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THE PROCESS OF FORMATION AND OF A QUASI-BOUND STATE

In this work the evolution of wave packets, which scatter on a field of a one-dimensional potential is considered. The scattering potential is taken as a system of two similar rectangular barriers and the wave packets are constructed on the base of the scattering wave functions being the eigenfunctions of stationary Schrodinger equation. The process of formation and decay of a quasi-bound state in the region between the barriers is considered. We investigate the time characteristics of this process such as a bound state formation time and its life or delay time. It is shown that when the carrier energy of wave packets coincides with the energy value of the resonance transmission, then in the region between the barriers a quasi-bound state is formed.
In particular, the dependence of the time characteristics from the width of rectangular barriers of the scattering potential is considered. When the width the barriers tends to infinite, i.e. when the scattering potential transforms to a simple quantum well, the quasi-bound formation time takes a finite value while the decay time tends to infinite. This result means that the tunneling of wave packets from the region out of the scattering potential into its middle part even for the case of infinite wide barriers occurs within a finite time. The latter is the demonstration of the well-known Hartman effect. The life time infinity for the infinite wide barriers shows that the wave perturbation arrived due to the tunneling in the region between the barriers remains locked in the scattering potential volume. So, we conclude that a bound state is a result of

evolution of wave packets constructed on the base of the scattering functions.

Keywords: *scattering problem, bound state formation, appearance and delay times, evolution, wave packet, quasi-bound state, barriers, perturbation.*

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ՁԵՎԱՎՈՐՄԱՆ ՊՐՈՑԵՍՆԵՐ ԵՎ ԿՎԱԶԻԿԱՊՎԱԾ ՎԻՃԱԿ

Սույն աշխատանքում դիտարկվում է ալիքային փաթեթի էվոլյուցիան, որը ցրվում է միաչափ պոտենցիալի դաշտի վրա: Ցրող պոտենցիալը ընդունվում է որպես երկու ուղղանկյուն համանման ալիքների համակարգ, իսկ ալիքային փաթեթը կառուցվում է ցրվող ալիքային ֆունկցիաների հիման վրա, որոնք հանդիսանում են Շրեդինգերի ստացիոնար հավասարման սեփական ֆունկցիաներ: Դիտարկվել է ձևավորման և քվազի կապված վիճակի տրոհման պրոցեսը միջարգելքային տիրույթում:

Մենք ուսումնարկել ենք այդ պրոցեսի ժամանակային բնութագրիչները, ինչպիսին են կապված վիճակի ձևավորման ժամանակը և նրա կյանքի տևողությանը կամ ուշացման ժամանակը: Ցույց է տրվել, որ երբ ալիքային փաթեթի տանող էներգիան համընկնում է ռեզոնանսային անցման էներգիայի արժեքի հետ, ապա միջարգելքային տիրույթում առաջանում է քվազիկապված վիճակ: Մասնավորապես դիտարկվել է ժամանակային բնութագրերի կախվածությունը ուղղանկյուն արգելքների լայնությունից: Երբ արգելքի լայնությունը ձգտում է անվերջության, այսինքն՝ երբ ցրող պոտենցիալը ձևավորվում է հասարակ քվանտային փոսում, քվազիկապված վիճակի ստեղծման ժամանակը ընդունում է վերջավոր արժեքներ, իսկ մարման ժամանակը ձգտում է անվերջության: Այդ արդյունքը նշանակում է, որ ալիքային փաթեթի թունելավորումը ցրող պոտենցիալի տիրույթից դեպի նրա միջին մասը, նույնիսկ անվերջ լայնությամբ արգելքների դեպքում տեղի է ունենում վերջավոր ժամանակի ընթացքում: Վերջինս հայտնի Հարթմանի էֆեկտի ցուցադրումն է:

Բանալի բառեր՝ *ցրման խնդիրը, կապված վիճակի գոյացում, տրոհման ժամանակ, էվոլյուցիա, ալիքային փաթեթ, քվազի-կապված վիճակ, արգելք, գրգռում:*

