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The Focusing of a Wave Superposition Field Generated by a System of Coherent Point Sources

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Abstract. The paper discusses the problem of a superposition wave field created by a system of coherent point sources. For the general case, the conditions for a point amplification of the superposition field are found. The focusing problem is discussed for a system of point sources located on circles with centers lying on the same straight line parallel to the planes of the circles.

Keywords: diffraction, focusing, Fresnel pattern

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It is well known that the problem of describing the wave field generated by a system of coherent radiating point sources is one of the main problems of wave theory [1, 2]. Let us consider a system of point sources generating spherical waves. We will discuss the problem in a general form, so it is assumed that the location of sources in space can be arbitrary;

$$U(\vec{R},t) = \sum_{p=1}^{N} \frac{A_p}{|\vec{R}-\vec{r}_p|} cos\left[\omega t - k\left|\vec{R}-\vec{r}_p\right| + \gamma_p\right], \tag{1}$$

where N is a number of the point sources, A_p is the sphere wave amplitude generated by p -th source. The space vector $\vec{R}$ shows the observation point and the vectors $\vec{r}_p$ $(p = 1,2,\cdots,N)$ show the source locations. The quantities ω and k are the frequency and the wave number of the generated wave field and we assume that the dispersion low $\omega(k)$ can be in any form, for example, has a linear form $\omega(k) = ck$, where c is the propagation velocity of waves. The quantity γ_p $(p = 1,2,\cdots,N)$ is the initial phase magnitude of a wave generating by the p-th source.

In the case of when the locations of the source have three dimensional characters, it is more useful to change the vectors $\vec{r}_p$ of one index to vectors of three indexes;

$$\vec{r}_p \to \vec{r}_{mph}. \tag{2}$$

The initial phases and the wave amplitudes are considered as a quantity of three indexes as well:

$$\gamma_p \to \gamma_{mph} \text{ and } A_p \to A_{mph}. \tag{3}$$

If the system of sources is a periodic lattice, then for the vectors $\vec{r}_{mph}$we can write:

$$\vec{r}_{mph} = m \cdot \vec{a} + p \cdot \vec{b} + h \cdot \vec{c}, \tag{4}$$

where $\vec{a}, \vec{b}, \vec{d}$ are the bases vectors of a periodic lattice and

$$m = 1,2,\cdots,N_a,\ p = 1,2,\cdots,N_b,\ h = 1,2,\cdots,N_c. \tag{5}$$

Note that in accordance with (5) the number of the sources equals to $N_a \cdot N_b \cdot N_c$. Using (4), (5) the superposition field (1) can be written:

$$U(\vec{R},t) = \sum_{m=1}^{N_a} \sum_{p=1}^{N_b} \sum_{p=1}^{N_c} \frac{A_{mph}}{|\vec{R}-\vec{r}_{mph}|} cos\left[\omega t - k\left|\vec{R}-\vec{r}_{mph}\right| + \gamma_{mph}\right]. \tag{6}$$

Recently, the above mention problem was consider in works [3, 4], where, in particular, the description of the superposition field (1) in the framework of the Fresnel approximation was done.

Let us denote by $\vec{R}_0$ the vector which shows the main observation direction and the basic observation distance (see Fig. 1):

$$\vec{e}_0 = \vec{R}_0/R_0 \text{ and } R_0 = \left|\vec{R}_0\right|. \tag{7}$$

As it was shown in Fig. 1 the points O and O' are the initial and end points of the vector $\vec{R}_0$. We will call the point O' as the main observation point. Note that for the vectors $\vec{R}$ and $\vec{r}_p$ the point O is the initial point as well. In the figure we introduced the vector

$$\Delta\vec{R} = \vec{R} - \vec{R}_0, \tag{8}$$

which specifies the shift of the observation point relative to the main observation point.

