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The Equation of Electron Diffusion in the Momentum Space in Graphene

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Abstract. Small fluctuations of the electron system from the equilibrium state due to electron-acoustic phonon intraband, intravalley random scatterings in graphene have been analyzed. In the linearization approximation of the Boltzmann transport equation a second-order linear partial differential equation for the time and energy dependences of the symmetric component of the fluctuations of the electron distribution function has been obtained. This equation can be considered as the Fokker-Planck equation in the momentum space, which describes the chaotic movement of the electron along the energy axis, i.e. the electron diffusion in the momentum space.

Keywords: graphene, fluctuations, distribution function, electron-phonon scattering

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1. Introduction

Since its discovery [1], graphene has been one of the most studied materials because of its unique properties which are very useful in various fields of micro- and nanoelectronics (see, e.g., [2-4]). From this point of view, the study of current noises in graphene is one of the important areas of modern research in solid state physics. Here, as in bulk semiconductors and metals, as well as in various nanostructures, the main unsolved problems are associated with low-frequency current noise (1/f-noise) (see, e.g., [5-8]).

The development of the theory of electron mobility fluctuations could serve as good basis for explanation of the basic mechanisms of 1/f-noise in semiconductors. This is confirmed by some experimental results (see, e.g., [9, 10]). The main sources of carrier mobility fluctuations are: random intraband scattering, generation-recombination transitions, and the shot effect [11, 12], out of which electron-phonon intraband scattering has significant importance.

It was established [13] that in general cases electron mobility fluctuation $\tilde{\mu}$ is the result of the fluctuations of the electron quasi-momentum relaxation time τ_k ($\tilde{\tau}_k$-source) and the occupancy of energy levels of the conduction band n_k ($\tilde{n}_k$-source). As shown in [13] for the theory of 1/f-noise the $\tilde{n}_k$ component of the mobility fluctuations is of basic interest [12, 13]. The occupancy fluctuations $\tilde{n}_k$, in turn, are expressed by $\tilde{f}_{\mathbf{k}}^{0}(t)$ fluctuations of the equilibrium distribution function of conduction electrons $f_{\mathbf{k}}^{0}$. Here $\mathbf{k}$ is the electron quasi-wave vector, and by the index "0" it is emphasized the important fact that fluctuations are equilibrium fluctuations which arise in equilibrium semiconductors, too.

The function $\tilde{f}_{\mathbf{k}}^{0}(t)$ can be represented as a sum of symmetric $\tilde{f}_{\mathbf{k}}^{0,s}(t)$ (i.e. $\tilde{f}_{\mathbf{k}}^{0,s} = \tilde{f}_{-\mathbf{k}}^{0,s}$) and asymmetric $\tilde{f}_{\mathbf{k}}^{0,a}(t)$ (i.e. $\tilde{f}_{\mathbf{k}}^{0,a} = -\tilde{f}_{-\mathbf{k}}^{0,a}$) components. Then, $f_{\mathbf{k}}^{0}$ can be represented as:

$$f_{\mathbf{k}}^{0} = \overline{f}_{\mathbf{k}}^{0} + \tilde{f}_{\mathbf{k}}^{0} = \overline{f}_{\mathbf{k}}^{0} + \tilde{f}_{\mathbf{k}}^{0,a} + \tilde{f}_{\mathbf{k}}^{0,s}\ , \qquad (1)$$

where $\overline{f}_{\mathbf{k}}^{0}$ is the statistical average distribution function of the equilibrium electron system which is determined by the Fermi-Dirac (or Boltzmann) statistics.

It was stated that the electron mobility fluctuations $\tilde{\mu}(t)$ are caused by the symmetric component $\tilde{f}_{\mathbf{k}}^{0,s}(t)$. Moreover, asymmetric component $\tilde{f}_{\mathbf{k}}^{0,a}(t)$ has no contribution to the mobility fluctuations [13]. Hence, to find out the features of the time dependence of the mobility fluctuations $\tilde{\mu}(t)$, first of all, it is necessary to find out the time dependence of the $\tilde{f}_{\mathbf{k}}^{0,s}(t)$.

