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Description of the Classical Diffraction Experiment of a System of Many Slits and the Interference Pattern of Two Sources within the Framework of a Single Scheme

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Abstract. The problem of an approximate description of the wave field of many sources is considered. It is shown that, if the problem is considered more comprehensively, the description of the interference and diffraction experiments can be given within the framework of a single scheme. It is shown that the character of wave amplification in these two classical experiments is different. In the case of an interference experiment, the field amplification is fixed at a point, while in the case of a diffraction experiment the field amplification is fixed in the observation direction. The problem of determining the intensity maximum of a diffraction grating in the Fresnel pattern is studied under the condition that the maximum is satisfied in the Fraunhofer pattern. A condition under which a diffraction grating behaves like a lens is obtained.

Keywords: interference, diffraction, Fresnel and Fraunhofer patterns

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1. Introduction

It is known that one of the central problems of wave theory is the problem of describing the wave field in regions far from sources and surfaces generating, re-emitting or absorbing this field [1, 2]. Usually, in many problems of diffraction, the influence of the generated field on the process of wave generation by secondary sources is assumed to be small. In the general case, the solution of the diffraction problem is mathematically difficult, which, even in an approximate form, as a rule, must be performed numerically [3–12]. The most well-known method of approximate consideration of a diffracted field is the so-called Fresnel pattern, an important special case of which is the Fraunhofer pattern.

The validity of the use of one or another approximation is usually commented on by the proximity or remoteness of the observation area from the area of sources and surfaces affecting the propagation of waves. At the same time, it is well known that such a comment must necessarily be connected with the radiation wavelength. If for one wavelength this observation distance falls, for example, in the middle observation zone, where the Fresnel pattern can be applied, then the same distance for another wavelength value can fall, for example, already in the near zone. Therefore, in the diffraction theory, besides the space parameters of the problem, also the so-called wave parameter is introduced. In addition to the restrictions imposed on the space parameters of the problem, the applicability condition for the approximate description of the field is also substantiated by the restriction imposed on the values of the wave parameter. On the basis of these values of the wave parameter, the concepts of near, middle and far wave observation zones are introduced.

Let us consider the above-mentioned in the context of two classical problems of wave theory, namely, the problem of interference and the problem of diffraction grating. It is known that the first of them is solved in the Fresnel pattern, and the second – in the Fraunhofer pattern (see Fig. 1).

In both problems, it is required to consider a superposition of spherical waves, however, in the case of interference the number of sources is two, and in the case of a diffraction grating the number of sources can reach several thousand. It is obvious that, in its essence, the problem of interference is a particular case of the diffraction grating problem, of course, if the latter is considered in the Fresnel pattern.

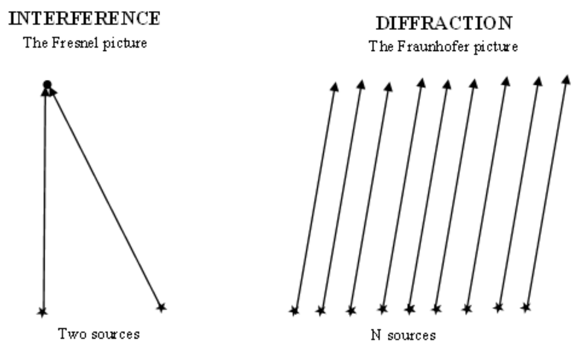

Fig. 1. Interference of waves from two sources (near observation region) and diffraction of waves from N sources (far observation region).

At the same time, despite the fact that both experiments mentioned above represent a demonstration of the same phenomenon, even their approximate description traditionally is not given within the framework of a single mathematical approach. That is, patterns of interference and diffraction experiments seem to be presented to some extent in isolation from each other. For the most part, this very circumstance prompted the implementation of this work. As it will be shown below, the presentation of interference and diffraction experiments can be placed within the framework of one single scheme.

Despite the widespread use, the above-mentioned Fraunhofer method for describing a superposition field based on the construction of parallel rays, in our opinion, requires a more detailed review [13]. This is primarily due to the fact that, according to Euclidean geometry, parallel lines do not have a point of intersection, i.e., parallel rays never converge. Obviously, it then follows that in Fraunhofer constructs the question of mutual amplification or weakening of waves at a point is generally meaningless. A meaning emerges only in the context of directions of observation, and it appears this fact is failed to be mentioned anywhere. In the framework of this work, this issue will also be a subject of discussion.

It is important to note that in addition to the presence of a certain information content, the work presented below is mostly cognitive in nature, since the main emphasis here is on the importance of the generality of the description (integrity of perception) of these two experiences. So far, the interference and diffraction experiments, which are demonstrations of one phenomenon (superposition of waves), have been described separately. In the light of what has been said, within the framework of this work, we substantiate the expediency of introducing the concept of a viewing angle into the picture of an approximate description of a superposition field.

2. Longitudinal and transverse spatial parameters for an approximate description of a spherical field

At its core, the problem of diffraction (interference) is the problem of describing a superposition field of coherent spherical waves. It follows from this, inter alia, that the problem of an approximate description of a diffracted field is ultimately boiled down to the problem of an approximate description of a spherical field [13-15]. In this regard, some results related to the approximate description of a spherical field in the Fresnel and Fraunhofer patterns will be presented below.