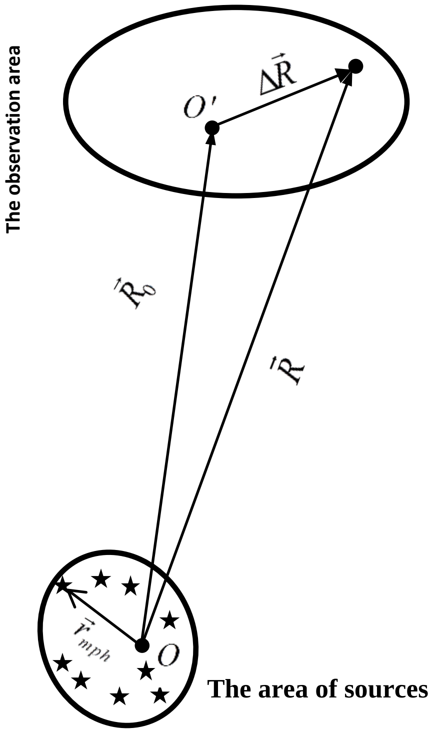

Fig. 1. The space parameters of a superposition field.

Let us suppose that the sources arrange into the area near the point O (the area of sources) and the wave field is described in the region close the main observation point O' (the observation area**).** We consider the problem by introducing longitudinal and transverse spatial parameters relative to the vector $\vec{R}_0$;

$$\Delta\vec{R} = \vec{\rho} + \vec{\mu}, \vec{r}_{mph} = \vec{\xi}_{mph} + \vec{\eta}_{mph}, \tag{9}$$

where

$$\vec{\rho} \perp \vec{R}_0 , \vec{\xi}_{mph} \perp \vec{R}_0 \text{ and } \vec{\mu} \parallel \vec{R}_0 , \vec{\eta}_{mph} \parallel \vec{R}_0. \tag{10}$$

As it was shown in the works [3, 4] when

$$\rho << R_0, \xi_{mph} << R_0, \mu << R_0, << R_0, \tag{11}$$

where $\rho = |\vec{\rho}|$, $\xi_{mph} = \left|\vec{\xi}_{mph}\right|$,$\mu = |\vec{\mu}|$, $\eta_{mph} = \left|\vec{\eta}_{mph}\right|$ and

$$\frac{\rho^2}{2R_0}k \sim 2\pi, \frac{\xi_{mph}^2}{2R_0}k \sim 2\pi, \tag{12}$$

the wave field (6) can be presented in the form of:

$$U(\vec{R},t) = \sum_{m=1}^{N_a} \sum_{p=1}^{N_b} \sum_{p=1}^{N_c} \frac{A_{mph}}{R_0} cos\left[\omega t + \vec{r}_{mph} \cdot \vec{k} + \frac{1}{2} \frac{\vec{\rho} \cdot \vec{\xi}_{mph} - \xi_{mph}^2}{R_0} k + \gamma_{mph}\right], \tag{13}$$

where

$$\vec{k} = k \cdot \vec{R}_0 / R_0. \tag{14}$$

The given sum is the expression of the superposition field (6) in the Fresnel approximation. In the case when

$$\frac{\rho^2}{2R_0} k << 2\pi, \frac{\xi_{mph}^2}{2R_0} k << 2\pi \tag{15}$$

the sum (13) takes the form:

$$U(\vec{R},t) = \sum_{m=1}^{N_a} \sum_{p=1}^{N_b} \sum_{p=1}^{N_c} \frac{A_{mph}}{R_0} cos\left[\omega t + \vec{r}_{mp\square} \cdot \vec{k} + \gamma_{mph}\right], \tag{16}$$

which corresponds to the superposition field (6) in the Fraunhofer approximation.