Damping peculiarities of small deviations (fluctuations) of electron system from equilibrium state due to electron-acoustic phonon random scattering have been analysed in Ref. [14]. A second-order linear partial differential equation for $\tilde{f}_{\mathbf{k}}^{0,s}(t)$ was obtained via linearization of the Boltzmann equation. Another equation which describes the time dependence of electron lattice mobility fluctuations was obtained. These results are very important for further study of the mechanisms of 1/f-noise in semiconductors. Since the methods developed and used in Ref. [14] are somewhat universal, they can be used in two-dimensional materials, including graphene.

2. Derivation of the equation

The methods based on the solution of the Boltzmann transport equation are powerful theoretical tools for the study of the transport phenomena in different materials (see, e.g., [2, 3, 15-17]). It was shown that a semi-classical approach for the transport properties of graphene based on the Boltzmann equation is accurate and very effective [3, 18].

In some special cases to solve the Boltzmann equation it is necessary to determine the scattering mechanisms. Important scattering mechanisms in graphene are (see, e.g., [2, 3, 19-24]): short range scattering due to localized defects, charged impurity scattering and deformation potential scattering by acoustic and optical phonons. The role of optical phonon scattering is significant above 300K [24].

One can notice that for relatively pure graphene the main scattering mechanism of interest can be the electron-acoustic phonon intraband scattering [2,3,19-24]. Thus, in this paper the electron-acoustic phonon intraband, intravalley scattering is considered on the basis of the following physical model. Assume that at the time instance $t=0$, the electron absorbs (or emits) acoustic phonon with energy $\hbar\omega_q$ and changes its state of equilibrium and this causes fluctuations $\tilde{f}_{\mathbf{k}}^{0,s}(t)$. At random time instances the electron randomly changes its energy by absorbing or emitting phonons with random energies $\hbar\omega_q$. Therefore, the movement of the electron along the energy axis is a chaotic movement which is known as diffusive movement of an electron along the energy axis [25]. This movement is described by the symmetric component of the function $\tilde{f}_{\mathbf{k}}^{0,s}(t)$.

Using the Boltzmann transport equation one can determine the time dependence $\tilde{f}_{\mathbf{k}}^{0,s}(t)$.

In our case the Boltzmann equation has the following form [15, 25]:

$$\frac{\partial f_{\mathbf{k}}^{0}(t)}{\partial t} = J_{e-ph}\left[g_{\mathbf{q}}^{0}(t),\, f_{\mathbf{k}}^{0}(t) \right]. \tag{2}$$

Here J_{e-ph} is the collision integral for electron-phonon scattering, $g_{\mathbf{q}}^{0}(t)$ is the phonon distribution function, $\mathbf{q}$ is the quasi-wave vector of phonons.

For the more probable case of electron-single phonon scattering J_{e-ph} is represented as [15, 25]

$$\begin{aligned} J_{e-ph} = \sum_{\mathbf{q}} \mathrm{W}_q \Big\{ \Big[f_{\mathbf{k}+\mathbf{q}}^{0}(1-f_{\mathbf{k}}^{0})(1+g_{\mathbf{q}}^{0}) - (1-f_{\mathbf{k}+\mathbf{q}}^{0}) f_{\mathbf{k}}^{0} g_{\mathbf{q}}^{0} \Big] \delta(\varepsilon_{\mathbf{k}+\mathbf{q}} - \varepsilon_{\mathbf{k}} - \hbar\omega_{\mathbf{q}}) - \\ - \Big[(1-f_{\mathbf{k}-\mathbf{q}}^{0}) f_{\mathbf{k}}^{0}(1+g_{\mathbf{q}}^{0}) - f_{\mathbf{k}-\mathbf{q}}^{0}(1-f_{\mathbf{k}}^{0}) g_{\mathbf{q}}^{0} \Big] \delta(\varepsilon_{\mathbf{k}-\mathbf{q}} - \varepsilon_{\mathbf{k}} + \hbar\omega_{\mathbf{q}}) \Big\}. \end{aligned} \tag{3}$$

