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On the Role of Scalar Fields in the Evolutionary Development of the Early Universe

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Abstract: In most constructions of the inflationary regime of the development of the Universe, the presence of a specific scalar field (inflaton) is assumed, which contributes to the expansion of space at an enormous pace. In the present work, we study the dynamics of a conformally bound scalar field for the de Sitter case, in the presence of a closer connection between the potential and the space curvature R.

Keywords: Inflation, scalar-tensor theory of gravity, Higgs boson, cosmological constant, inflaton

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1. Introduction

Cosmological models are essential for understanding the Universe and its evolution. They are based on the principles of theoretical physics and are validated through observations in astronomy. The birth of scientific cosmology can be traced back to Albert Einstein's general theory of relativity, which introduced the concept of spacetime geometry being influenced by gravitational fields. This theory led to the development of the Einstein field equations, which describe the dynamics of spacetime.

To determine the solutions that approximate the geometry of our universe, scientists sought to understand the different possible configurations of spacetime. The study of cosmology gained significant impetus after the observation of the cosmic microwave background (CMB) anisotropy, which provided crucial evidence supporting the theory of inflation.

Inflation is a phase of rapid expansion believed to have occurred in the Early Universe. It helps explain various observed features of the Universe, such as its overall homogeneity and isotropy. Inflation is typically attributed to the potential energy of a scalar field known as the inflaton. However, within the framework of the standard model of particle physics, there is no suitable candidate for the inflation field [1]. This has led researchers to explore extensions beyond the standard model.

Two intriguing inflation models are the Higgs-inflation and R-inflation models. In the Higgs-inflation model, the inflation field is associated with the Higgs boson of the standard model. The Higgs boson, with its large non-minimal coupling to the Ricci (R) scalar (a measure of the curvature of spacetime), can potentially drive inflation. This model is appealing due to its simplicity and consistency with CMB observations.

The R-inflation model is another interesting case. It involves modifications to gravity, specifically the Einstein action, by including additional terms involving the Ricci scalar. These modifications provide a mechanism for inflation and have been studied extensively in the context of cosmology.

Both the Higgs-inflation and R-inflation models offer intriguing possibilities for explaining the Early Universe's inflationary period. They highlight the need to extend our understanding beyond the standard model of particle physics to account for the observed phenomena in cosmology.

In this paper, the role of the inflationary field by a conformally coupled scalar field is studied, which arises under the conformal transformation of the modified Jordan-Dicke-Brans theory obtained by Bekenstein [2], taking into account the cosmological constant.

2. Scalar field equations

The basic idea of inflation is that the hot stage was preceded by an inflationary stage, in which the product a(t)H(t) is far from being a growing function of time, where a(t) is called the scale factor of the Universe, H(t) is parameter of Hubble. For this condition to be satisfied, it is required that the scale factor grows with time in a faster than linear fashion, that is, the Universe expands with acceleration [1]. In contrast to Starobinsky's model [3-7], we will propose that the inflationary regime arises due to the presence of a special scalar field in the Universe. In contrast to the operation (3) in ref. [1], the action functional is presented in the form

$$w = \int -\left(\frac{1}{2k} - \frac{\psi^2}{12}\right)(R + 2\Lambda) + \frac{1}{2}(\Delta\psi)^2\Big\}\sqrt{-g}d^4x, \quad (1)$$

where ψ is a scalar field, Λ- cosmological constant, R-Ricci scalar.

Accordingly, the equations describing the cosmological evolution take the form

$$R - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu} = kT_{\mu\nu}, \quad (2)$$

$$T_{\mu\nu} = \nabla_\mu\psi\nabla_\nu\psi - \frac{1}{2}g_{\mu\nu}(\nabla\psi)^2 - \frac{1}{6}\nabla_\mu\nabla_\nu\psi^2 - \frac{1}{6}g_{\mu\nu}\nabla^2\psi^2 + \frac{\psi^2}{6}\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda g_{\mu\nu}\right), \quad (3)$$

where $\nabla^2\psi = g^{\mu\nu}\nabla_\mu\psi\nabla_\nu\psi$, $(\nabla\psi)^2 = g^{\mu\nu}\nabla_\mu\nabla_\nu\psi$.