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ПРОЦЕСС ФОРМИРОВАНИЯ И КВАЗИСВЯЗАННОЕ СОСТОЯНИЕ

В данной работе рассматривается эволюция волновых пакетов, рассеивающих на поле одномерного потенциала. Потенциал рассеяния принимается в виде системы из двух одинаковых *прямоугольных барьеров, а волновые пакеты строятся на основе волновых функций рассеяния, являющихся собственными функциями стационарного уравнения Шредингера. Рассмотрен процесс формирования и распада квази-связанного состояния в области между барьерами. Мы исследуем временные характеристики этого процесса, такие как время формирования связанного состояния и его время жизни или время задержки. Показано, что когда энергия несущей волновых пакетов совпадает со значением энергии резонансного прохождения, то в области между барьерами образуется квазисвязанное состояние.*

В частности, рассматривается зависимость временных характеристик от ширины прямоугольных барьеров рассеивающего потенциала. Когда ширина барьеров стремится к бесконечности, то есть когда потенциал рассеяния преобразуется в простую квантовую яму, время квази-связанного образования принимает конечное значение, а время затухания стремится к бесконечности. Этот результат означает, что туннелирование волновых пакетов из области потенциала рассеяния в ее среднюю часть даже для случая бесконечно широких барьеров происходит за конечное время. Последнее является демонстрацией известного эффекта Хартмана. Бесконечность времени жизни для бесконечных широких барьеров показывает, что волновое возмущение, возникающее из-за туннелирования в области между барьерами, остается заблокированным в объеме потенциала рассеяния. Итак, мы заключаем, что связанное состояние является результатом эволюции волновых пакетов, построенных на основе функций рассеяния.

Ключевые слова: *задача рассеяния, образование связанного состояния, время распада, эволюция, волновой пакет, квазисвязанное состояние, барьер, возмущение.*

Introduction

The problem of description a wave motion with non-harmonic time dependence has a long time history and places an essentially important role for different application problems. Usually, for the case of linear media, the wave motions of complicated dynamics are considered by means their decomposition on separate wave motions, each of them has more simple form of the time dependence. Namely, the simple wave motions, which are also called eigen wave modes, have the harmonic form of the time dependence. So, determination of the frequency (energy) spectrum and the space form of harmonic wave motions, which can realize in a given media, is important as well [1-3].

In the general case of complicated dynamic waves the description suggests that the staring or initial form of the wave perturbation is known and it is given from out. In contrast to that the harmonic waves always repeat the same space form and the mean problem is to find this form. The latter is found by solving the corresponding stationary wave equation. It can be called as an initial form of the wave perturbation for harmonic wave motion as well, since being initially given this form is always repeated with certain frequency in time.

So, for the complicated wave motion the problem of determination of the initial condition form is formally absent. However, to get successfully physics picture of the wave process the correct initial condition should be taken. In other words the correct initial condition is the correct problem statement. Otherwise by taking any initial form for a wave perturbation and solving a non-stationary wave equation one can consider in principle possible wave process. The following question is seen to be opened: is this process is the object of interesting and if it is not so, when how the initial condition should be chosen to provide the consideration of the studied physical situation [4]. The problem becomes more difficult, when for the initial condition expert the starting form of a perturbation the space form of its derivation should be given as well. The mentioned situation can take place, for example, for acoustic waves.

Another important issue for description of complicated wave processes is the introduction of the corresponding time characteristics. The problem is how by mean of a finite number of parameters to describe the dynamic of the system such as a wave field, which has an infinite number of freedom degrees. This problem is an object of intensive discussion for many years and means questions connect with the wave process tunneling through non-unformal regions of a media. So, in accordance with the well-known Hartman effect the tunneling speed can takes with an infinitely large speed, more them velocity of light [5-13].

This work is devoted to the tunneling problem of a wave perturbation through a one-dimensional potential from two rectangular barriers. The given problem was considered by many authors [14-24], which basically investigated the reflection and transmission times in dependence of the barrier width and the separation distance between them. In the opaque limit for the barriers the existence of the Hartman effect, like to the case of a single barrier, was found. It is shown that the behavior of the transmission coefficient and of the tunneling phase-time near a resonance is given by expressions with “Breit-Wigner type” denominators [14].