Here $\varepsilon_{\mathbf{k}}$ is the electron energy in a state which is described by the quasi-wave vector $\mathbf{k}$, and the factor W_q in the case of electron-acoustic phonon deformation interaction in graphene is defined as (see, e.g., [3, 23])

$$\mathrm{W}_q \equiv \frac{\pi D_{ac}^2 q^2}{\rho A \omega_q}(1+\cos\theta)\,, \tag{4}$$

where ρ is the mass density, D_{ac} is the acoustic deformation potential constant, and A is the area. Using the linear dispersion law $\omega_q = v_s q$, for W_q one can write:

$$\mathrm{W}_q \equiv \frac{\pi D_{ac}^2 q}{\rho A v_s}(1+\cos\theta) = \mathrm{W}_0 q(1+\cos\theta)\,. \tag{5}$$

where v_s is the sound velocity in graphene and

$$\mathrm{W}_0 \equiv \frac{\pi D_{ac}^2}{\rho A v_s}\,. \tag{6}$$

Taking into account the quasi-momentum conservation law ($\mathbf{k}' = \mathbf{k} \pm \mathbf{q}$) Eq. (2) can be represented as:

$$\begin{aligned}\frac{df_{\mathbf{k}}^{0}(t)}{dt} = \sum_{\mathbf{k}'} \mathrm{W}_{\mathbf{k}'-\mathbf{k}} \Big[f_{\mathbf{k}'}^{0}\left(1-f_{\mathbf{k}}^{0}\right)\left(g_{\mathbf{k}'-\mathbf{k}}^{0}+1\right) - f_{\mathbf{k}}^{0}\left(1-f_{\mathbf{k}'}^{0}\right) g_{\mathbf{k}'-\mathbf{k}}^{0} \Big] \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} - \hbar\omega_{\mathbf{k}'-\mathbf{k}}) - \\ - \sum_{\mathbf{k}'} \mathrm{W}_{\mathbf{k}-\mathbf{k}'} \Big[f_{\mathbf{k}}^{0}\left(1-f_{\mathbf{k}'}^{0}\right)\left(g_{\mathbf{k}-\mathbf{k}'}^{0}+1\right) - f_{\mathbf{k}'}^{0}\left(1-f_{\mathbf{k}}^{0}\right) g_{\mathbf{k}-\mathbf{k}'}^{0} \Big] \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} + \hbar\omega_{\mathbf{k}-\mathbf{k}'})\,.\end{aligned} \tag{7}$$

Here the peculiarities

$$\omega_{\mathbf{q}} = v_s \left|\mathbf{k}'-\mathbf{k}\right| \equiv \omega_{\mathbf{k}'-\mathbf{k}}\,, \tag{8}$$

$$\mathrm{W}_{\mathbf{q}} = \mathrm{W}_{\mathbf{k}'-\mathbf{k}} = \mathrm{W}_0 \left|\mathbf{k}'-\mathbf{k}\right|(1+\cos\theta)\,. \tag{9}$$

are used. Here and above θ is the angle between vectors $\mathbf{k}$ and $\mathbf{k}'$.