As a result of simplifications

$$\frac{1}{6}\nabla_\mu\nabla_\nu\psi^2 = \frac{1}{3}\nabla_\mu\nabla_\nu\psi + \frac{1}{3}\nabla_\mu\nabla_\nu\psi$$
$$\frac{1}{6}g_{\mu\nu}\nabla^2\psi^2 = \frac{1}{3}g_{\mu\nu}g^{\alpha\beta}\nabla_\alpha\nabla_\beta\psi + \frac{1}{3}g_{\mu\nu}\nabla^2\psi, \quad (4)$$

Finally, we get:

$$R - \frac{1}{2}g_{\mu\nu} - \Lambda g_{\mu\nu} = k\left(1 - \frac{k\psi^2}{6}\right)^{-1}\tau^c_{\mu\nu}$$
$$\tau^c_{\mu\nu} = \frac{2}{3}\nabla_\mu\psi\nabla_\nu\psi - \frac{1}{6}g_{\mu\nu}(\nabla\psi)^2 - \frac{\psi}{3}\nabla_\mu\psi\nabla_\nu\psi + g_{\mu\nu}\nabla^2\psi, \quad (5)$$

Eventually,

$$R = -4\Lambda\left(1 - \frac{k\psi^2}{12}\right), \quad (6)$$

$$\dot{H} + 2H^2 = \frac{2}{3}\Lambda\left(1 - \frac{k\psi^2}{12}\right). \quad (7)$$

and the scalar field equation

$$\ddot{\psi} + 3H\dot{\psi} + \frac{2\Lambda}{3}\psi\left(1 - \frac{k\psi^2}{12}\right) = 0, \quad (8)$$

where $H = \dot{a}/a$.

3. Representation of ψ(a) in dimensionless applications

From (7) and (8) equations

$$\ddot{\psi}+3H\dot{\psi}+\psi\left(\dot{H}-2H^2\right)=0, \tag{9}$$

which can be represented as

$$\frac{d}{dt}(\dot{\psi}+H\psi)+2H(\dot{\psi}+H\psi)=0, \tag{10}$$

Indeed,

$$\ddot{\psi}+\dot{H}\psi+\dot{\psi}H+2H\dot{\psi}+2H^2\psi=0, \tag{11}$$

$$\ddot{\psi}+3H\dot{\psi}+\psi(2H^2+\dot{H})=0, \tag{12}$$

$$ln(\dot{\psi}+H\psi)=2\frac{\dot{a}}{a}=-2\,ln\,a=ln\,a^{-2}, \tag{13}$$

Here we can use laws of ln and get

$$(\dot{\psi}+\frac{\dot{a}}{a}\psi)a^2=const, \tag{14}$$

That's mean

$$\dot{\psi}+H\psi=\frac{c}{a^2}, \tag{15}$$

$$\dot{\psi}+\frac{\dot{a}}{a}\psi=\frac{c}{a^2},\quad \frac{1}{a}\frac{d}{dt}(\psi a)=\frac{c}{a^2},$$
$$\frac{d}{dt}(\psi a)=\frac{c}{a}, \tag{16}$$

which is graphically illustrated in Figure 1 and indicates the expansion of the Universe.

4. Representation in dimensionless variables

Introducing dimensionless quantities [8, 9]

$$\tau={}^{t}/_{t_0},\quad \psi=\frac{x}{\sqrt{k}},\quad \dot{\psi}=\frac{y}{t_0\sqrt{k}},\quad \ddot{\psi}=\frac{1}{t_0^2\sqrt{k}}\frac{dy}{d\tau}, \tag{17}$$

we represent the system of basic equations in the form

$$\frac{dx}{d\tau}=y, \tag{18}$$

$$\frac{dy}{d\tau}+3(Ht_0)y+\frac{2x\Lambda t_0^2}{3}\left(1-\frac{x^2}{12}\right)=0, \tag{19}$$

$$\frac{d(Ht_0)}{d\tau}+2(Ht_0)^2-\frac{2}{3}\Lambda t_0^2\left(1-\frac{x^2}{12}\right)=0. \tag{20}$$