Below we consider a time evolution of a wave process initially having the form of two wave packets falling from the left and right sides on a one-dimension scattering potential. Under certain conditions, when the carrier energy of the wave packets equals to the energy of a potential resonance transmission, the wave process brings to formation of a quasi-bound state into the value of the scattering potential. In particular, we consider the genesis and collapse process of a bound state into the value of a simple rectangular well which locates inside of the scattering potential, namely in the region between two identical rectangular potentials. Such scattering system, when the width of the rectangular potentials takes an infinite large value, transforms to a simple potential well. We investigate the time characteristics of the formation and delay process of a quasi-bound state as functions of the width of the scattering system barriers. In particular, in the limit of infinite wide barriers, when the scattering potential having from two identical rectangular barriers takes a quantum well form, the process time evolution is considered.

In outlines the time evolution of the above described process is seems to be clear. Some part of the falling wave perturbation reflects from the scattering system. The remaining part enters into the region of the barriers where a quasi-bound state can appear. As the time characteristics of the discussed wave process we consider the entry time of the wave packets into the scattering potential volume and the life time or decay time of the appeared quasi-bound state. The entry time is defined as the difference between the achievement times of the maximum value of the wave perturbation in the well center, which are calculated for two cases of scattering potential present and it absent. The delay process of a quasi-bound is directly started after it appearance process, i.e. the final time moment of the appearance time is the start time moment from which the life time is calculated. The final of the delay process is considered the time moment when the diverging wave perturbation immediately at the borders of the scattering potential takes the maximum value.

2. The bases of the scattering wave functions.

It is well known that a quantum particle motion in a potential field $U(x)$ is described by mean of the time-dependent Schrodinger equation, which for the case of a one-dimensional motion has the form of:

$$i\hbar\frac{\partial}{\partial t}\Phi\left(x,t\right)=\left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+U(x)\right]\Phi\left(x,t\right), \qquad (1)$$

where $\Phi\left(x,t\right)$ is the wave function. This equation has to be collaterally considered with an initial condition defining the space dependence of the wave function for an initial time moment:

$$\Phi\left(x,0\right)=\Phi_0\left(x\right), \qquad (2)$$

where $\Phi_0\left(x\right)$ is given. The standard requirement imposed on a wave function is the normalization condition;

$$\int_{-\infty}^{+\infty}\Phi\left(x,t\right)\Phi^*\left(x,t\right)dx=1. \qquad (3)$$

As a function of time the dependence of the function $\Phi\left(x,t\right)$ is usually divided on two types. First of them is the harmonic form of the time dependence and the second one is all others. For time harmonic solitons the wave function is considered as

$$\Phi\left(x,t\right)=\exp\left\{-iE\left(k\right)t/\hbar\right\}\Psi(x,k),$$

where $\Psi(x,k)$ satisfies to the stationary Schrodinger equation:

$$\frac{d^2\Psi(x)}{dx^2}+\left(k^2-u(x)\right)\Psi(x,k)=0 \qquad (4)$$

and $k=\sqrt{2mE(k)}/\hbar$, $u(x)=2mU(x)/\hbar^2$. The function $u(x)$, which is usually called as a potential, can take both positive and negative values. Further the potentials vanishing at infinites will be considered only;

$$u(x\to\pm\infty)=0.$$

The above given equation can have two type solutions. First of the corresponds to an infinite motion, when the wave function is normalized to delta-function and second one describes the finite motion or bound states, when the wave function is normalized to a finite quantity. Note that for the last case the energy can take discrete values only and for an existence of bound states in same places the potential have to have negative values.

The stationary wave functions of infinite and finite motions have different natures, which are seen from the difference of their normalization conditions;

$$\int_{-\infty}^{\infty} \psi(x,k)\psi(x,-k')\,dx = \delta(k-k'), \quad \int_{-\infty}^{\infty} \phi(x,\chi_n)\phi(x,\chi_m)\,dx = \delta_{nm}.$$

Here we denoted the wave functions of an infinite motion as ψ and for a bound state as ϕ. Note that for the bound states in Eq. (4) the quantity k^2 takes a negative value, so that $k^2 = -\chi^2$ and $k = i\chi$, where χ is a real quantity.

