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CORRIGENDUM TO 'ON WEIGHTS WHICH ADMIT REPRODUCING KERNEL OF SZEGÖ TYPE'

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In *Journal of Contemporary Mathematical Analysis (Armenian Academy of Sciences)* volume 55 (5), pages 320 – 327, 2020, the paper 'On weights which admit reproducing kernel of Szegö type' was published. The author found a mistake which he wants to fix.

Theorem 5.2. is miscited. Instead of

$$\int_{\partial\Omega_2} f \mathrm{d}S = \int_{\partial\Omega_1} (f \circ \Phi) \det |J_{\mathbb{C}}\Phi|^{\frac{2N}{N+1}} \mathrm{d}S$$

we should have

$$\int_{\partial\Omega_2} f \mathrm{d}\sigma_{F_2} = \int_{\partial\Omega_1} (f \circ \Phi) \det |J_{\mathbb{C}}\Phi|^{\frac{2N}{N+1}} \mathrm{d}\sigma_{F_1},$$

where we integrate using Fefferman measure instead of Lebesgue measure.

Theorem 5.2. Let Ω_1, Ω_2 be domains of one of types 1-3 introduced above and $\Phi : \Omega_1 \to \Omega_2$ be a biholomorphic mapping. Then for any integrable function $f : \partial\Omega_2 \to \mathbb{C}$ we have

$$\int_{\partial\Omega_2} f \mathrm{d}\sigma_{F_2} = \int_{\partial\Omega_1} (f \circ \Phi) |\det J_{\mathbb{C}}\Phi|^{\frac{2N}{N+1}} \mathrm{d}\sigma_{F_1},$$

where $J_{\mathbb{C}}\Phi$ is the complex Jacobian matrix of Φ.

Theorem 5.3. remains true, since integrating in Lebesgue measure and integrating in Fefferman measure define the same topologies, i.e. for any domain Ω_j which satisfies assumptions of the theorem, there exist positive constants d_j, D_j, such that for any positive almost everywhere f we have

$$d_j \int_{\partial\Omega_j} f \mathrm{d}\sigma_F \leq \int_{\partial\Omega_j} f \mathrm{d}S \leq D_j \int_{\partial\Omega_j} f \mathrm{d}\sigma_F. \tag{0.1}$$

The proof however needs some changes.

Theorem 5.3.: Let Ω_1, Ω_2 be of type 1, 2 or 3. Let $\Phi : \Omega_1 \to \Omega_2$ be a biholomorphism. Then

(i) for any g measurable and non-negative almost everywhere we have:

$$\int_{\partial\Omega_2} g\mu \mathrm{d}S < \infty \Leftrightarrow \int_{\partial\Omega_1} (g \circ \Phi)(\mu \circ \Phi) \mathrm{d}S < \infty$$

In particular, $g \in L^2H(\partial\Omega_2, \mu)$ if and only if $g \circ \Phi \in L^2H(\partial\Omega_1, \mu \circ \Phi)$.

(ii) μ is S-admissible on $\partial\Omega_2$ if and only if $\mu \circ \Phi$ is S-admissible on $\partial\Omega_1$.

Proof: (i) By the fact that $u := |\det J_{\mathbb{C}}\Phi|^{\frac{2N}{N+1}}$ is smooth function on compact set $\overline{\Omega_1}$, we have

$$C_1 \int_{\partial\Omega_1} (g\circ\Phi)(\mu\circ\Phi)\mathrm{d}S \leq \int_{\partial\Omega_1} (g\circ\Phi)(\mu\circ\Phi)|\det J_{\mathbb{C}}\Phi|^{\frac{2N}{N+1}}\mathrm{d}S \leq C_2 \int_{\partial\Omega_1} (g\circ\Phi)(\mu\circ\Phi)\mathrm{d}S,$$

where $C_1 := \min_{w\in\overline{\Omega}} u(w) > 0$ and $C_2 := \max_{w\in\overline{\Omega}} u(w)$. By theorem 5.2. and inequality (0.1) we have

$$\int_{\partial\Omega_1} (g \circ \Phi)(\mu \circ \Phi)\mathrm{d}S \leq D_1 \int_{\partial\Omega_1} (g \circ \Phi)(\mu \circ \Phi)\mathrm{d}\sigma_{F_1} \leq \frac{D_1}{C_1} \int_{\partial\Omega_1} (g \circ \Phi)(\mu \circ \Phi)u\mathrm{d}\sigma_{F_1}$$
$$= \frac{D_1}{C_1} \int_{\partial\Omega_2} g\mu\mathrm{d}\sigma_{F_2} \leq \frac{D_1}{C_1 d_2} \int_{\partial\Omega_2} g\mu\mathrm{d}S.$$

Similarly

$$\int_{\partial\Omega_2} g\mu\mathrm{d}S \leq D_2 \int_{\partial\Omega_2} g\mu\mathrm{d}\sigma_{F_2} = D_2 \int_{\partial\Omega_1} (g \circ \Phi)(\mu \circ \Phi)u\mathrm{d}\sigma_{F_1}$$
$$\leq D_2C_2 \int_{\partial\Omega_1} (g \circ \Phi)(\mu \circ \Phi)\mathrm{d}\sigma_{F_2} \leq \frac{D_2C_2}{d_1} \int_{\partial\Omega_1} (g \circ \Phi)(\mu \circ \Phi)\mathrm{d}S.$$

So we showed that there exist positive constants c, C, such that

$$c \int_{\partial\Omega_1} (g \circ \Phi)(\mu \circ \Phi)\mathrm{d}S \leq \int_{\partial\Omega_2} g\mu\mathrm{d}S \leq C \int_{\partial\Omega_1} (g \circ \Phi)(\mu \circ \Phi)\mathrm{d}S \tag{0.2}$$

If the integral on the right-hand side is finite, then the integral in the middle is also finite. If the integral in the middle is finite, then the integral on the left-hand side is also finite.

To complete the proof of (i) we just need to recall that composition of two holomorphic functions is also a holomorphic function.

(ii) Since Φ is biholomorphism, we need only to show implication in one direction.

If μ is S-admissible on $\partial\Omega_2$, then for any compact set $X \subset \Omega_2$, $w \in X$ and any $f \in \tilde{B}(\partial\Omega_2, \mu)$ we have

$$|f(w)| \leq C_X \sqrt{\int_{\partial\Omega_2} |f|^2\mu\mathrm{d}S}. \tag{0.3}$$

By using (0.2) for inequality (0.3) we gain

$$|(f \circ \Phi)(\tilde{w})| \leq C_X\sqrt{C}\sqrt{\int_{\partial\Omega_1} |f \circ \Phi|^2(\mu \circ \Phi)\mathrm{d}S},$$

for $\Omega_1 \supset Y := \Phi^{-1}(X)$, $\tilde{w} := \Phi^{-1}(w) \in Y$, so (CB) is satisfied for $C_Y := C_X\sqrt{C_2}$. ■

Also in an example of non-admissible weight all instances of $\overline{\Omega} \setminus A_n$ should be replaced with $\overline{\Omega \setminus A_n}$.

Other parts of the text should remain unchanged.