Let the locations of the source and the observation point be indicated by the vectors $\vec{r}$ and $\vec{R}$ emanating from the point O (see Fig. 2). Then, as it is known, for the case of a field harmonic in time, the wave field at the observation point is described by the expression

$$U(\vec{R},t) = \frac{a}{|\vec{R}-\vec{r}|} cos\left[\omega t - k\left|\vec{R}-\vec{r}\right|\right], \quad (1)$$

where a is the amplitude, ω is the frequency of the wave and k is the wave number.

Let us assume that the observations of the wave field (1) are carried out in the vicinity of some point O', the position of which is determined by the vector $\vec{R}_0$emanating from the point$O$. Furthermore, we will call the direction of the vector $\vec{R}_0$ the main direction of observation, the $R_0 = \left|\vec{R}_0\right|$ value – the main observation distance, and the O' point – the central observation point. Let us assume that the description of the field (1) is done on a straight line l, passing through the point O' and perpendicular to the vector $\vec{R}_0$. We will refer to the line l as the line of view. On Fig. 2 the observation point is indicated by the vector $\vec{\rho}$ emanating from the pointO'

$$\vec{R} = \vec{R}_0 + \vec{\rho}. \quad (2)$$

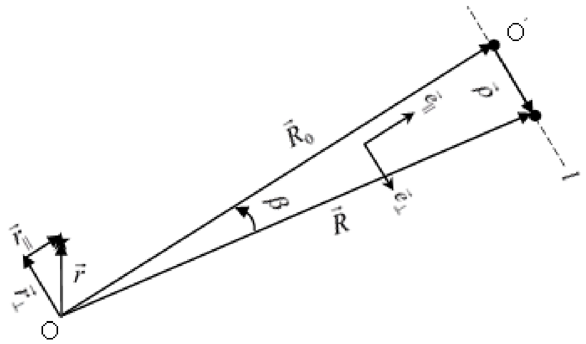

Fig. 2. Longitudinal and transverse spatial parameters for approximate description of a spherical field.

Dimensionless unit vectors $\vec{e}_\parallel$ and $\vec{e}_\perp$, respectively parallel and perpendicular to the vector $\vec{R}_0$, are also shown on Fig. 2

$$\vec{e}_\parallel = \vec{R}_0/R_0 \text{ and } \vec{R}_0 \cdot \vec{e}_\perp = 0. \quad (3)$$

Vectors $\vec{r}_\parallel$, $\vec{r}_\perp$ are the vector $\vec{r}$ components, respectively parallel and perpendicular to the vector $\vec{R}_0$

$$\vec{r} = \vec{r}_\perp + \vec{r}_\parallel. \quad (4)$$

Using the definition (4), the vectors $\vec{r}_\parallel$ and $\vec{r}_\perp$ can be represented as

$$\vec{r}_\parallel = \eta \cdot \vec{e}_\parallel \text{ and } \vec{r}_\perp = \xi \cdot \vec{e}_\perp, \quad (5)$$

where η is the coordinate of the source on the axis in the direction of the vector $\vec{R}_0$ with the origin at the point O, ξ is the coordinate of the source defined by the straight line l and the vector $\vec{e}_\perp$with the origin at the point O'. Further, for convenience, we will assume that the line of view l is parallel to the vector $\vec{e}_\perp$, so the vector $\vec{\rho}$can be written as

$$\vec{\rho} = \rho \cdot \vec{e}_\perp, \quad (6)$$

where $\rho$is the observation coordinate on the observation line.

Using (2) - (6), let us rewrite the wave field (1) as

$$U(\vec{R},t) = \frac{a \cdot cos\left[\omega t - k\sqrt{(R_0-\eta)^2+(\rho-\xi)^2}\right]}{\sqrt{(R_0-\eta)^2+(\rho-\xi)^2}} \tag{7}$$

and consider the following values

$$\sigma = \frac{(\rho-\xi)^2}{R_0}k, \quad \tau = \left(\frac{\rho-\xi}{R_0}\right)^2. \tag{8}$$

Following the terminology adopted in the theory of diffraction, the dimensionless value σ will be called the wave parameter. At the same time, the dimensionless value τ will be referred to herein as the observation ratio. The spherical field (1) rewritten in the form (7) corresponds to its representation by means of spatial parameters of the problem, longitudinal R_0, η and transverse ρ, ξ to the main direction of observation.

As the shown by analysis in [13-15], if the values of the parameters (8) are such that

$$\sigma \sim 2\pi, \quad \tau << 1 \tag{9}$$

and in a way that

$$\sigma \cdot \tau^2 << 1, \tag{10}$$

then the wave field (1) (see also (7)) can be presented in the following form

$$U(\vec{R},t) = \frac{a}{R_0} cos\left[\omega t - k\left((R_0-\eta) + \frac{(\rho-\xi)^2}{2R_0}\right)\right], \tag{11}$$

which, taking into account (8), can also be written as

$$U(\vec{R},t) = \frac{a}{R_0} cos\left[\omega t - k(R_0-\eta) - \frac{\sigma}{2}\right]. \tag{12}$$

This expression is nothing else than the form of a spherical field in the Fresnel pattern.

Further, the first condition (9) will be called the necessary, and relation (10) – the sufficient condition for applying the Fresnel pattern

$$\tau << 1, \ \sigma \cdot \tau^2 << 2\pi. \tag{13}$$

Obviously, the necessary condition $\tau << 1$ imposes quite definite restrictions on the spatial parameters of problem

$$\rho^2 << R_0^2, \eta^2 << R_0^2 \text{ and } \xi^2 << R_0^2. \tag{14}$$

Note, that the from two last conditions it follows that $|\vec{r}|^2 << R_0^2$.

