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UDC 62-50 AUTOMATION AND CONTROL SYSTEMS

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ON APPLICATION OF L_1 ADAPTIVE CONTROL TO MULTIVARIABLE CONTROL SYSTEMS

Part 2. Uniform Systems

Some issues concerning the stability of uniform adaptive control systems for rejection of external disturbances are discussed. Based on the properties of positive real transfer matrices, it is shown that such systems are stable for arbitrary large values of the adaptation gain, even in the case of the systems with right half plane zeros. Some specific features of uniform adaptive systems are revealed and explained.

Keywords: multivariable control system, uniform system, adaptive control, stability, characteristic transfer function, positive real system.

The paper examines the application of L_1 adaptive control to multivariable control systems [1,2]. L_1 adaptive control was developed to address some of the deficiencies apparent in *Model Reference Adaptive Control* (MRAC), as a loss of robustness in the presence of fast adaptation [3,4].The first part of the paper was devoted to the application of L_1 adaptive control to general-type *square,* i.e. having the same number of inputs and outputs, *Multiple-Input Multiple-Output* (MIMO) control systems [5,6] subjected to external disturbances. In Part 2, a special class of *uniform* MIMO systems is discussed.

Uniform Systems. In various technical applications, such as aerospace engineering, chemical industry and many others, the so-called *uniform* MIMO systems occur very often [6]. The separate channels of uniform MIMO systems have identical transfer functions $w(s)$, and the cross-connections are *rigid,* i.e. are characterized by a real-valued numerical matrix $R = \{r_{ij}\}$. To get a better insight in the structural features of uniform systems and their difference from the general-type MIMO control systems, two matrices, as well as extended (for $N = 2$) block diagrams of both classes of multivariable control systems are shown in Fig. 1.

The transfer matrix $W(s) = \{r_{ij} w(s)\}$ of the uniform system can be written as:

$$W(s) = w(s)R . \tag{1}$$

In what follows, we will assume that the transfer function $w(s)$ is completely controllable and observable, strictly stable, and, maybe, with *Right Half Plane* (RHP) zeros.

As a basic model of linear N -dimensional uniform systems with constant parameters, let us consider the system that can be expressed in the following standard state-space form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{2}$$

where $x(t)$ is an n_x -dimensional state vector; $u(t)$ and $y(t)$ are N -dimensional vectors of inputs and outputs; A, B, C are constant matrices of appropriate sizes.

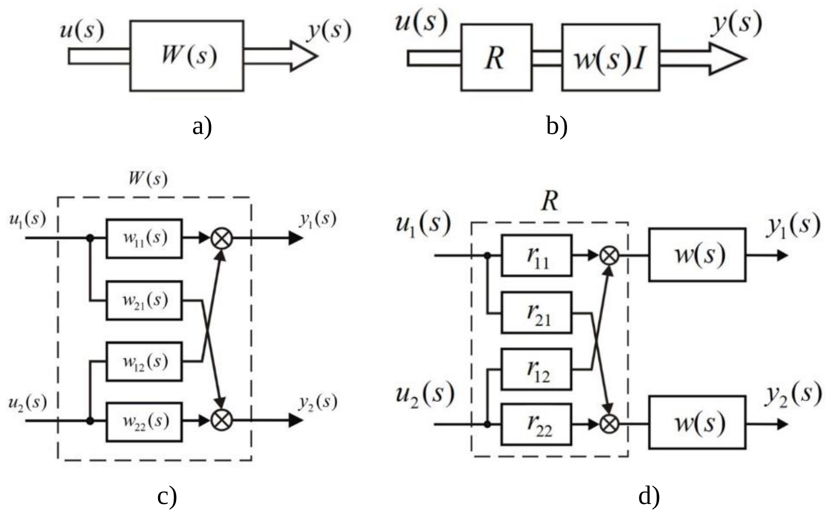


Fig. 1. Block diagrams of the MIMO system: (a), (c) Matrix and extended block diagrams of general-type systems; (b),(d) Matrix and extended block diagrams of uniform systems

Generally, the transfer matrix $W(s)$ (1) of uniform systems, which constitute a particular class of general-type MIMO systems, is connected with the matrices A, B, C in (2) by the common formula [5]:

$$W(s) = C(sI - A)^{-1}B, \tag{3}$$

where I is an identity matrix.