As seen from the above written conditions the wave function of infinite motion has no a dimension, i.e. it is dimensionless quantity (note that the delta function has a dimension inversed of its argument dimension, for example, $\left[k\right] = 1/meter$ and $\left[\delta(k)\right] = meter$). In contrast to that the wave function of a bound state has a dimension equaling to $1/\sqrt{meter}$.

For the one-dimensional scattering theory the more interesting solutions of Eq. (4) are the so-called scattering wave functions corresponding to the left and right scattering problems, which have the following forms of an asymptotic behavior:

$$\psi_l(x,k) = \frac{1}{\sqrt{2\pi}} \begin{cases} \exp\{ikx\} + R(k)\exp\{-ikx\}, & x \to -\infty, \\ T(k)\exp\{ikx\}, & x \to +\infty, \end{cases} \tag{5}$$

$$\psi_r(x,k) = \frac{1}{\sqrt{2\pi}} \begin{cases} S(k)\exp\{-ikx\}, & x \to -\infty, \\ \exp\{-ikx\} + P(k)\exp\{ikx\}, & x \to +\infty, \end{cases} \tag{6}$$

where $k > 0$ and $T(k)$, $R(k)$ and $S(k)$, $P(k)$ are the transmission and reflection amplitudes of the left and right scattering problems, correspondingly . Note that in Eq. (5) and Eq. (6) the factor $1/\sqrt{2\pi}$ provides the normalization condition on delta function of the scattering wave functions (see, for example, [25]);

$$\int_{-\infty}^{\infty} \psi_l(x,k)\psi_l(x,-k')\,dx = \delta(k-k'), \quad \int_{-\infty}^{\infty} \psi_r(x,k)\psi_r(x,-k')\,dx = \delta(k-k'). \tag{7}$$

It is important to mention as well that the left and right scattering functions are orthogonal to each other;

$$\int_{-\infty}^{+\infty} \psi_l(x,k)\psi_r(x,-k')\,dx = 0. \tag{8}$$

The functions $\psi_l(x,k)$ and $\psi_r(x,k)$ are independent solutions of Eq. (4), so its arbitrary solution can be presented by them in a linear combination form. For the transmission and reflection amplitudes of the left and right scattering problems the following relations are take place (see, for example, [26-29]);

$$1 - R(k)R(-k) = T(k)T(-k), \tag{9}$$

$$1 - P(k)P(-k) = S(k)S(-k), \tag{10}$$

$$P(k)T(-k) + R(-k)S(k) = 0, \tag{11}$$

$$S(k) = T(k)\,. \tag{12}$$

Note that for a case, when in Eq. (4) the potential $u(x)$ is a real function the action sign change of the parameter k is equivalent to the complex conjugation action (for example, $\psi_l(x,-k) = \psi_l^*(x,k)$, $R(-k) = R^*(k)$ and so on).

Below we will consider the real potentials only. As it follows from relations (11), (12) in this case the reflection amplitudes of the left and right scattering problems are differ from each other by a phase factor;

$$R(k) = -P^*(k)T(k)/T^*(k). \tag{13}$$

Presenting $T(k) = |T(k)|\exp\{\mathrm{i}\,\varphi_T(k)\}$, where $|T(k)|$ and $\varphi_T(k)$ are the module and phase of the transmission amplitudes, it is easy to see that the last equation takes the form of:

$$R(k) = -P^*(k)\exp\{\mathrm{i}\,2\varphi_T(k)\}. \tag{14}$$

From this equation it follows that

$$|R(k)| = |P(k)| \text{ and } \varphi_R(k) + \varphi_p(k) = \pi + 2\varphi_T(k)\,, \tag{15}$$

where $|R(k)|, |P(k)|$ and $\varphi_R(k)$, $\varphi_p(k)$ are the modules and phases of the reflection amplitudes of the left and right scattering amplitudes;

$$R(k) = |R(k)|\exp\{\mathrm{i}\,\varphi_R(k)\},\ P(k) = |P(k)|\exp\{\mathrm{i}\,\varphi_P(k)\}\,.$$