In general, due to the phonon absorption or emission the equilibrium of both electron and phonon systems is disturbed. The equilibrium state of the phonon system restores due to the phonon-phonon scattering which generally are more intense and fast processes. Therefore, one can suppose that the equilibrium of the phonon system restores very quickly: it is assumed that the absorption or emission of a phonon by the electron practically does not change the equilibrium of the phonon system [26]. Hence, fluctuations of the phonon system are neglected. It is supposed that the phonon distribution function $g_{\mathbf{q}}^{0}$ (or $g_{\mathbf{k}-\mathbf{k}'}^{0}$) is time-independent quantity and determined by the Bose-Einstein distribution:

$$g_{\mathbf{k}-\mathbf{k}'}^{0} = \bar{g}_{\mathbf{k}-\mathbf{k}'}^{0} = \frac{1}{\exp\left(\dfrac{\hbar\omega_{\mathbf{k}-\mathbf{k}'}}{k_B T}\right)-1}\,. \tag{10}$$

Substituting Eq. (1) into Eq. (7) and linearizing it in the case of small deviations, for the function $\tilde{f}_{\mathbf{k}}^{0,s}(t)$ one obtains the following equation:

$$\begin{aligned}\frac{d\tilde{f}_{\mathbf{k}}^{0,s}(t)}{dt} = \sum_{\mathbf{k}'} \mathrm{W}_{\mathbf{k}'-\mathbf{k}} \Big[\tilde{f}_{\mathbf{k}'}^{0,s}\left(g_{\mathbf{k}'-\mathbf{k}}^{0}+1\right) - \tilde{f}_{\mathbf{k}}^{0,s} g_{\mathbf{k}'-\mathbf{k}}^{0} \Big] \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} - \hbar\omega_{\mathbf{k}'-\mathbf{k}}) - \\ - \sum_{\mathbf{k}'} \mathrm{W}_{\mathbf{k}-\mathbf{k}'} \Big[\tilde{f}_{\mathbf{k}}^{0,s}\left(g_{\mathbf{k}-\mathbf{k}'}^{0}+1\right) - \tilde{f}_{\mathbf{k}'}^{0,s} g_{\mathbf{k}-\mathbf{k}'}^{0} \Big] \delta(\varepsilon_{\mathbf{k}'} - \varepsilon_{\mathbf{k}} + \hbar\omega_{\mathbf{k}-\mathbf{k}'})\,.\end{aligned} \tag{11}$$

Substituting (10) into (11) one gets:

$$\frac{d\tilde{f}^{0,s}_{\varepsilon_{\mathbf{k}}}(t)}{dt}=\sum_{\mathbf{k}'}\mathrm{W}_{\mathbf{k}'-\mathbf{k}}\left[\tilde{f}^{0,s}_{\varepsilon_{\mathbf{k}'}}\frac{\exp\left(\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_BT}\right)}{\exp\left(\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_BT}\right)-1}-\tilde{f}^{0,s}_{\varepsilon_{\mathbf{k}}}\frac{1}{\exp\left(\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_BT}\right)-1}\right]\times \tag{12}$$

$$\times\left[\delta(\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}-\hbar\omega_{\mathbf{k}'-\mathbf{k}})-\delta(\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}+\hbar\omega_{\mathbf{k}-\mathbf{k}'})\right].$$

In the quasielastic approximation δ-functions can be represented by Taylor series and one obtains:

$$\delta(\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}-\hbar\omega_{\mathbf{k}'-\mathbf{k}})-\delta(\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}+\hbar\omega_{\mathbf{k}'-\mathbf{k}})=-2\frac{d\delta(\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}})}{d\varepsilon_{\mathbf{k}'}}\hbar\omega_{\mathbf{k}'-\mathbf{k}}+\cdots \tag{13}$$

Confining to the first term of series (13), from Eq. (12) one obtains:

$$\frac{d\tilde{f}^{0,s}_{\varepsilon_{\mathbf{k}}}(t)}{dt}=-2\sum_{\mathbf{k}'}\mathrm{W}_{\mathbf{k}'-\mathbf{k}}\left[\tilde{f}^{0,s}_{\varepsilon_{\mathbf{k}'}}\frac{\exp\left(\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_BT}\right)}{\exp\left(\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_BT}\right)-1}-\tilde{f}^{0,s}_{\varepsilon_{\mathbf{k}}}\frac{1}{\exp\left(\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_BT}\right)-1}\right]\times \tag{14}$$

$$\times\frac{d\delta(\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}})}{d\varepsilon_{\mathbf{k}'}}\hbar\omega_{\mathbf{k}'-\mathbf{k}}\ .$$