Disturbance Rejection by Means of Adaptive Control. In this section, we will follow the results presented in [1]. Let an N -dimensional *strictly stable* uniform system be described in state-space by the following equations:

$$\dot{x}(t) = Ax(t) + B(u(t) + \sigma(t)), \quad x(0) = x_0,$$
$$y(t) = Cx(t), \tag{4}$$

where $\sigma(t)$ is an N-dimensional time-dependent vector of unknown external bounded ($|\sigma(t)| \le \Delta_0$) disturbances that should be rejected by adaptive control, and all other matrices and vectors have the dimensions as in (2).

The state predictor has the same structure as the systems in (4):

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B(u(t) + \hat{\sigma}(t)), \quad \hat{x}(0) = x_0,$$
$$\hat{y}(t) = C\hat{x}(t), \tag{5}$$

and the only difference is that the unknown disturbance vector $\sigma(t)$ is replaced by its estimate $\hat{\sigma}(t)$.

The disturbance rejection process is governed by the following adaptation law [1]

$$\dot{\hat{\sigma}}(t) = \Gamma B^T P \varepsilon(t), \tag{6}$$

where

$$\varepsilon(t) = x(t) - \hat{x}(t) \tag{7}$$

is the *prediction error*, P ($P = P^T > 0$) is the solution of the Lyapunov equation

$$A^T P + PA = -Q \tag{8}$$

for an arbitrary symmetric positive definite matrix Q ($Q = Q^T > 0$), and the positive scalar $\Gamma > 0$ is called the *adaptation gain* [1,2].

The control signal $u(t)$ of the system is given in operator form as:

$$u(s) = Q(s)(k_g r(s) - \hat{\sigma}(s)), \tag{9}$$

where $r(s)$ is an N-dimensional reference signal, k_g is an $N \times N$ static matrix, and $Q(s)$ is the transfer matrix of a low-pass filter having the form

$$Q(s) = q(s)I, \tag{10}$$

where $q(s)$ is a strictly proper transfer function [usually, satisfying the condition $q(0) = 1$]. Its state-space realization assumes zero initialization.

The general block diagram of the adaptive control system (4) with the disturbance rejection law (6) is depicted in Fig. 2 and represents a linear MIMO system with an integral feedback [1].

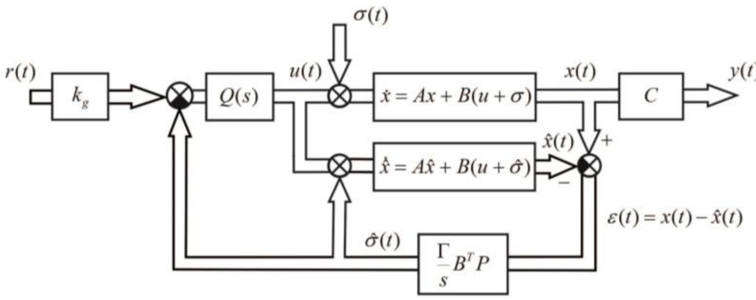


Fig. 2. Block diagram of the adaptive system with the state predictor and the adaptive disturbance rejection law (6)

In [1], it is shown that the block diagram in Fig. 2 is described by the following matrix equation:

$$y(s) = w(s)q(s)Rk_g r(s) + w(s)R\left[I - q(s)\left[I + W_0(s)\right]^{-1} W_0(s) \right]\sigma(s) \ , \qquad (11)$$

where

$$W_0(s) = \frac{\Gamma}{s} W_B(s) \, ; \; W_B(s) = B^T P(sI - A)^{-1} B \, , \qquad (12)$$

and the formulas of the transfer matrices $W(s)$ (1) and $Q(s)$ (10) are used.

The matrix equation (11) implies that dynamics of the uniform adaptive system can be represented by two independent block diagrams in Fig. 3 and 4, where the first one describes the system behavior with respect to the reference signal $r(t)$, and the second, with respect to the disturbance vector $\sigma(t)$.

Stability analysis of the uniform adaptive system. In essence, stability properties of the uniform systems in Fig. 2, 3, and 4 are the same as for the general-type MIMO systems discussed in [1]. Particularly, the block diagram in Fig. 3 represents an open-loop and stable uniform system, since both transfer functions $q(s)$ and $w(s)$ are assumed stable. Besides, that system does not depend on the adaptation gain Γ.