If the necessary condition imposes restrictions on the spatial parameters of the problem, then the sufficient condition limits the wavelength. Using the relation $k = 2\pi/\lambda$ and also taking into account (8), let is consider the value of

$$\delta = \frac{(\rho-\xi)^4}{\lambda R_0^3} = \frac{\sigma \cdot \tau^2}{2\pi}, \tag{15}$$

which we will call the approximation parameter. Taking into account **Error! Reference source not found.**, **Error! Reference source not found.** the following can be derived

$$\frac{(\rho-\xi)^4}{\lambda R_0^3} << 1. \qquad (16)$$

As it can be seen from (16), for the fixed values of the problem's spatial parameters under which the condition (14) holds, an increase in the wavelength leads to a definite improvement in the sufficient condition for applying the Fresnel pattern.

Since the values ρ, ξ are parameters independent on each other, it is clear that condition (16) breaks down into two independent conditions

$$\frac{\rho^4}{R_0^3}k << 2\pi, \frac{\xi^4}{R_0^3}k << 2\pi. \qquad (17)$$

Introducing the values

$$\sigma_\rho = \frac{\rho^2}{R_0}k, \sigma_\xi = \frac{\xi^2}{R_0}k, \qquad (18)$$

and also taking into account (8) the conditions (17) can be rewritten as

$$\sigma_\rho \cdot \tau^2 << 2\pi, \ \sigma_\xi \cdot \tau^2 << 2\pi. \qquad (19)$$

Further, the values σ_ρ, σ_ξ will be referred to herein as the wave parameters of the observation point and the source location point, respectively. Along with the wave parameters σ_ρ, σ_ξ we will also consider the parameters of approximation by the position of the source and the observation point

$$\delta_\rho = \frac{\sigma_\rho \cdot \tau^2}{2\pi} = \frac{\rho^4}{\lambda R_0^3}, \delta_\xi = \frac{\sigma_\xi \cdot \tau^2}{2\pi} = \frac{\xi^4}{\lambda R_0^3}. \qquad (20)$$

Using Eqs. (19), (20) the sufficient condition of the approximation application can be represented as

$$\delta_\rho << 1, \delta_\xi << 1. \qquad (21)$$

As it is known the Fraunhofer approximation is a special case of the Fresnel approximation when

$$\sigma << 2\pi. \qquad (22)$$

Indeed, it is easy to see that taking into account the necessary condition ($\tau << 1$), this condition automatically implies a sufficient condition for the fulfillment of the Fresnel pattern (see (13)). It can be noticed that condition (22), as well as the Fresnel condition (see (16) - (19)) splits into two conditions

$$\sigma_\rho << 1, \ \sigma_\xi << 1, \qquad (23)$$

and the wave field (12) can be presented by the following expression

$$U(\vec{R},t) = \frac{a}{R_0}cos[\omega t - k(R_0 - \eta)]. \qquad (24)$$

Thus, according to the results presented above, the condition imposed on the wave parameter in the approximate theory of Fresnel diffraction, in fact, contains two conditions. In conjunction with the value of the wavelength, the first one characterizes the distance to the source, and the second one – the distance to the observation point from the axis of the main direction of observation. As it can be seen from conditions (19) and (23), the closer the source and observation point are to the observation axis, the more satisfactory the approximate description of the field becomes.

3. The viewing angle and equiphase surfaces in the Fresnel and Fraunhofer approximations

In addition to the spatial parameters of the problem, the angleβ between the vectors $\vec{R}$ and $\vec{R}_0$ is also indicated on Fig. 2. The angle β will be hereinafter referred to as the viewing angle of the pattern. As we will see below, it is the viewing angle that appears in the description of the interference experiment in the Fresnel pattern and it differs from the angle of determination of the intensity maxima in the diffraction experiment in the Fraunhofer pattern.

As it directly follows from Fig. 2, there is a relation between the viewing angle β, the coordinate of the observation point ρ on the line l, and the main observation distance R_0

$$\beta = arctan(\rho/R_0). \tag{25}$$

In an approximate description, considering the necessary condition $\rho^2 << R_0^2$ (see (14)), this relationship turns into

$$\beta = \rho/R_0, \tag{26}$$

where the angle β is calculated in radians.

Using (26) and (18), as well as the relationship $k = 2\pi/\lambda$, it is easy to see that the first condition (17) can be written as

$$\beta^4 << \frac{\lambda}{R_0}. \tag{27}$$

It follows from this ratio that the Fresnel pattern satisfactorily describes the wave field at small values of the viewing angle. It is also easy to see that the condition for applying the Fraunhofer pattern (22) can be written as

$$\beta^2 << \frac{\lambda}{R_0}. \tag{28}$$

Conditions (27), (28) obviously show that the Fraunhofer pattern is a special case of the Fresnel pattern. Indeed, if $\lambda << R_0$, then $\sqrt[2]{\lambda/R_0} << \sqrt[4]{\lambda/R_0}$.

As mentioned above, the possibility of an approximate description of the field depends on the values of the wave parameters of the problem, which in turn, in addition to the wavelength, depend on the main observation distance R_0 and transverse spatial parameters ρ, ξ. It is easy to see that if the source is located directly on the observation axis, then the source wave parameter is equal to zero ($\sigma_\xi = 0$, see (18)).