Using Eq. (9), (14) can be represented as shown below:

$$\frac{d\tilde{f}^{0,s}_{\varepsilon_{\mathbf{k}}}(t)}{dt}=-2\hbar\nu_s\mathrm{W}_0\times$$

$$\times\sum_{\mathbf{k}'}\left|\mathbf{k}'-\mathbf{k}\right|^2(1+\cos\theta)\left[\tilde{f}^{0,s}_{\varepsilon_{\mathbf{k}'}}\frac{\exp\left(\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_BT}\right)}{\exp\left(\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_BT}\right)-1}-\tilde{f}^{0,s}_{\varepsilon_{\mathbf{k}}}\frac{1}{\exp\left(\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_BT}\right)-1}\right]\times \tag{15}$$

$$\times\frac{d\delta(\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}})}{d\varepsilon_{\mathbf{k}'}}.$$

Taking into account the spin degeneracy and using the following transition for graphene

$$\sum_{\mathbf{k}'}(\cdots)\Leftrightarrow\frac{A}{2\pi^2}\int_{BZ}d\mathbf{k}'(\cdots)\to\frac{A}{2\pi^2}\int_{k_{\min}}^{k_{\max}}\int_0^{2\pi}k'dk'd\theta\ , \tag{16}$$

then integrating over angle θ , (15) can be represented as shown below:

$$\frac{d\tilde{f}^{0,s}_{\varepsilon_{\mathbf{k}}}(t)}{dt}=-2\hbar\nu_s\mathrm{W}_0\frac{A}{2\pi^2}2\pi\times$$

$$\times\int_{k_{\min}}^{k_{\max}}dk'k'\left(k'^2+k^2-k'k\right)\left[\tilde{f}^{0,s}_{\varepsilon_{\mathbf{k}'}}\frac{\exp\left(\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_BT}\right)}{\exp\left(\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_BT}\right)-1}-\tilde{f}^{0,s}_{\varepsilon_{\mathbf{k}}}\frac{1}{\exp\left(\frac{\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}}}{k_BT}\right)-1}\right]\times \tag{17}$$

$$\times\frac{d\delta(\varepsilon_{\mathbf{k}'}-\varepsilon_{\mathbf{k}})}{d\varepsilon_{\mathbf{k}'}}.$$

Taking into account that in graphene $\varepsilon_{\mathbf{k}}=\nu_F\hbar k$, hence

$$k = \frac{\varepsilon_{\mathbf{k}}}{v_F \hbar}\,, \tag{18}$$

and using the dimensionless quantity x:

$$\frac{\varepsilon_{\mathbf{k}}}{k_B T} \equiv x\,, \qquad \frac{\varepsilon_{\mathbf{k}'}}{k_B T} \equiv x'\,, \tag{19}$$

then substituting W_0 from (6), from Eq. (17) one obtains the following equation:

$$\frac{d\tilde{f}_x^{0,s}(t)}{dt} = -\frac{2\hbar D_{ac}^2 \left(k_B T\right)^2}{\rho\left(v_F \hbar\right)^4} \int_0^\infty dx' x' \left(x'^2 + x^2 - x'x\right) \times \\ \times \left[\tilde{f}_{x'}^{0,s} \frac{\exp\left(x'-x\right)}{\exp\left(x'-x\right)-1} - \tilde{f}_x^{0,s} \frac{1}{\exp\left(x'-x\right)-1} \right] \frac{d\delta(x'-x)}{dx'}\,. \tag{20}$$

Here v_F is the Fermi velocity.