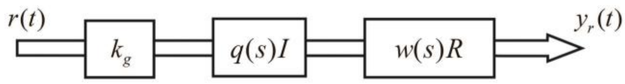


Fig. 3. Equivalent block diagram of the uniform adaptive system with respect to the input reference signal $r(t)$

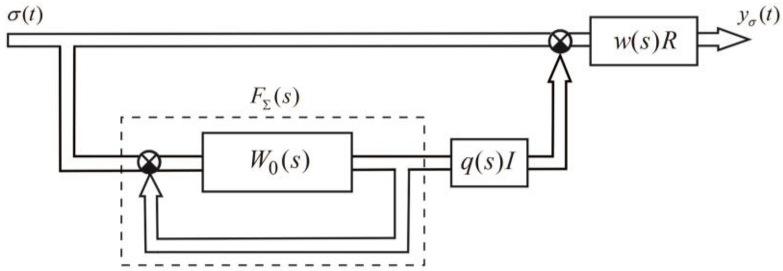


Fig. 4. An equivalent block diagram of the uniform adaptive system with respect to the disturbance $\sigma(t)$

As for the block diagram in Fig. 4, it contains a negative feedback loop with the open-loop transfer matrix $W_0(s)$ (12) and the following closed-loop transfer matrix:

$$F_\Sigma(s) = \left[I + W_0(s)\right]^{-1} W_0(s), \tag{13}$$

the roots of the characteristic equation

$$\det\left[I + W_0(s)\right] = 0 \tag{14}$$

of which depend on Γ. Note also that the system with the $N \times N$ transfer matrix $W_B(s)$ in (12) is, due to its form, a *Positive Real* (PR) transfer matrix [1,3,4,7].

Any uniform system with the transfer matrix $W(s) = \{r_{ij} w(s)\} = w(s)R$ can be treated as a general-type MIMO system, which results in the state-space vector $x(t)$ of order $n_x = n_x^0 \cdot N^2$ [1], where n_x^0 is the order of the transfer function $w(s)$ in (1).

Another choice is based on the following equations of the uniform system:

$$\begin{gathered} \dot{x}_i(t) = A_0 x_i(t) + b_0 \varphi_i(t), \\ \varphi_i(t) = \sum\nolimits_{j=1}^{N} r_{ij} u_j(t), \quad y_i(t) = c_0^T x_i(t), \quad (i = 1, 2, ..., N), \end{gathered} \tag{15}$$

where the triple (A_0, b_0, c_0) corresponds to the state-space representation of the transfer function $w(s)$ in (1). The vector $x(t)$ in this case has the form

$$x(t) = \left[x_1^T(t) \;\; x_2^T(t) \;\; x_3^T(t) \ldots x_{N-1}^T(t) \;\; x_N^T(t) \right]^T, \tag{16}$$

and its order $n_x = n_x^0 \cdot N$ is N times as small as that in the general case. It can be shown that in this case, the transfer matrix $W_0(s)$ (12) is not diagonal (as in the case of general-type systems [1]), but preserves the structure inherent in uniform systems:

$$W_0(s) = \frac{\Gamma}{s} w_0(s) R_0 , \tag{17}$$

where the transfer function $w_0(s)$ is given by the following expression

$$w_0(s) = b_0^T P_0 (sI - A_0)^{-1} b_0 , \tag{18}$$

and the numerical matrix of cross-connections R_0 is a positive definite symmetric matrix equal to

$$R_0 = R^T R . \tag{19}$$

In equation (18), the positive definite symmetric matrix P_0 solves the Lyapunov equation:

$$A_0^T P_0 + P_0 A_0 = -Q_0 . \tag{20}$$

Note that the inspection of equations (15) and (18) shows that the transfer function $w_0(s)$ belongs to PR functions, the properties of which are thoroughly discussed in technical literature [1-4,7].

Based on the *Characteristic Transfer Function* (CTF) method [6], the transfer matrix $W_0(s)$ (12) can be represented in the canonical form:

$$W_0(s) = L diag\{q_i^0(s)\} L^{-1}; \quad q_i^0(s) = \lambda_i \frac{\Gamma}{s} w_0(s) , \quad (i = 1, 2, ..., N), \tag{21}$$

where $q_i^0(s)$ are called the CTFs of $W_0(s)$ (we will assume them *distinct*), the *unitary* (i.e. $L^{-1} = L^*$) modal matrix L is composed of the orthonormal set of eigenvectors l_i of the symmetric matrix R_0 (19), and λ_i are *real-valued* and *positive* eigenvalues of R_0.