5. Representation in (Ht_0) dimensionless variables

Two equations defining R_{00} and R_{11} [10, 11] from system (5) are presented in the form

$$\left(1-\frac{k\psi^2}{6}\right)(3H^2-\Lambda)=\frac{k\dot{\psi}^2}{2}+k\psi\dot{\psi}H, \qquad (21)$$

$$3\left(1-\frac{k\psi^2}{6}\right)\left(2\dot{H}+3H^2-\Lambda\right)=-\frac{k\dot{\psi}^2}{2}+k\psi\left(\ddot{\psi}+2\dot{H}\dot{\psi}^2\right), \qquad (22)$$

Substituting equation (8) into (22) and taking into account (21) we'll get (7), which confirms that the equations are not independent. From (21) in dimensionless variables is determined by the expression (Ht_0)

$$3\left(1-\frac{x^2}{6}\right)H^2t_0^2-xyHt_0-\left[\Lambda t_0^2\left(1-\frac{x^2}{6}\right)+\frac{y^2}{2}\right]=0, \qquad (23)$$

$$(Ht_0)=\frac{\frac{xy}{2}\pm\sqrt{3\Lambda t_0^2\left(1-\frac{x^2}{6}\right)^2+\frac{3y^2}{2}}}{3\left(1-\frac{x^2}{6}\right)}. \qquad (24)$$

Considering that equation (8) can be attributed to a simple physical analogy - mechanical rolling in a potential well with a profile [12]

$$V(x)=2\Lambda t_0^2\left(1-\frac{x^2}{12}\right)^2, V_x^{'}(x)=-\frac{2x}{3}\Lambda t_0^2\left(1-\frac{x^2}{12}\right), \qquad (25)$$

and time-dependent friction coefficient $3H\dot{\psi}$, it is advisable to choose a function $y=\frac{dx}{d\tau}$ as

$$y^2=2\Lambda t_0^2\left(1-\frac{x^2}{12}\right)^2 sh^2X(x)\ . \qquad (26)$$

where $X(x)$ will be determined by comparison $\frac{dy}{d\tau}$ from (19) and (26).

Using the expression $Ht_0$from (24) subject to (26)

$$3Ht_0=\sqrt{\Lambda t_0^2}\left[\frac{xshX(x)}{\sqrt{2}}\pm\sqrt{3}chX(x)\right], \qquad (27)$$

we get from (26)

$$\frac{dy}{d\tau}=-\frac{2}{3}\Lambda t_0^2\left(1-\frac{x^2}{6}\right)xsh^2X(x)+y^2cthX(x)\frac{dX(x)}{dx}, \qquad (28)$$

in and out (19)

$$\frac{dy}{d\tau}=y\sqrt{\Lambda t_0^2}\left[\frac{x}{\sqrt{2}}shX(x)\pm\sqrt{3}chX(x)\right]-\frac{2\Lambda t_0^2}{3}x\left(1-\frac{x^2}{12}\right), \qquad (29)$$

as a result, we get

$$\frac{dX(x)\Big/\sqrt{\Lambda t_0^2}}{d\tau}=\frac{x\left(sh^2X-1\right)}{3\sqrt{2}chX}\mp\sqrt{3}shX. \qquad (30)$$

$$V(x)=2\Lambda t_0^2\left(1-\frac{x^2}{12}\right)^2, \qquad (31)$$

6. Physical picture of the potential energy of a scalar field

In this model, there are two vacuum states. The point $x = 0$, in which V(x) has a local maximum, just as the indicated states are equilibrium points, but $x = 0$ the field is unstable in equilibrium and small fluctuations lead to the fact that the field begins to grow in amplitude, rolling into one of the minimum. The field is called Higgs [13]. Theories of this type are called theories with spontaneous symmetry breaking [14]. At the point $x \neq 0$ the symmetry is broken, and the system goes into this state spontaneously.