It is clear that in this case, for fixed values of R_0, λ the applicability of the approximation will depend on the coordinate of the observation point on the line of view. For fixed values of ρ, λ, when the applicability of the approximation is considered depending on the main observation distance, then, as a rule, this approach is called approximation by the observation area. If the observation is carried out at fixed values of R_0, λ, and the change in value of $\sigma_\rho$is associated only with the value of ρ, then one usually it is referred to as approximation by the field or area of view.

Assuming that the source is located at the origin of coordinates ($\eta = 0$,$\xi = 0$), we will consider the phase of a spherical wave as a function of two variables R_0, ρ, proceeding from the exact expression (7), as well as from approximate expressions (11) and (24). Since the wave phase value φ at a point is determined by the distance of the wave run L to the given point, i.e.

$$\varphi = L/\lambda, \tag{29}$$

it is then clear that the study of the character of equiphase points is reduced to the problem of equidistant points. In other words, the surface of equiphase points coincides with the surface of equidistant points.

As it follows from **Error! Reference source not found.** in the case of exact expression (7), the line of equiphase points will be determined by the equation

$$\sqrt{R_0^2 + \rho^2} = L, \tag{30}$$

and in the case of Fresnel and Fraunhofer patterns (see (11), (24), respectively) by the following equations

$$R_0 - \rho^2/(2R_0) = L, \tag{31}$$

$$R_0 = L. \tag{32}$$

Obviously, on the plane R_0, ρ the equation (30) defines a circle, equation (31) a parabola, and equation (32) a straight line (see Fig. 3).

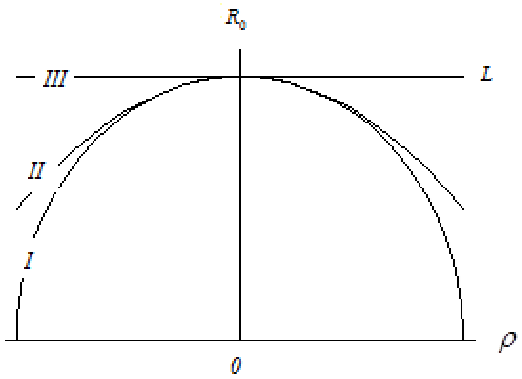

Fig. 3. Equiphase curves of a spherical wave (I) on the planeR_0, ρ, Fresnel (II) and Fraunhofer (III) approximations, respectively.

As it can be seen from the presented figure, at small values of ρ, the equiphase lines of the three cases differ very little from each other. In other words, the closer the observation point is to the central observation point O', the more adequate the approximate description of the field becomes. Within the limits of a small section of the observation plane including the pointO', all three surfaces, namely the sphere, the paraboloid, and the plane, appear almost indistinguishable. Their differences manifest as they move away from the pointO'. It is also clear that for large deviations of the observation point from the point $O^{'}$, the parabolic surface in its topological form is closer to a sphere than to a flat surface. Thus, in the Fresnel pattern, a spherical wave is approximated by a paraboloid wave, which in turn is approximated by a plane wave in the Fraunhofer pattern.

4. Approximate description of the superposition wave field of many sources

In the light of the problem of an approximate description of the wave field, the question of the field raised above in the context of one source may not seem so insignificant. Yet, for the superposition field of many sources, this issue is crucial. Let the positions of sources in space be determined by vectors $\vec{r}_p (p = 1,2,\cdots,N)$. Then the superposition field at the observation point $\vec{R}$ is determined by the following sum

$$U(\vec{R},t) = \sum_{p=1}^{N} \frac{a}{|\vec{R}-\vec{r}_p|} cos\left[\omega t - k\left|\vec{R} - \vec{r}_p\right|\right], \tag{33}$$

where we assumed that the initial phases of wave generation and their amplitudes are equal to each other.

Suppose here, as in the case of a single source (see above), a description of the wave field is required on a straight line l that is perpendicular to the main observation vector $\vec{R}_0$ and that it is at a distance R_0 from the origin of coordinates. For an approximate consideration of (33), we introduce the transverse and longitudinal components of the vectors $\vec{r}_p$ with respect to the vector $\vec{R}_0$ (see (4), (5))

$$\vec{r}_p = \xi_p \vec{e}_\perp + \eta_p \vec{e}_\parallel. \tag{34}$$

Taking into account the above, when the vectors are$\vec{r}_p$ in the same plane with the vector $\vec{R}_0$and the line of view l , the wave field expression (33) can be represented as follows (see (7))

$$U(\vec{R}, t) = \sum_{p=1}^{N} \frac{a \cdot cos\left[\omega t - k\sqrt{R_p^2 + (\rho - \xi_p)^2}\right]}{\sqrt{R_p^2 + (\rho - \xi_p)^2}}, \tag{35}$$

where R_p is the distance from the p source to the line of view l

$$R_p = R_0 - \eta_p. \tag{36}$$

Based on the above consideration, let us write down the necessary and sufficient conditions for an approximate description of the exact expression (35)

$$\left(\frac{\rho}{R_0}\right)^2 << 1, \left(\frac{\eta_p}{R_0}\right)^2 << 1, \left(\frac{\xi_p}{R_0}\right)^2 << 1 \tag{37}$$

and

$$\delta_\rho << 1, \delta_p^\xi << 1, \tag{38}$$

where

$$\delta_\rho = \frac{\rho^4}{\lambda R_0^3}, \qquad \delta_p^\xi = \frac{\xi_p^4}{\lambda R_0^3}. \tag{39}$$

If conditions (37), (38) are satisfied for all sources and observation points, then superposition field (33) can be written in the Fresnel pattern as follows (see (11))

$$U(\vec{R}, t) = \frac{a}{R_0} \sum_{p=1}^{N} cos\left[\omega t - k\left(R_p + \frac{(\rho - \xi_p)^2}{2R_0}\right)\right]. \tag{40}$$