Allowing for (21), the characteristic equation (14) takes the form

$$\det\left[I + W_0(s)\right] = \prod_{i=1}^{N} \left[1 + \lambda_k \frac{\Gamma}{s} w_0(s)\right] = 0, \tag{22}$$

and the stability of the uniform adaptive system in Figure 2 is defined by N characteristic equations:

$$1 + \lambda_i \frac{\Gamma}{s} w_0(s) = 0, \quad (i = 1, 2, ..., N) . \tag{23}$$

Following the arguments presented in [1], we can claim that the CTFs $q_i^0(s)$ in (21) have a relative degree 1 or 2 and are minimum-phase, even if the transfer function $w(s)$ in (1) has RHP zeros. Besides, the phases of $q_i^0(j\omega)$ are always less or equal to $\pm 180^\circ$, which means that the Nyquist plots of $q_i^0(j\omega)$ cannot encircle the critical point $(-1, j0)$, irrespectively of the value of the gain Γ. Finally, the root loci of the CTFs $q_i^0(s)$ will tend to infinity, as $\Gamma \to \infty$, along the negative real semi-axis or along the asymptotes that are parallel to the imaginary axis and lie in the left half plane.

Note that if the matrix of cross-connections of the uniform system is orthogonal, i.e. $R^T = R^{-1}$, the matrix R_0 (19) becomes an identity matrix (i.e. $R_0 = I$), and the stability of the adaptive system will not depend at all on the matrix R and the number of channels N. Indeed, if $R^T = R^{-1}$, then, instead of (21), we have the expression

$$W_0(s) = \frac{\Gamma}{s} w_0(s) I \, , \tag{24}$$

and the stability of the uniform adaptive system will be determined by the stability of a single closed-loop one-dimensional system with the characteristic equation

$$1 + \frac{\Gamma}{s} w_0(s) = 0 \, . \tag{25}$$

Example. Consider a three-dimensional uniform system with the following transfer function of separate channels:

$$w(s) = \frac{20}{(s+0.5)\ (s+2)\ (s+10)} \ , \tag{26}$$

the numerical matrix of cross-connections

$$R = \begin{bmatrix} 0.9 & 0.03 & -0.01 \\ -0.05 & 0.866 & 0.5 \\ 0.02 & -0.5 & 0.866 \end{bmatrix}, \tag{27}$$

and the transfer matrix of low-pass filter $Q(s) = [1/(0.01s+1)]I$.

Based on the structure of the uniform system, we can write:

$$\begin{gathered} \dot{x}_i(t) = A_0 x_i(t) + b_0 \varphi_i(t) \, , \\ \varphi_i(t) = \sum_{j=1}^{N} r_{ij} u_j(t), \quad y_i(t) = c_0^T x_i(t) \quad (i = 1, 2, 3), \end{gathered} \tag{28}$$

where

$$A_0 = \begin{bmatrix} -0.5 & 1.0 & 0 \\ 0 & -2.0 & 1.0 \\ 0 & 0 & -10.0 \end{bmatrix};\ b_0 = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix};\ c_0 = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}. \qquad (29)$$

The solution P_0 to the Lyapunov equation (20) for $Q_0 = I$ is

$$P_0 = \begin{bmatrix} 1.0 & 0.4 & 0.0381 \\ 0.4 & 0.45 & 0.0407 \\ 0.0381 & 0.0407 & 0.0541 \end{bmatrix}, \qquad (30)$$

and the symmetric positive definite matrix of cross-connections R_0 (19) in the transfer matrix $W_0(s)$ (17) is

$$R_0 = \begin{bmatrix} 0.8110 & -0.0240 & -0.0057 \\ -0.0240 & 1.0025 & -0.0010 \\ -0.0057 & -0.0010 & 1.0004 \end{bmatrix}. \qquad (31)$$

The eigenvalues λ_i of the matrix R_0 are

$$\lambda_1 = 0.8079,\quad \lambda_2 = 1.0006,\quad \lambda_3 = 1.0055 \qquad (32)$$

and the transfer function $w_0(s)$ calculated by the formula (18) is equal to

$$w_0(s) = \frac{0.86508(s+2.377)(s+0.8754)}{(s+0.5)(s+2)(s+10)}. \qquad (33)$$

The Nyquist plot of the transfer function $w_0(s)$ (33) for positive frequencies $\omega \geq 0$ is shown in Fig. 5 and verifies that $w_0(s)$ is positive real.