Below we consider the evolution problem of wave packets constructed on the bases of scattering wave functions (5), (6);

$$\Phi(x,t) = \int_0^\infty \left[v_l(k)\psi_l(x,k) + v_r(k)\psi_r(x,k)\right]\exp\{-iE(k)t/\hbar\}dk\,, \tag{16}$$

where $E(k) = \hbar^2 k^2/2m$ and $v_l(k)$, $v_r(k)$ are the coefficients of the expansion spectrum of the wave process $\Phi(x,t)$ conducted on the basis of the scattering functions $\psi_l(x,k)$, $\psi_r(x,k)$. Note that in the case of choice of any another basis of orthogonal functions, for example, Fourier waves;

$$\frac{1}{\sqrt{2\pi}}\exp\{i(kx - E(k)t/\hbar)\},\quad \frac{1}{\sqrt{2\pi}}\exp\{-i(kx + E(k)t/\hbar)\},$$

the expansion coefficients will depend on t, unless of course the case when $u(x) = 0$ everywhere.

$$v_l(k) = \int_{-\infty}^{+\infty} \Phi_0(x)\psi_l^*(x,k)dx\,,\ v_r(k) = \int_{-\infty}^{+\infty} \Phi_0(x)\psi_r^*(x,k)dx\,. \tag{17}$$

It is easy to check that for the function $\Phi(x,t)$ satisfying to the condition (3) the spectral coefficients would be chosen so that

$$\int_0^\infty [v_l(k)v_l^*(k) + v_r(k)v_r^*(k)]dk = 1\ . \tag{18}$$

In accordance with (16)-(18) the evolution of the wave perturbation $\Phi(x,t)$ is defined by the form of the spectral functions $v_l(k)$, $v_r(k)$.

3. Multiple scattering and resonance tunneling

Any one dimensional potential relatively to any point can be mentally divided on two parts, which locate to the left and to the right from this point. The scattering amplitudes of a one-dimensional potential can be presented by means of the scattering amplitudes corresponding to these parts (see, for example, [29]). So, the system transmission amplitude has the form of:

$$T = \frac{t_I t_{II}}{1 - p_I r_{II}} , \tag{19}$$

where the indexes I and II correspond to the left and right parts (barriers) of the scattering potential. So, p_I is the reflection amplitude of the first barrier determined from the right scattering problem and r_{II} is the reflection amplitude of the second barrier determined for the left scattering problem. The formula (19) has a quite transparent physical significance relating to the effect of multiple reflections of a wave inside a layered structure. Expanding this expression as a series in powers of $p_I r_{II}$ one can get:

$$T = t_I t_{II} + t_I \left(r_{II} p_I \right) t_{II} + t_I \left(r_{II} p_I \right)^2 t_{II} + \cdots = \sum_{j=0}^{\infty} t_I \left(r_{II} p_I \right)^j t_{II} . \tag{20}$$

It is easily seen that the first term in the sum (the $n = 0$ term) is the contribution to the total transmission amplitude from the amplitude of the process in which no reflection from the barriers takes place. The second term corresponds to the amplitude of the process in which a wave, on passing through the first barrier of the scattering potential, is reflected from its second barrier and, then, after reflection from the first barrier, the wave passes through the second barrier. Clearly, the n-th term of the sum represents the transmission of a wave perturbation through the structure accompanied by n-fold re-reflection of the wave between the barriers.

Now we consider a scattering potential presenting a system from two identical barriers spaced from each other by some distance a (see Fig. 1).

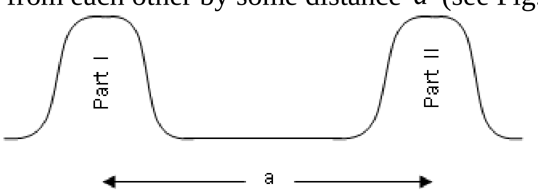

Fig. 1 A scattering potential form of two identical barriers.