The complex Higgs field can form interesting cosmologically significant objects, the so-called vortex filaments, that is, topologically stable objects. At the center of the thread $|\langle x\rangle|=0$, but at a great distance $|\langle x\rangle|=\sqrt{12}$. These objects have transverse dimensions $L \sim \sqrt{\Lambda t_0}$ and they are macroscopically longitudinal. They can play an important role in the formation of the large-scale structure of the Universe, being the seeds of growing density in homogeneities.

7. Results and Discussions

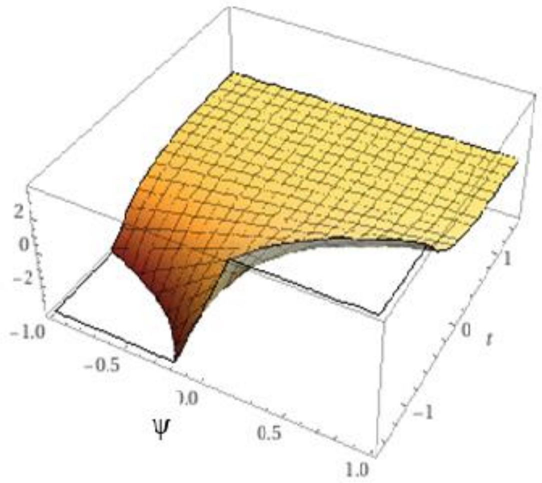

Fig. 1. Expansion of the Universe over time.

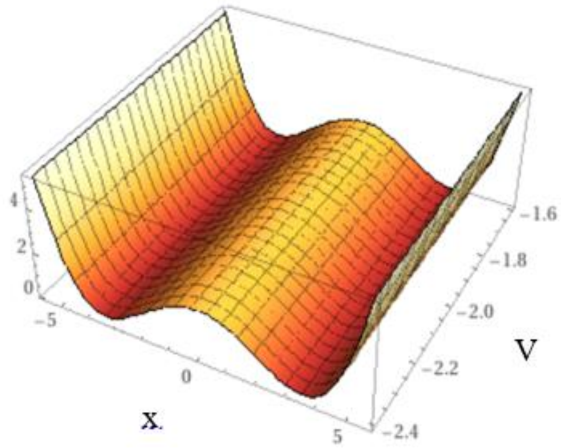

Fig. 2. Dependence of the scalar field on time.

The plot in Fig. 1 depicts the evolution of Universe over time with acceleration. The plot in Figure 2 illustrates the evolution of the field as it transitions between the two minimum points. The specific shape and behavior of the plot depend on the potential energy function associated with the Higgs field and the dynamics of the field equations in the model.

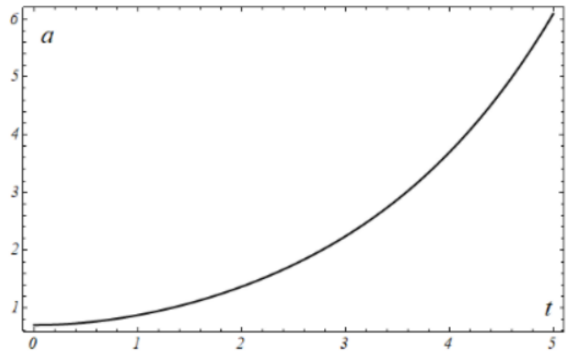

Fig. 3. The dependence of the scaling factor on time.

In Fig. 3 the dependence of the scaling factor on time was shown, suggests that the Universe is undergoing accelerated expansion. In the context of cosmology, the scaling factor represents the relative size of the Universe at different points in time.

8. Conclusion

In summary, we have shown that the Higgs field exhibits two vacuum states, with one being a local maximum and the others being local minimum-energy states. When the Higgs field is at the local maximum point, it is in equilibrium, but this equilibrium is unstable. Small fluctuations or perturbations in the field cause it to undergo a process known as symmetry breaking, where it rolls into one of the minimum-energy states.

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