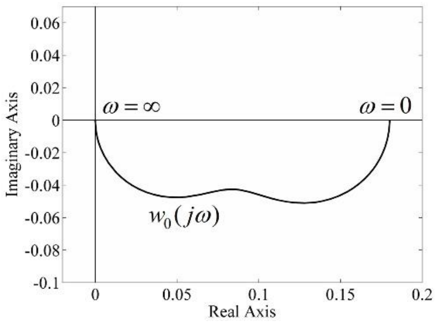
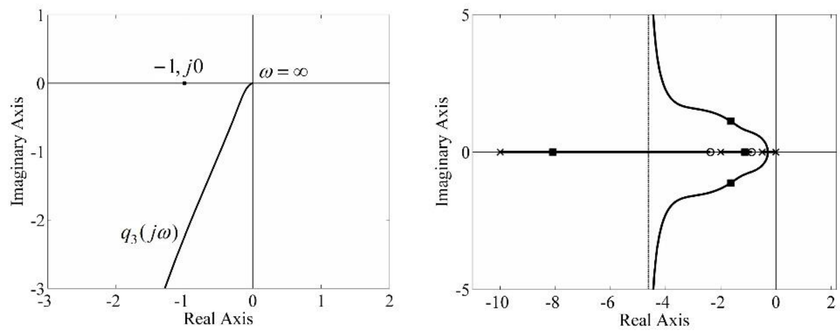


Fig. 5. Nyquist plot of the transfer function $w_0(s)$ (33) for $\omega \geq 0$

The CTFs $q_i^0(s)$ of the transfer matrix $W_0(s)$ (21) correspond to three common single-input single-output systems with real-valued parameters, where the eigenvalues λ_i (32) can be regarded as usual gains:

$$q_i^0(s) = \lambda_i \Gamma \frac{0.86508(s+2.377)(s+0.8754)}{s(s+0.5)(s+2)(s+10)}, \qquad (34)$$
$$i = 1, 2, 3.$$

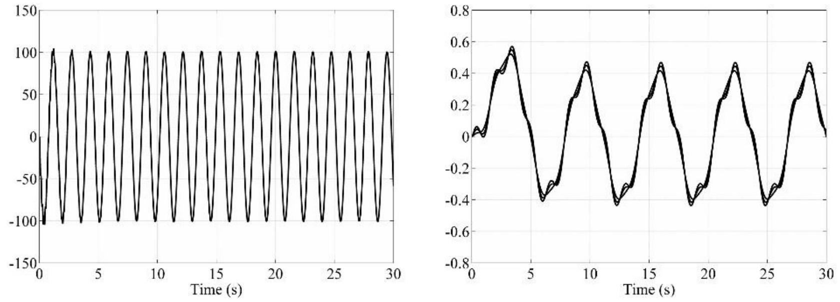
Note that all λ_i (32) in $q_i^0(s)$ (34) are real-valued and quite close to 1. Therefore, the root loci of the CTFs $q_i^0(s)$ (34) completely coincide (only the roots of the closed-loop CTFs slightly differ) and the difference of the Nyquist plots of $q_i^0(s)$ is actually imperceptible. The Nyquist plot and root loci of the CTF $q_3^0(s)$ (the CTF with the largest eigenvalue $\lambda_3 = 1.0055$) for $\Gamma = 20$ are shown in Fig. 6.



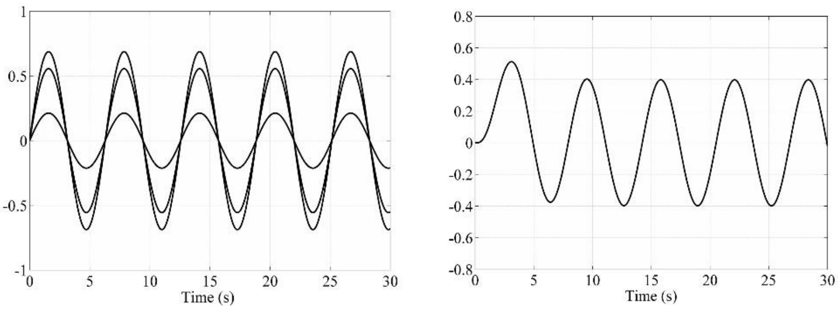
a) b)