Applying the general relation (13) to the first barrier ($p_I = -r_I^* t_I / t_I^*$) and taking into account that the barriers of the scattering potential are identical ($t_I = t_{II} = t$ and $r_I = r_{II} \exp\{-i2ka\} = r$), for (19) one can write down:

$$T(k) = \frac{|t|^2 \, \mathrm{e}^{2i\varphi_t}}{1 + |r|^2 \, \mathrm{e}^{2i\phi}} . \tag{21}$$

where φ_t is the phase of the transmission amplitude of a single barrier, which can take the value in the interval $-\pi/2 < \varphi_t < \pi/2$;

$$t = |t|\exp[i\varphi_t] \text{ and } \phi = ka + \varphi_t \; . \tag{22}$$

As it follows from Eq. (21), when

$$\phi = \pi/2 + n\pi \; , \; n = 0,1,\cdots \tag{23}$$

then the resonance tunneling of the system from two identical barriers takes place. Indeed, in this case the exponent in the dominator of Eq. (21) equals to -1. Taking into account that for any value of k the equality $1-|r|^2 = |t|^2$ takes place, one can cheek that Eq. (23) defines the resonance values of k_n, i.e. when $|T(k_n)| = 1$. By using Eq. (22) the condition (23) defining the resonance transmission for the scattering potential form two identical barriers can be written:

$$k_n a + \varphi_t(k_n) = \pi/2 + n\pi \; . \tag{24}$$

Like to Eq. (20), the transmission amplitude of a double-barrier system (see Eq. (21)) can be written:

$$T(k) = \sum_{j=0}^{\infty} T_j(k), \;\; T_j(k) = |t|^2 \, \mathrm{e}^{2i\varphi_t(k)} \left(|r|^2 \, \mathrm{e}^{2i\phi(k)+\pi}\right)^j . \tag{25}$$

Note, that in the series (25) the index mentions the corresponding contribution to the transmission amplitude due to the process of multiple (n-times) re-reflections arising between the barriers. Rewriting the transmission and reflections coefficients of a single barrier as exponents;

$$|t|^2 = e^{\delta(k)}, \; |r|^2 = e^{\sigma(k)}, \tag{26}$$

the partial transmission amplitudes $T_j(k)$ can be presented as well (see below):

$$T_j(k) = \mathrm{e}^{2i\varphi_t(k)+\delta(k)} \left(\mathrm{e}^{2i\phi(k)+\sigma(k)+\pi}\right)^j . \tag{27}$$

Now we construct a wave packet from the scatting wave functions (5) and consider its behavior in the aria right to the scattering potential;

$$\Phi(x,t) = \int_{k_0-\Delta k}^{k_0+\Delta k} \nu(k) T(k) \exp i \left\{ kx - \frac{\hbar k^2}{2m} t \right\} dk \;\; , \tag{28}$$

where k_0 is the value of the carrier wave number and Δk defines the width of the spectral interval. Taking into account Eq. (25) it is easy to see that Eq.(28) can be written:

$$\Phi(x,t) = \sum_{j=0}^{\infty} \Phi_j(k) \, , \; \Phi_j(x,t) = \int_{k_0-\Delta k}^{k_0+\Delta k} \nu(k) T_j(k) \exp i \left\{ kx - \frac{\hbar k^2}{2m} t \right\} dk \, . \tag{29}$$

So, in accordance with Eq. (29) due to the multiple reflections the wave packet is divided on many wave packets.

Considering $\Delta k << k_0$ and expanding the functions $\varphi_t(k)$, $\phi(k)$, $\delta(k)$ $\sigma(k)$ in the Taylor series near k_0 for

$$\mathrm{e}^{2i\varphi_t(k)+\delta(k)} \left(\mathrm{e}^{2i\phi(k)+\sigma(k)+\pi}\right)^j , \; |t|^2 = e^{\delta(k)}, \; |r|^2 = e^{\sigma(k)} \; . \tag{30}$$

Now, we consider the behavior of $T(k)$ near some point and we made a notation; $r_I = r_{II}\exp\{-i2k(L+d)\} = r$

$$\varphi = kL + kd + \varphi_t\,. \tag{31}$$

It is easy to see that when

$$\varphi = \pi/2\,, \tag{32}$$

The scattering potential is considered as a system of two identical rectangular potentials having magnitude U_0 and width d, which divide from each other by the free motion region of length L. It is easy to see that that this scattering potential transforms to a simple rectangular well when $d \to \infty$ (see Fig. 2).