If the sufficient condition (38) is replaced with a stronger condition

$$\frac{\rho^2}{\lambda R_0} = \frac{\sigma_\rho}{2\pi} << 1, \frac{{\xi_p}^2}{\lambda R_0} = \frac{\sigma_p^\xi}{2\pi} << 1, \tag{41}$$

then the form of the superposition field in the Fraunhofer pattern is derived from expression (40) as follows

$$U(\vec{R}, t) = \frac{a}{R_0} \sum_{p=1}^{N} cos\left[\omega t - kR_p\right]. \tag{42}$$

As can be seen from the result obtained, only the longitudinal spatial parameters of the problem appear in the Fraunhofer pattern [16].

Let us now discuss the question of determining the superposition field intensity maxima points. It is obvious that oscillation at a point will occur with an amplitude of

$$A = N\frac{a}{R_0}, \tag{43}$$

if the phase difference of all waves arriving at a given point is a multiple 2π

$$\varphi_p - \varphi_j = 2\pi n_{pj}, \tag{44}$$

where $n_{pj} = 0, \pm 1, \pm 2, \cdots$.

Using the exact wave field expression (35), the intensity maximum condition (44) can be written as follows

$$k\left(\sqrt{R_p^2 + \left(\rho - \xi_p\right)^2} - \sqrt{R_j^2 + \left(\rho - \xi_j\right)^2}\right) = 2\pi n_{pj}. \tag{45}$$

Considering (40), in the Fresnel pattern, this condition turns into

$$k(R_p - R_j) + k\frac{\xi_p^2 - \xi_j^2 - 2\rho(\xi_p - \xi_j)}{2R_0} = 2\pi n_{pj}. \tag{46}$$

It is important to note that under the condition **Error! Reference source not found.** one can take

$$k\frac{\xi_p^2 - \xi_j^2 - 2\rho(\xi_p - \xi_j)}{2R_0} << 1.$$

It is easy to see that in this case (46) turns into

$$k(R_p - R_j) = 2\pi n_{pj}, \tag{47}$$

which is in accordance with (42) is the intensity maximum condition in the Fraunhofer pattern.

Here attention should be paid to two circumstances, the first of which is related to the approximate condition (46). It is important to note that the maximum intensity condition in the Fresnel pattern includes the situation when

$$k(R_p - R_j) = 2\pi m_{pj},\ \ k\frac{\xi_p^2 - \xi_j^2 - 2\rho(\xi_p - \xi_j)}{2R_0} = 2\pi h_{pj}, \tag{48}$$

where $m_{pj}, h_{pj} = 0, \pm 1, \pm 2, \cdots$. Obviously, if $m_{pj} + h_{pj} = n_{pj}$, then (48) automatically implies the fulfillment of (46). At the same time, it is clear that the first condition (48) is nothing but the condition for the maximum intensity in the Fraunhofer pattern. It follows from the above that, from a practical point of view, it is valuable to consider the diffraction experiment in the Fresnel pattern, when the conditions for the maximum in the Fraunhofer pattern are met (see **Error! Reference source not found.**). We will analyze this issue in more detail in the context of the interference problem of waves from two sources (see below).

The second circumstance is connected with formula (47) and, in particular, its underlying Fraunhofer method for determining the maxima of the superposition field based on the construction of parallel rays. The essence of this method is well known and it entails the following. A system of parallel straight lines is drawn from the points of location of the sources parallel to some chosen direction. If the differences in the paths of the waves in the given direction are multiples of the wavelength, then the waves are amplified in that direction. Otherwise, the waves partially or completely dampen each other in that direction. In this regard, it is very often possible to meet the statement that parallel beams amplify each other. In the context of the above, the following question seems reasonable: if we talk about amplification in a direction, should we assume that all rays emanating from different sources are simultaneously amplified? And if so, is the amplification uniform, i.e., are all beams amplified by the same amount? If the amplification is non-uniform, then which of the rays is amplified the most, and which the least? Clearly, such an interpretation of the results of the diffraction experiment raises many questions. As it will be shown below, the enhancement in the Fraunhofer diffraction pattern corresponds to the effect of enhancement in the direction of observation, while in the Fraunhofer interference pattern a point enhancement occurs [16].

5. About the observation angle and the viewing angle in the interference pattern

The discussion presented above is rather general, the only restriction for which is that the sources, the direction of observation and the direct line of view are located in the same plane. Let us apply the results presented above to the problem of the interference of the fields of two sources of spherical waves, the classical treatment of which is given in the Fresnel pattern (see (40), where $N = 2$).

For definiteness, suppose the origin of coordinates is located directly in the middle of the segment connecting the sources through which the axis$X$ passes (see Fig. 4). Then, the vectors $\vec{r}_1$ and$\vec{r}_2$ can be presented as

$$\vec{r}_1 = -d \cdot \vec{e}_x/2, \text{ and } \vec{r}_2 = d \cdot \vec{e}_x/2, \tag{49}$$

where $\vec{e}_x$ is the unit dimensionless vector indicating the positive direction of the axis$X$ and $d > 0$ is the distance between the sources.
The angle α is also shown on the figure. It is the angle between the axis X and the main observation vector $\vec{R}_0$. The angle α will be referred to as the angle of observation. It is easy to show that

$$\vec{e}_x = \vec{e}_\parallel \, cos\, \alpha + \vec{e}_\perp \, sin\, \alpha. \tag{50}$$

Note that the value $\alpha = \pi/2$ corresponds to the case of frontal observation ($\vec{e}_\parallel = \vec{e}_y$, where $\vec{e}_y$ is the unit dimensionless vector indicating the positive direction of the axis). Fig. 4 also shows the viewing angle β between the vectors $\vec{R}_0$ and $\vec{R}$, which, in sum with the observation angle determines the actual viewing angle $\alpha + \beta$.