Fig. 6. Nyquist (a) and root loci (b) plots of the CTF $q_3^0(s)$ *for* $\Gamma = 20$

The results of simulation of the three-dimensional uniform adaptive system with the help of *Simulink* for $\Gamma = 5000$, two types of reference input signals (sinusoidal signals with unit amplitudes, zero phase shifts, and period $T = 6.28\ s$, as well as unit step signals applied simultaneously at $t = 1.0\ s$), and sinusoidal disturbances with amplitudes $A_d = 100$ and period $T_d = 1.57\ s$ are shown in Figures 7 and 8. Note that the amplitudes of disturbances are 100 times as large as the amplitudes (or unit step values) of reference signals $r(t)$.

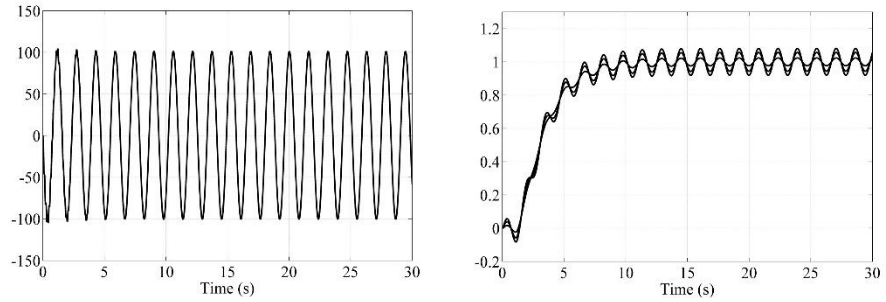


a) *Control signal* $u(t)$ b) *Output signal* $y(t)$



c) *Control signal* $u(t)$ d) *Output signal* $y(t)$

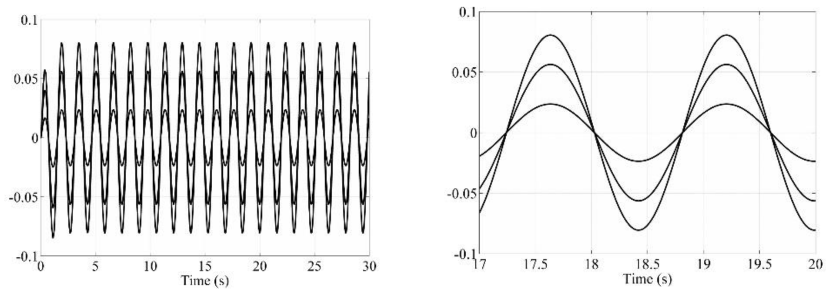
Fig. 7. Simulation results. Sinusoidal reference inputs $r_i(t) = \sin(t)$ *:(a),(b) sinusoidal disturbances* $\sigma_i(t) = 100\sin(4t)$ *; (c),(d) zero disturbances* $\sigma_i(t) = 0$



a) b)

Fig. 8. Simulation results. Unit step reference inputs, sinusoidal disturbances $\sigma_i(t) = 100\sin(4t)$ $(i = 1, 2, 3)$ *: (a) Control signal* $u(t)$ *; (b) Output signal* $y(t)$

In Fig. 7(c), (d), to get a better understanding of behavior of the adaptive system, the results of simulation with the same sinusoidal reference signals but without disturbances (i.e. $\sigma(t)=0$) are shown. In fact, as the principle of superposition is valid for the discussed adaptive system, the output signals of the system with sinusoidal disturbances are the sum of the output signals of the system with $\sigma(t)=0$ and the same system with disturbances but zero reference signals $r(t)=0$. The same is true for unit step reference signals (Fig. 8). The results of simulation for $r(t)=0$ and sinusoidal disturbances $\sigma_i(t)=100\sin(4t)$ are presented in Fig. 9. The control signal $u(t)$ in this case is actually the same as in Fig. 7(a) and 8(a) due to the large ratio of amplitudes of disturbances and reference signals.



a) b)

Fig. 9. Simulation results. Zero reference inputs $r(t)=0$, sinusoidal disturbances $\sigma_i(t)=100\sin(4t)$; (a) Output signal $y(t)$; (b) Output signal $y(t)$ (enlarged)

Finally, it should be noted that an increase in Γ will bring to smaller deviations of $y(t)$ from the ideal outputs, i.e. to higher performance of the L_1 adaptive system.