Below we consider the conditions, when the oscillations of the wave field (13) occur with the maximum value. As one can see if

$$\vec{r}_{mph} \cdot \vec{k} + \frac{1}{2} \frac{\vec{\rho} \cdot \vec{\xi}_{mph} - \xi_{mph}^2}{R_0} k + \gamma_{mph} = 2\pi n \; (n = 0, \pm 1, \pm 2, \cdots), \tag{17}$$

when the wave field (13) takes the form:

$$U(\vec{R},t) = \left(\sum_{m=1}^{N_a} \sum_{p=1}^{N_b} \sum_{p=1}^{N_c} A_{mph}\right) \frac{cos[\omega t]}{R_0}. \tag{18}$$

It is obvious that this is the wave field form when the oscillations have a maximum value. Note, that when $A_{mp\square} = A$ the amplitude takes the value:

$$\sum_{m=1}^{N_a} \sum_{p=1}^{N_b} \sum_{p=1}^{N_c} A_{mph} = A \cdot N_a \cdot N_b \cdot N_c. \tag{19}$$

In the case of the Fraunhofer approximation (15) the conditions of maximum (17) takes the form:

$$\vec{r}_{mph} \cdot \vec{k} + \gamma_{mph} = 2\pi n \; (n = 0, \pm 1, \pm 2, \cdots). \tag{20}$$

Further consideration of the problem will be made under certain assumptions regarding the locations of the sources. Let the sources of the system be located on the set of a finite number of parallel planes. If the number of the sources locating on a first plane equals to N_1, the sources locating on a second plane equals to N_2 and so on, when for the number of sources of whole system one get:

$$N = N_1 + N_2 + \cdots N_M, \tag{21}$$

where M is a number of parallel plane.

Let a space vectors $\vec{a}_m$ having the same initial point be perpendicular to the mentioned planes, where m an index of plane numeration, so that $|\vec{a}_m - \vec{a}_{m-1}|$ is a distance the m-th and $m + 1$-th between neighboring planes. In that case the vectors $\vec{r}_{mph}$ (4) showing the source locations can be changed by two index vectors:

$$\vec{r}_{mph} \to \vec{r}_{mj}, \tag{22}$$

where the vector $\vec{r}_{mj}$ shows the location of the j-th source of the m-th plane;

$$\vec{r}_{mj} = \vec{a}_m + \vec{l}_{mj}. \tag{23}$$

Note that for considered system of sources $\vec{a}_m \perp \vec{l}_{mj}$, so that the vector $\vec{l}_{mj}$ shows the location of the j-th source of the m–th plane in the given plane.

In accordance with the above mention the superposition wave field (16) in the case of (22), (23) can be written:

$$U(\vec{R},t) = \sum_{m=1}^{M} \sum_{j=1}^{N_m} \frac{A_{mj}}{R_0} cos\left[\omega t + (\vec{a}_m + \vec{l}_{mj}) \cdot \vec{k} + \frac{1}{2}\frac{\vec{\rho}\cdot\vec{\xi}_{mj} - \xi_{mj}^2}{R_0} k + \gamma_{mj}\right]. \qquad (24)$$

Let us suppose the observation direction be perpendicular to the planes of sources. It will mean that

$$\vec{k} \parallel \vec{a}_m, \vec{k} \perp \vec{l}_{mj} \text{ and } \vec{l}_{mj} = \vec{\xi}_{mj}, \qquad (25)$$

so that (24) takes the form:

$$U(\vec{R},t) = \sum_{m=1}^{M} \sum_{j=1}^{N_m} \frac{A_{mj}}{R_0} cos\left[\omega t + a_m \cdot k + \frac{1}{2}\frac{\vec{\rho}\cdot\vec{l}_{mj} - l_{mj}^2}{R_0} k + \gamma_{mj}\right], \qquad (26)$$

where $a_m = |\vec{a}_m|$ ($k \uparrow\uparrow \vec{a}_m$). Note that the number of plane sources N_m may depend on the plane number m.