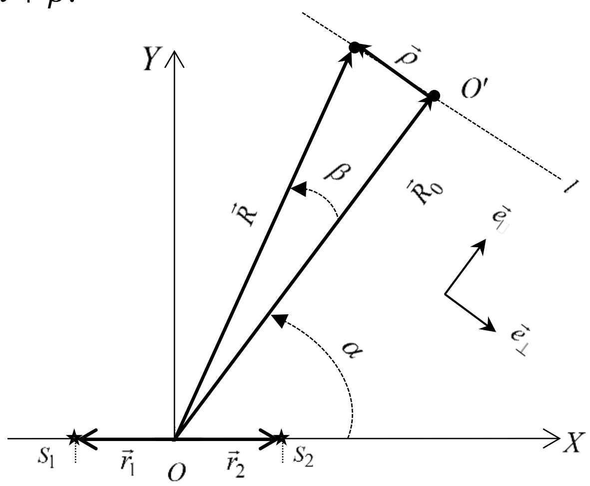

Fig. 4. Diagram of the interference experiment.

Here it seems necessary to comment on the expediency of introducing the concept of viewing angle. The thing is that when changing the observation angle α, we change the observation plane (in this case, the spatial position of the line l). When the observation angle is changed, the spatial position of the observation line does not change. Obviously, at a fixed value of α, a change of the viewing angle β leads to a change in the position of the observation point on the line l.

Using (49), (50) the following are derived

$$\vec{r}_1 = -\frac{d}{2}\vec{e}_{\parallel} \cos\alpha - \frac{d}{2}\vec{e}_{\perp} \sin\alpha,\ \vec{r}_2 = \frac{d}{2}\vec{e}_{\parallel} \cos\alpha + \frac{d}{2}\vec{e}_{\perp} \sin\alpha. \tag{51}$$

Based on these formulas for the longitudinal and transverse spatial parameters of sources, the following can be written (see (5) and (34))

$$\eta_1 = -\frac{d}{2}\cos\alpha, \eta_2 = \frac{d}{2}\cos\alpha, \xi_1 = -\frac{d}{2}\sin\alpha, \xi_2 = \frac{d}{2}\sin\alpha. \tag{52}$$

Considering the field amplification condition for two sources in the Fresnel pattern (see (46)) the following is derived,

$$k(R_1 - R_2) + k\frac{\xi_1^2 - \xi_2^2 - 2\rho(\xi_1 - \xi_2)}{2R_0} = 2\pi n_{12}, \tag{53}$$

where in accordance with **Error! Reference source not found.**

$$R_1 = R_0 - \eta_1, R_2 = R_0 - \eta_2, \tag{54}$$

and it can be seen that in case of (52), the formula **Error! Reference source not found.** turns into

$$k \cdot d\cos\alpha + k \cdot \frac{\rho \cdot d \sin\alpha}{R_0} = 2\pi n_{12}. \tag{55}$$

It is easy to check that in the case of frontal observation ($\alpha = \pi/2$) the equation (55) turns into the well-known form of the condition for the maximum of the interference pattern

$$\frac{\rho_n \cdot d}{R_0} = \lambda \cdot n, \tag{56}$$

where $k = 2\pi/\lambda$.

Taking into account the relationship (26), and using (56) it is easy to determine the values of the viewing angles at which the intensity maximum is observed

$$\beta_n = \frac{\lambda \cdot n}{d}. \tag{57}$$

It is interesting the maximum condition **Error! Reference source not found.**obtained in Fresnel approximation in the Fraunhofer approximation. This suggests that

$$k \cdot \frac{\rho \cdot d \sin\alpha}{R_0} << 2\pi.$$

Taking into account this condition, it is easy to see that **Error! Reference source not found.**) takes the form of $d\cos\alpha = 2\pi n_{12}$. It is easy to see that this equality is nothing more than the maximum condition for a in the far region of observation for a diffraction grating (see below). It is easy to see that in the case of the angle α_m ($d\cos\alpha_m = 2\pi m$) the condition **Error! Reference source not found.** takes the form

$$\frac{\rho_n \cdot \tilde{d}}{R_0} = \lambda \cdot n,$$

where $\tilde{d} = d\sin\alpha_m$. This equality exactly coincides with the intensity maximum condition for frontal observation **Error! Reference source not found.**, when the distance between the sources is considered as equal to $d\,sin\,\alpha_m$. It follows from the above that when the observation screen is perpendicular to the direction of one of the maxima in the far region, then the distribution of maxima in the near observation region has a periodic character.

Here it is necessary to pay special attention to the following circumstances. The fact is that when considering interference from two or many sources (diffraction grating) in the near region of observation, we are dealing with the effect of an intensity amplification at a point. At the same time, in optical experiments observed in a far region, we fix the amplification in the direction (see [16]). That is why lenses are used for observations in the far region. Lenses change the nature of the amplification, namely, the amplification in the direction is changed into amplification at the point.