Using the δ-integration, from Eq. (20) one gets:

$$\frac{d\tilde{f}_x^{0,s}(t)}{dt} = \frac{2\hbar D_{ac}^2 \left(k_B T\right)^2}{\rho\left(v_F \hbar\right)^4} \times \\ \times \frac{d}{dx'} \left\{ \left(x'^3 - x'^2 x + x' x^2\right) \left[\tilde{f}_{x'}^{0,s} \frac{\exp\left(x'-x\right)}{\exp\left(x'-x\right)-1} - \tilde{f}_x^{0,s} \frac{1}{\exp\left(x'-x\right)-1} \right] \right\} \Bigg|_{x=x'}\,. \tag{21}$$

The right-hand side of Eq. (21) is a 0/0 type uncertainty. Using the L'hopital's rule to calculate it, one obtains the following equation:

$$\frac{d\tilde{f}_x^{0,s}(t)}{dt} = \frac{\hbar D_{ac}^2 \left(k_B T\right)^2}{\rho\left(v_F \hbar\right)^4} \left[\frac{x^3}{2} \frac{d^2 \tilde{f}_x^{0,s}}{dx^2} + \left(\frac{x^3}{2} + 2x^2 \right) \frac{d\tilde{f}_x^{0,s}}{dx} + 2x^2 \tilde{f}_x^{0,s} \right]. \tag{22}$$

Using the following notation:

$$\frac{1}{\tau_M} \equiv \frac{\hbar D_{ac}^2 \left(k_B T\right)^2}{2\rho\left(v_F \hbar\right)^4}\,, \tag{23}$$

one can write the Eq. (22) in the following form:

$$\frac{d\tilde{f}_x^{0,s}(t)}{dt} = \frac{1}{\tau_M} \left[x^3 \frac{d^2 \tilde{f}_x^{0,s}}{dx^2} + \left(x^3 + 4x^2\right) \frac{d\tilde{f}_x^{0,s}}{dx} + 4x^2 \tilde{f}_x^{0,s} \right]. \tag{24}$$

3. Discussion and conclusions

Equation of the form (24) is a second-order linear partial differential equation of parabolic type which can be considered as the special case of the Fokker-Planck equation in **k**-space [27-30]. Eq. (24) describes the chaotic movement (diffusion) of the electron along the energy axis. Both diffusion and drift coefficients are not constant quantities and depend on electron energy x.

Eq. (24) was obtained on the basis of electron - acoustic phonon intraband scattering. Intraband scattering does not change the concentration of conduction electrons n. It must be a time-independent constant quantity:

$$\frac{dn}{dt} = \frac{d(\bar{n} + \tilde{n})}{dt} = \frac{d\tilde{n}}{dt} = 0\,, \tag{25}$$

hence it is obvious that Eq. (24) and its solution must satisfy the above mentioned condition.

To show that the function determined by Eq. (24) truly satisfies the above given condition one can use the following relation [2,3] between conduction electron concentration and electron distribution function:

$$n = \frac{2}{\pi\left(v_F\hbar\right)^2}\int_0^\infty d\varepsilon_{\mathbf{k}}\varepsilon_{\mathbf{k}} f^0_{\varepsilon_{\mathbf{k}}} \ . \tag{26}$$

Therefore

$$\tilde{n}(t) = b\int_0^\infty dx x\tilde{f}^{0,s}_x(t) \ , \tag{27}$$

where the dimensionless quantity x and the following notation were used:

$$b \equiv \frac{2\left(k_BT\right)^2}{\pi\left(v_F\hbar\right)^2} \ . \tag{28}$$

Using relation (27), multiplying Eq. (24) by $b\,x\,dx$, and making integration over dx within the range $[0,\infty)$, one obtains:

$$\frac{d\tilde{n}(t)}{dt} = \frac{b}{\tau_M}\int_0^\infty dx x\left[x^3\frac{d^2\tilde{f}^{0,s}_x}{dx^2} + \left(x^3+4x^2\right)\frac{d\tilde{f}^{0,s}_x}{dx} + 4x^2\tilde{f}^{0,s}_x\right]. \tag{29}$$

Using transformations of integration by parts it is not difficult to make sure that the right-hand side of Eq.(29) is equal to zero. So, one obtains:

$$\frac{d\tilde{n}(t)}{dt} = \frac{d}{dt}\left(b\int_0^\infty dx x\tilde{f}^{0,s}_x(t)\right) = 0 \ . \tag{30}$$

Therefore Eq. (24) satisfies the condition $n(t) = const$.