Next, we will discuss the considered above system, when within the same plane the sources are located on a circle;

$$l_m = |\vec{l}_{mj}|, \qquad (27)$$

where l_m is a circle radius (see Fig. 2). Let us assume that the centers of all planes lie on a one straight line. It is clear in the case of (25) this line will coincide with the main observation direction. If initial phase of generation γ_{mj} and amplitudes A_{mj} of all sources of a one plane are the same ($\gamma_{mj} = \gamma_m$,$A_{mj} = A_m$) the superposition field (26) can be presented as:

$$U(\vec{R},t) = \sum_{m=1}^{M} \frac{A_m}{R_0} N_m \, cos\left[\omega t + a_m \cdot k + \frac{2\cdot\vec{\rho}\cdot\vec{l}_m - l_m^2}{2R_0} k + \gamma_m\right]. \qquad (28)$$

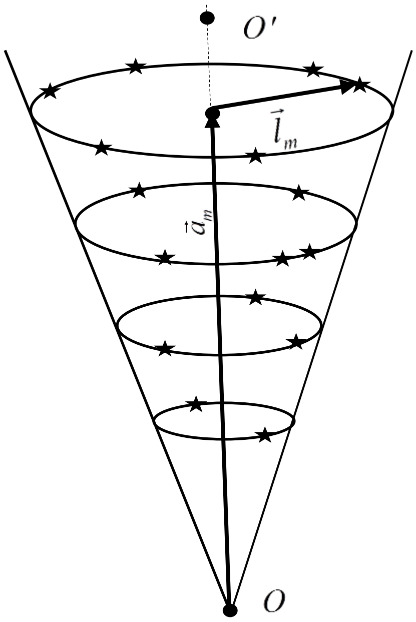

Fig. 2. A system of sources having axial symmetry.

As it follows from this formula when

$$a_m \cdot k + \frac{2\cdot\vec{\rho}\cdot\vec{l}_m - l_m^2}{2R_0} k + \gamma_m = 2\pi n \; (n = 0, \pm 1, \pm 2, \cdots), \qquad (29)$$

the wave oscillations occur with the maximum amplitudes;

$$U_{\max}(\vec{R},t)=\left(\sum_{m=1}^{M}\frac{A_m}{R_0}N_m\right)\cos\left[\omega t\right]. \tag{30}$$

One can see that when the following conditions take place:

$$a_m \cdot k = 2\pi n_1, \frac{2\cdot\vec{\rho}\cdot\vec{l}_m - l_m^2}{2R_0}k = 2\pi n_2, \gamma_m = 2\pi n_1, \tag{31}$$

where $n_1, n_2, n_3 = 0, \pm 1, \pm 2, \cdots$, the condition (29) is satisfied automatically. Note, that in the three following cases the condition (29) is satisfied automatically as well;

$$a_m \cdot k + \frac{2\cdot\vec{\rho}\cdot\vec{l}_m - l_m^2}{2R_0}k = 2\pi n_{12}, \gamma_m = 2\pi n_1, \tag{32}$$

$$a_m \cdot k + \gamma_m = 2\pi n_{13} \frac{2\cdot\vec{\rho}\cdot\vec{l}_m - l_m^2}{2R_0}k = 2\pi n_2, \tag{33}$$

$$a_m \cdot k = 2\pi n_1, \frac{2\cdot\vec{\rho}\cdot\vec{l}_m - l_m^2}{2R_0}k + \gamma_m = 2\pi n_{23}, \tag{34}$$

where $n_{12}, n_{13}, n_{23} = 0, \pm 1, \pm 2, \cdots$.

Let us consider the maximum condition corresponding to the case of (31), when $a_m = 0$, $\gamma_m = 0$ and the observation point arranges on the main observation line (). In this case, the first and third equations of (31) are executed automatically and the problem is reduced to consideration of the second equation only.

$$\frac{l_m^2}{2R_0}k = 2\pi n. \tag{35}$$

One can see that this equality defines the system concentric circles. As follows from result (35), when the sources are located on these circles a point focusing of the radiation takes place.

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