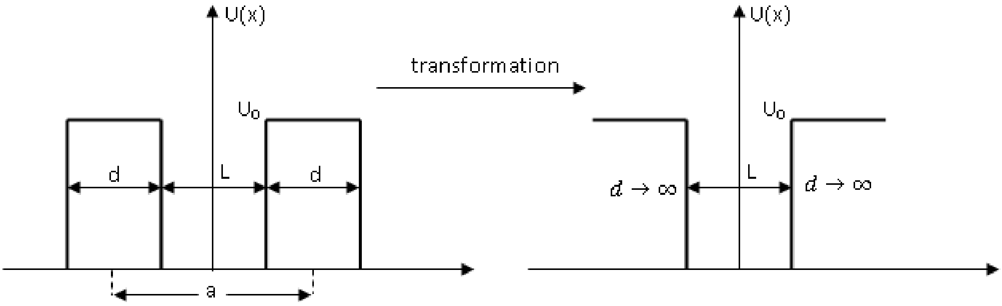

Fig. 2. The system of two barriers when $d \to \infty$ transforms to a simple well.

Let us consider the dynamics of the wave packet from the scattering wave functions having the asymptotic behavior form of Eq. (20) and the following form of a spectral composition. In the region of the sun-barrier scattering ($E(k) < U_0$) the modules and phases of the transmission and reflection amplitudes of a rectangular barrier have the forms of;

$$\frac{1}{|t|^2} = \cosh^2\{\chi d\} + \left(\frac{k^2-\chi^2}{2\chi k}\right)^2 \sinh^2\{\chi d\}\ , \tag{33}$$

$$\frac{|r|^2}{|t|^2} = \left(\frac{k^2+\chi^2}{2\chi k}\right)^2 \sinh^2\{\chi d\}, \tag{34}$$

$$\varphi_t(k) = -kd + arctg\left[\frac{k^2-\chi^2}{2\chi k}\,\mathrm{tgh}\{\chi d\}\right], \tag{35}$$

where $\chi = \sqrt{2m(U_0 - E(k))}/\hbar$.

By using Eq. (35) for the resonance condition (32) one can write (see, for example, [30, 31]):

$$ctg\{k_n L\} = \frac{k_n^2 - \chi_n^2}{2\chi_n k_n}\,\mathrm{tgh}\{\chi_n d\}\,. \tag{36}$$

Here the index n mentions that this equality can take place for certain values of k_n only and

$$\chi_n = \sqrt{2mU_0/\hbar^2 - k_n^2} \ .$$

As it follows from the above mentioned the resonance values of the wave number depend on the distance between the barriers, the width and the magnitude of potential of the barriers;

$$k_n = k_n(L, d, U_0), \qquad (37)$$

where, as it was mentioned, k_n are the magnitudes of the quasi wave number corresponding to the resonance tunneling.

It should be mentioned as well that when $\chi d \to \infty$ the condition (36) transforms to the equation determining the energy spectrum of the simple rectangular well [2];

$$ctg\{k_n L\} = \frac{k_n^2 - \chi_n^2}{2\chi_n k_n}. \qquad (38)$$

Since the transmission amplitudes of left and right scattering problems equals to each other (see Eq. (12)) and reflection amplitudes differ by the phase factor (see Eq. (14)), then for the same value of k_n the resonance tunneling can take place in two directions. If for a given value of k_n a particle falling from the left on a potential resonantly transmits, when for this value k_n it will resonantly transmit potential falling from the right as well;

$$|T(k_n)| = |S(k_n)| = 1 \text{ and } |R(k_n)| = |P(k_n)| = 0. \qquad (39)$$

Here we consider the wave packets with spectral composition of k near to the magnitude of k_n, i.e. taking magnitudes into the interval

$$k_n - \Delta k \le k \le k_n + \Delta k, \qquad (40)$$

where $\Delta k << k_n$. Expanding the modules and phases of scattering amplitudes $T(k)$, $R(k)$ and $P(k)$ on series of k near the resonance values one can write down:

$$T(k) = \exp\{i(\varphi_T(k_n) + \varphi_T'(k_n)(k - k_n))\}, \ R(k) = P(k) = 0, \qquad (41)$$

$$k^2 = (k_n + (k - k_n))^2 \approx k_n^2 + 2k_n(k - k_n). \qquad (42)$$

Note that near resonance modules of the transmission and reflection amplitudes take their maximum and minimum values (see Eq. (39)), so that near the resonance

$$\partial|T(k_n)|\partial k = \partial|R(k_n)|\partial k = \partial|P(k_n)|\partial k = 0.$$

In accordance with equations (41), (42) in the region of k near to k_n (40) the scattering wave functions can be presented:

$$\psi_l(x,k) = \frac{1}{\sqrt{2\pi}} \begin{cases} \exp\{ikx\}, & x \to -\infty, \\ \exp i\{kx + \varphi_T(k_n) + \varphi_T'(k_n)(k - k_n)\}, & x \to +\infty, \end{cases} \qquad (43)$$

$$\psi_r(x,k) = \frac{1}{\sqrt{2\pi}} \begin{cases} \exp i\{\varphi_T(k_n) + \varphi_T'(k_n)(k - k_n) - kx\}, & x \to -\infty, \\ \exp\{-ikx\} & x \to +\infty. \end{cases}$$

(44)

Below we consider the wave packets having the following spectral composition:

$$v_l(k)=\frac{e^{ik(L+d)}}{4\sqrt{\Delta k}}\begin{cases}0, & k<k_n-\Delta k,\\ 1, & k_n-\Delta k<k<k_n+\Delta k,\\ 0, & k>k_n+\Delta k,\end{cases} \quad (45)$$

$$v_r(k)=\frac{e^{-ik(L+d)}}{4\sqrt{\Delta k}}\begin{cases}0, & k<k_n-\Delta k,\\ 1, & k_n-\Delta k<k<k_n+\Delta k,\\ 0, & k>k_n+\Delta k.\end{cases} \quad (46)$$

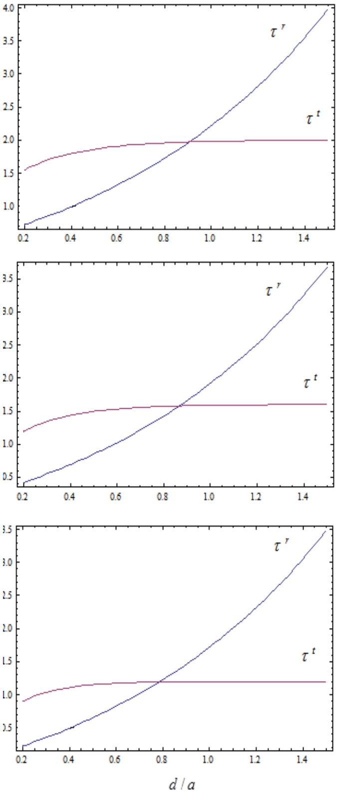

Fig. 2. The time characteristics of the considered wave process.

It is easy to check that the chosen forms of the spectral coefficients $v_l(k)$, $v_r(k)$ satisfy to the condition (18) and proved the certain form of initial perturbation $\Phi_0(x)$ (see Eq. (2)). Namely, at the initial time moment $t=0$ there are two wave packets with maximums the borders points of the scattering potential $x=-L-d$ and $x=L+d$, i.e. in the left point of the first barrier and in the right point of the second one.

In accordance with the above given statement of the wave evolution problem we did some calculations relating to the time characteristics of a quasi-bound state appearance τ^r and it`s decay τ^p. In the Figure we present the time as a dimensionless quantity in the units of u_g/a. The parameters of the quantum well was chosen as $2mU_0a^2/\hbar^2=7$, which has three bound states. It is easy to see that when the width of the barriers tends to infinite the appearance time tends to finite value, while the decay time limits to infinite.

Conclusion. As it follows from the obtained result any bound state formed into the potential volume is a standing wave packet which arises due to the certain wave scattering process.

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