Now let us calculate the numerical value of the coefficient τ_M which can be considered as some characteristic time. Using the following numerical values for given quantities [23]:

$$\rho = 7.6\times10^{-7}\ \mathrm{kg/m^3} \ , \qquad D_{ac} = 18\mathrm{eV} \ , \qquad v_F\hbar = 5.75\mathrm{eV\,\mathring{A}} \ ,$$

one can obtain the following numerical value for τ_M:

$$\tau_M \simeq 7.3\times10^{-9}\mathrm{s} \qquad \text{at} \qquad T = 300\mathrm{K} \ ,$$

$$\tau_M \simeq 6.6\times10^{-6}\mathrm{s} \qquad \text{at} \qquad T = 10\mathrm{K} \ .$$

For comparison, one can note that this value is much larger than the value of the electron quasimomentum relaxation time due to acoustic phonon scattering [23]: $\tau_{ac} \simeq 10^{-12}\mathrm{s}$ at $T = 300\mathrm{K}$.

The analysis of literature on second-order linear partial differential equations in two variables (see, e.g., [31]) shows that analytic solutions of equations of type Eq. (24) practically are not investigated. In the given case it is possible to find only either numerical solutions or approximate analytic solutions of Eq. (24) based on the known approximate methods [31].

Eq. (24) can be represented as follows:

$$\frac{d\tilde{f}^{0,s}_x(t)}{dt} = h_1(x)\frac{d^2\tilde{f}^{0,s}_x}{dx^2} + h_2(x)\frac{d\tilde{f}^{0,s}_x}{dx} + h_3(x)\tilde{f}^{0,s}_x \ , \tag{31}$$

where

$$h_1(x) \equiv \frac{x^3}{\tau_M}, \qquad h_2(x) \equiv \frac{\left(x^3+4x^2\right)}{\tau_M}, \qquad h_3(x) \equiv \frac{4x^2}{\tau_M} \ . \tag{32}$$

It is known that equations of type Eq. (31) have particular solutions with separable variables, which can be represented as [31]:

$$\tilde{f}^{0,s}_x(t) = \exp\left(-\lambda t\right)u\left(x\right) \ , \tag{33}$$

where the function $u(x)$ can be found by solving the following ordinary linear differential equation with parameter λ:

$$h_1(x)\frac{d^2u(x)}{dx^2}+h_2(x)\frac{du(x)}{dx}+\left[h_3(x)+\lambda\right]u(x)=0\,. \tag{34}$$

However, the analysis of literature shows (see, e.g., [32]) that analytic solutions of equations of type Eq. (34) in the general case are also practically not investigated. Moreover, it is obvious that the solution of Eq. (31) (or Eq. (24)) does not have to be of form Eq. (33). The solution of Eq. (24) is the subject of further research.

4. Conclusion

The main conclusions of the paper are as follows:

i. In the linearization approximation of the Boltzmann transport equation the time and energy dependences of the symmetric component of the fluctuations of the electron distribution function in graphene in the case of electron - acoustic phonon intraband, intravalley scattering is given by the second-order linear partial differential equation (24) which can be considered as the Fokker-Planck equation in $\mathbf{k}$-space. This equation describes the chaotic movement (diffusion) of the electron along the energy axis.
ii. Eq. (24) satisfies the condition of conservation of the concentration of conduction electrons. This is a necessary condition, since intraband scattering does not change the concentration of conduction electrons.
iii. In Eq. (24) both diffusion and drift coefficients are not constant quantities and depend on electron energy. The numerical value of the characteristic coefficient τ_M is much larger than the value of the electron quasimomentum relaxation time due to acoustic phonon scattering.

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