Indeed, for the diffraction experiment, the description in the Fraunhofer pattern is applicable. Application of the Fraunhofer pattern condition to two sources means that the intensity maximum condition will be determined according to the formula $k(R_1 - R_2) = 2\pi n_{12}$ (see. **Error! Reference source not found.**), which in turn is transformed into

$$d\,cos\,\alpha = \lambda n_{12}. \tag{58}$$

This formula is nothing else than an equation that determines the angles of amplification in a diffraction experiment (see [1, 2]). As it can be seen from formula (57), the angle that determines the amplification effect in the interference experiment is the viewing angle β, while in formula (58) the angle that determines the amplification effect in the diffraction experiment is the observation angle α (see below).

When deriving the well-known formulas related to the description of the interference pattern, the Fresnel pattern used in the theory of diffraction was used (see (37), (38)). Further, to consider the connection between the Fresnel conditions directly and the parameters of the interference problem, we note that the source approximation parameters can be replaced by a single parameter (see (52))

$$\delta_{1,\zeta} \sim \delta_{2,\zeta} \sim \delta_d = \frac{d^4}{16\lambda R_0^3}. \tag{59}$$

Obviously, in this case the sufficient condition for the Fresnel pattern will be presented as

$$\delta_\rho << 1,\ \delta_d << 1. \tag{60}$$

Despite the fact that the Fresnel pattern is an approximation, in addition to the Fraunhofer pattern, it can also include the following situations

$$\delta_d << \delta_\rho << 1, \tag{61}$$
$$\delta_\rho << \delta_d << 1, \tag{62}$$
$$\delta_\rho \sim \delta_d << 1. \tag{63}$$

Remarkably, optical interference experiments correspond to situations (61), because the distances between the sources are on the order of a millimeter, and the region of observation of the maxima is on the order of a centimeter.

6. Description of the diffraction experiment in the Fresnel approximation

Let us now consider the situation when N sources are located on the axis X, and $N > 2$ (see Fig. 5). We again consider the problem of an approximate description of the wave field on the line l, perpendicular to the main direction of observation $\vec{R}_0$. The figure also shows the viewing angle β and the angle of observation α.

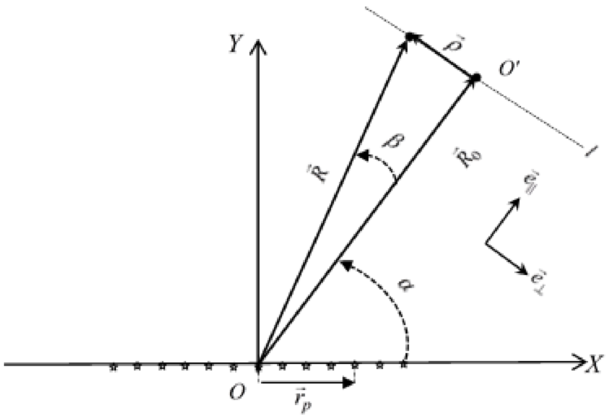

Fig. 5. Diagram of the diffraction experiment.

Suppose the sources on the axis $X$are located at certain distances from each other, i.e., periodically. Then

$$\vec{r}_p = x_p \vec{e}_x, x_p = x_1 + (p-1)d, \tag{64}$$

where $p = 1,2,\cdots,N$, x_1- the coordinate of the first source, d is the period of the diffraction grating. Further, we will consider the coordinate of the first source equal to

$$x_1 = -\frac{(N-1)d}{2}. \tag{65}$$

Note that by choosing the coordinate of the first source in the form (65), we place the origin of coordinates at the center of the diffraction grating. Using (64), (65) (see also (52)), the longitudinal and transverse coordinates of the sources can be represented as

$$\eta_p = -\frac{N-2p+1}{2} d\cos\alpha, \xi_p = -\frac{N-2p+1}{2} d\sin\alpha. \tag{66}$$

Further, on the viewing axis l, we will consider the intensity of the wave field

$$I(N) = \frac{1}{T}\int_0^T U^2(\vec{R},t)dt, \tag{67}$$

where$T = 2\pi/\omega$ is the oscillation period. Using the field expression in the Fresnel pattern (40) for (67), the following can be obtained

$$I(N,\rho) = \frac{1}{2}\left(\frac{a}{R_0}\right)^2 \sum_{j=1}^{N}\sum_{i=1}^{N} cos^2\left[\frac{L_{ji}\cdot k+\mu_{ji}}{2}\right], \tag{68}$$

where

$$L_{ji} = (j-i)kd\cos\alpha,$$
$$\mu_{ji} = \frac{k}{2R_0}\left(\xi_j^2 - \xi_i^2 - 2\rho(\xi_j - \xi_i)\right). \tag{69}$$

It is then easy to conclude that the intensity of the field in the Fraunhofer pattern will be determined by the expression

$$I(N,\rho) = \frac{1}{2}\left(\frac{a}{R_0}\right)^2 \sum_{j=1}^{N}\sum_{i=1}^{N} cos^2\left[\frac{L_{ji}\cdot k}{2}\right], \tag{70}$$

i.e., when in (68) it is assumed that all $\mu_{ji} << 2\pi$ (see also (42)). As it follows from (69), (70), if $L_{ji} = 2\pi n_{ji}$, i.e. when

$$(j-i)d\cos\alpha = \lambda n_{ji}, \tag{71}$$

then the intensity of the field in the Fraunhofer pattern has a maximum value

$$I\frac{1}{2}^2\left(\frac{a}{R_0}\right)^2_{max}. \quad (72)$$

Note that in the Fraunhofer pattern, the field intensity does not depend on the coordinate of the observation point ρon the viewing axis. It should be noted that not only in the directions of the maximums, but also in other directions in the far region of observation, the field intensity does not depend on the coordinate of the observation point on the screen. Indeed, as it follows from (42) the wave field in the far region does not depend on ρ.

