010537	0.000308347	0.004428	**0**	**1**	**0**	**0**	**0**

Table 2. Minimum squared Hellinger's distance, and respective portfolios for 09/11/2020-24/02/2022 data.

H^2	mean	St. dev.	w_1 (CNY)	w_2 (EUR)	w_3 (JPY)	w_4 (RUB)	w_5 (USD)
0.009792	-0.000399068	0.004562	**0**	**0**	**1**	**0**	**0**
0.01271	-0.000353284	0.004275	0.067402	0.010023	0.881002	0.000124	0.041449
0.01336	-0.0003075	0.00401	0.104651	0.309609	0.585595	0.000145	0
0.012596	-0.000261716	0.003638	0.121618	0.119939	0.457377	0.138651	0.162415
0.014297	-0.000215932	0.003431	0.139052	0.11113	0.328202	0.139576	0.282039
0.016957	-0.000170148	0.003512	0.357526	0.238879	0.242254	0.087413	0.073928
0.01809	-0.000124363	0.003409	0.473943	0.173895	0.193334	0.077161	0.081667
0.019066	$-7.85793*10^{-5}$	0.003431	0.695153	0.00981	0.28526	0.009777	0
0.019705	$-3.27951*10^{-5}$	0.003398	0.792802	$7.25*10^{-5}$	0.16872	0.038406	0
0.018514	$1.2989*10^{-5}$	0.003424	0.894114	$7.25*10^{-5}$	0.074431	0.031382	0
0.02073	$5.87731*10^{-5}$	0.003484	1	0	0	0	0

Note that we do not see substantial changes in minimal Hellinger's distance (0.01054 vs 0.00979), but we see explicit change in respective portfolios ((0,0,0,1,0) vs (0,0,1,0,0)).

So in the first period closer to normality was ruble, and in the second period - Japanese yen. This kind of corner solutions are not widespread (for example, we got a different result for a bigger period in [1]); however, they demonstrate our point explicitly.

IV. Conclusion

We found that Hellinger's distance did account for structural changes within the market, so it can reasonably be used as a market invariant specific for each market, and sensitive to extreme changes in pairwise distributions.

Further questions to investigate here are exact bounds on sensitivity and investigation of (squared) Hellinger's distance averaged over some period of time both for given portfolio or for overall minimal Hellinger's distance. Moreover, we conjecture that more extreme changes in market structure would change the value of minimal Hellinger's distance substantially, rather than changing portfolio weights only, and will try to establish this in our future works.

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ՖԻՆԱՆՍԱԿԱՆ ՇՈՒԿԱՆԵՐՈՒՄ ԿԱՌՈՒՑՎԱԾՔԱՅԻՆ ՓՈՓՈԽՈՒԹՅՈՒՆՆԵՐԻ ՀԱՅՏՆԱԲԵՐՈՒՄԸ ՄԵՏՐԻԿԱՅԻ ՎՐԱ ՀԻՄՆՎԱԾ ՊՈՐՏՖԵԼՆԵՐԻ ՕԳՆՈՒԹՅԱՄԲ

ՄԵՍՐՈՊՅԱՆ ՄԵՍՐՈՊ

ԵՊՀ, 3-րդ կուրսի ուսանող,
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ԲԱՐԴԱԽՉՅԱՆ ՎԱՐԴԱՆ

Ֆիզիկամաթեմատիկական գիտությունների թեկնածու,
ԵՊՀ դասախոս, մաթեմատիկայի և մեխանիկայի ֆակուլտետ,
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Հոդվածում առաջարկում ենք ուսումնասիրել ֆինանսական շուկաների կառուցվածքային (էական) փոփոխությունները նորմալ բաշխմանը ամենամոտ բաշխումով պորտֆելների միջոցով: Կան բազում եղանակներ՝ մեկ ժամանակային շարքում կառուցվածքային փոփոխությունները նկատելու և ստուգելու, բայց մի քանի ժամանակային շարքերի՝ միմյանց նկատամբ կառուցվածքային փոփոխությունները ստուգելու համար պետք է օգտագործվեն ֆունկցիոնալ տեսքեր: Այստեղ մենք դիտարկում ենք գործիքների եկամտաբերությունների գծային կոմբինացիաների հնարավորությունները: Ամբողջապես օգտագործելով Հելլինգերի հավանականային մետրիկայի հնարավորությունները՝ մենք կառուցում ենք պորտֆել՝ Գաուսյան բաշխմանը ամենամոտ բաշխումով, նախօրոք ընտրված պարամետրերով: Վերջիններս գտնվում են՝ լուծելով Մարկոցի միջին-վարիացայի պորտֆելի խնդիրը: Չնայած Հելլինգերի հեռավորությունը չի կարող նկատել շուկայի ընդհանուր փոփոխությունները, ինչպիսին է, օրինակ, ընդհանուր եկամտաբերության անկումը, այն շատ զգայուն է առանձին գործիքների կախվածությունների նկատմամբ: Արտարժույթի շուկայի օրինակով մենք ցույց ենք տալիս մեթոդի ունակ լինելը: Այսպիսով՝ հույս ունենք, որ ստացված հեռավորությունը, լինելով շուկայական կառուցվածքային փոփոխության դիագնոստիկ մեթոդ, հետագայում կվերածվի շուկայական ինվարիանտի կամ կլինի վերջինիս մոտարկում:

Բանալի բառեր՝ Հելինգերի հեռավորություն, արժեթղթերի պայուսակի օպտիմիզացիա, հավանականային մետրիկաներ:

НАХОЖДЕНИЕ СТРУКТУРНЫХ ИЗМЕНЕНИЙ В ФИНАНСОВЫХ РЫНКАХ С ПОМОЩЬЮ МЕТРИЧЕСКИХ ПОРТФЕЛЕЙ

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В статье предлагаем анализировать структурные изменения финансовых рынков через портфели с распределением наиболее близким к нормальному.

Существует множество способов проверки структурных изменений в единичных временных рядах, но для проверки структурных изменений по отношению друг к другу мы должны рассматривать функциональные формы. В данной статье мы рассматриваем конкретно линейные комбинации доходностей в портфелях. В полной мере используя возможности вероятностой метрики Хеллингера, вы конструируем модель портфеля наиболее близкого по распределению к Гауссовскому, с параметрами, полученными с эффективного фронта решения задачи портфели Марковица. Хотя расстояние Хеллингера не может фиксировать одновременные изменения всего рынка, такие как, например, общее изменение доходности, она очень чувствительна к зависимостям между отдельными активами. На примере рынка валют мы показываем действенность предложенного метода. Таким образом, мы надеемся, что данный метод диагностики в дальнейшем может перерасти в инвариант для рынков разных ценных бумаг, или хотя бы может быть приближением такого.

Ключевые слова: Расстояние Хеллингера, структурные изменения в рынках, портфели, основанные на метрике, вероятностные метрики.

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