REFERENCES

1. On Application of L_1 Adaptive Control to Multivariable Control Systems. Part I. General-Type Multivariable Systems / **T. Hovanissyan, N. Vardanyan, E. Kharisov, O. Gasparyan, et al** // Известия НАН РА и НПУА. Сер. ТН. – 2014. – Т.67, №4. – С. 434-445.
2. **Hovakimyan N. and Cao C.** L1 Adaptive Control Theory-Guaranteed Robustness with Fast Adaptation.- SIAM, Philadelfia, 2010. -340p.
3. **Astrom K. and Wittenmark B.** Adaptive Control.- Addison-Wesley Longman Publishing Co., Inc., Boston, MA, 1994. -575p.
4. **Ioannou P. and Sun J.** Robust Adaptive Control.- Prentice Hall, 1996. -834p.
5. **Skogestad S. and Postlethwaite I.** Multivariable Feedback Control. Analysis and Design. - John Wiley and Sons Ltd., Chichester, Sussex, UK, 2005. -595p.
6. **Gasparyan O.N.** Linear and Nonlinear Multivariable Feedback Control: A Classical Approach. - John Wiley & Sons, UK, 2008. -354p.
7. **Slotine J.-J. E. and Li W.** Applied Nonlinear Control.- Prentice Hall, Englewood Cliffs, USA, 1991. -476p.

National Polytechnic University of Armenia. The material is received 21.08.2014.

Տ.Ն. ՀՈՎՀԱՆՆԻՍՅԱՆ, Ն.Հ. ՎԱՐԴԱՆՅԱՆ, Ե. Ռ. ԽԱՐԻՍՈՎ, Օ.Ն. ԳԱՍՊԱՐՅԱՆ, Ն.Գ. ՀՈՎԱԿԻՄՅԱՆ

ԲԱԶՄԱՉԱՓ ԿԱՌԱՎԱՐՄԱՆ ՀԱՄԿԱՐԳԵՐՈՒՄ L1 ՀԱՐՄԱՐՎՈՂ ԿԱՌԱՎԱՐՄԱՆ ԿԻՐԱՌՄԱՆ ՎԵՐԱԲԵՐՅԱԼ

Մաս 2. Միատիպ համակարգեր

Դիտարկված են հարցեր, որոնք առնչվում են արտաքին վրդովմունքները չեզոքացնելուն ուղղված միատիպ հարմարվող կառավարման համակարգերի կայունությանը: Հիմնվելով դրական իրական փոխանցման մատրիցների հատկությունների վրա՝ ցույց է տրված, որ այդպիսի համակարգերը կայուն են հարմարման գործակցի կամայական մեծ արժեքների, նույնիսկ աջակողմյան զրոներով համակարգերի դեպքում: Բացահայտված և վերլուծված են միատիպ հարմարվող համակարգերի որոշ առանձնահատկություններ:

Առանցքային բառեր. կառավարման բազմաչափ համակարգ, միատիպ համակարգ, հարմարվող կառավարում, կայունություն, բնութագրիչ փոխանցման ֆունկցիա, դրական իրական համակարգ:

Т.Н. ОГАННИСЯН, Н.А. ВАРДАНЯН, Е.Р. ХАРИСОВ, О.Н. ГАСПАРЯН, Н.Г. ОВАКИМЯН

О ПРИМЕНЕНИИ L1 АДАПТИВНОГО УПРАВЛЕНИЯ В МНОГОМЕРНЫХ СИСТЕМАХ УПРАВЛЕНИЯ

Часть 2. Однотипные системы

Рассмотрены некоторые вопросы, связанные с устойчивостью однотипных адаптивных систем управления, предназначенных для компенсации внешних возмущений. Основываясь на свойствах положительных действительных передаточных матриц, показано, что подобные системы устойчивы при произвольно больших значениях коэффициента адаптации, даже в случае систем с правосторонними нулями. Выявлены и объяснены некоторые специфические свойства однотипных адаптивных систем.

Ключевые слова: многомерная система управления, однотипная система, адаптивное управление, устойчивость, характеристическая передаточная функция, положительная действительная система.