It is easy to see that the intensity maximum condition obtained in the Fraunhofer pattern for the interference experiment (58) coincides with the maximum condition for the diffraction experiment (71). Indeed, when $d\cos\alpha = \lambda n_{ji}$, then condition (71) is satisfied for all j, i. Further, we will consider the field pattern in the Fresnel pattern, when the field intensity in the Fraunhofer pattern satisfies the maximum condition. According to (68)-(71), in this case, the intensity distribution on the straight line l will be expressed by the condition

$$I(N,\rho) = \frac{1}{2}\left(\frac{a}{R_0}\right)^2 \sum_{j=1}^{N}\sum_{i=1}^{N} cos^2\left[\frac{k}{4R_0}\left((\xi_j^n)^2 - (\xi_i^n)^2 - 2\rho(\xi_j^n - \xi_i^n)\right)\right], \quad (73)$$

where

$$\xi_p^n = -\frac{N-2p+1}{2} d\sin\alpha_n, \quad (74)$$

where the angle α_n is determined based on the intensity maximum condition in the Fraunhofer pattern: $d\cos\alpha_n = \lambda n$.

It should be noted that even for the case of periodically located emitters (74), series (73) cannot be analytically calculated. It is easy to guess that it is a discrete analogue of the well-known Fresnel integral in the theory of diffraction. Below we will explore frontal observation, i.e. when $\alpha_n = \pi/2$ and, therefore, the line l is parallel to the source location axis X. Using (74) one can see that in this case the expression (73) turns into

$$I(N,\rho) = \frac{1}{2}\left(\frac{a}{R_0}\right)^2 \sum_{j=1}^{N}\sum_{i=1}^{N} cos^2\left[\frac{x_j^2 - x_i^2 - (x_j - x_i)\cdot\rho}{4R_0} k\right], \quad (75)$$

where

$$x_p = -\frac{N-2p+1}{2} d. \quad (76)$$

Note that the expression for the field intensity remains valid for any arrangement of emitters, and not only for periodic ones (see (76)). As it follows from (75), at the central observation point ($\rho = 0$)

$$I(N,0) = \frac{1}{2}\left(\frac{a}{R_0}\right)^2 \sum_{j=1}^{N}\sum_{i=1}^{N} cos\left[\frac{x_j^2 - x_i^2}{2R_0} k\right]. \quad (77)$$

It is easy to see that the effect of full amplification of the waves (see (72)), i.e. when

$$I(N,0) = I_{max}, \quad (78)$$

is possible only if the following condition is held

$$\frac{x_j^2 - x_i^2}{2R_0} k = 2\pi n_{ji}. \quad (79)$$

Inserting (76) into (79) one can get

$$\frac{x_j^2-x_i^2}{2R_0}k = (j-i)(N+1-j-i)\frac{d^2k}{2R_0}. \quad (80)$$

As it can be seen from this expression, if

$$j+i = N+1, \quad (81)$$

then the amplification condition (79) between the waves emitted by the j -th and i -th sources is satisfied automatically. Indeed, equation (81) corresponds to sources ($x_j = -x_i$) symmetrically located and therefore

$$\frac{x_j^2-x_i^2}{2R_0}k = 0. \quad (82)$$

It is noteworthy that in the interference experiment, where the number of sources is equal to two, the location of the sources fully corresponds to this case ($x_1 = -x_2$).

7. On the problem of determining the distribution of maximums in the near observation region

To meet the full amplification condition (78), it is necessary that (80) be satisfied between the waves of all emitters, and not only those symmetrically located relative to the origin (81). Suppose

$$\frac{d^2k}{2R_0} = 2\pi h, \quad (83)$$

where $h = 1,2,\cdots$. Then, as it can be seen from **Error! Reference source not found.**, **Error! Reference source not found.**

$$n_{ji} = h(j-i)(N+1-j-i) \quad (84)$$

and, consequently, the waves of all emitters amplify each other. Thus, if condition (83) is satisfied, the diffraction grating will behave as an intensifying lens.

Taking into account that $k = 2\pi/\lambda$ the expression **Error! Reference source not found.** can be presented in the form of

$$d = \sqrt{2\lambda R_0 h}. \quad (85)$$

As follows from the above, when condition (85) is satisfied, the field of a system of periodically located sources has the following point of maximum amplification (see (64), (76))

$$x_p = -\frac{N-2p+1}{2}\sqrt{2\lambda R_0 h}. \quad (86)$$

Restricting it to the case of $h = 1$, let us present (85) in the following form

$$R_0 = \frac{d^2}{2\lambda}. \quad (87)$$

Suppose the radiation wavelength is $\lambda = 0.4\mu\text{m}$ and the period of the diffraction grating is $d = 10^{-4}$ m. Then, as it follows from (87), the point of maximum field amplification will be at a distance of $R_0 = 1.25$ cm from the diffraction grating. In the end, we note that the obtained result **Error! Reference source not found.** directly reflects the well-known Talbot Effect [17, 18].

8. Conclusion

Thus, it is shown that with a more comprehensive presentation of the theory of approximate description of the superposition field, the description of interference and diffraction experiments can be given within the framework of a single scheme.

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