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UDC. 62-50 **AUTOMATION AND CONTROL SYSTEMS**

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ON APPLICATION OF L1 ADAPTIVE CONTROL TO MULTIVARIABLE CONTROL SYSTEMS

Part 1. General-Type Multivariable Systems

Some issues concerning the stability of multivariable square (i.e. having the same number of inputs and outputs) adaptive control systems for rejection of external disturbances are discussed. Based on the properties of positive real transfer matrices, it is shown that such systems are stable for arbitrary large values of the adaptation gain, even in the case of systems with right half plane zeros.

Keywords: multivariable control system, adaptive control, reference model, stability, positive real system.

The paper examines application of L_1 adaptive control to multivariable control systems [1]. L_1 adaptive control was developed to address some of the deficiencies apparent in *Model Reference Adaptive Control* (MRAC), as loss of robustness in the presence of fast adaptation [2, 3].The first part of the paper is devoted to application of L_1 adaptive control to general-type *square*, i.e. having the same number of inputs and outputs, *Multiple-Input Multiple-Output* (MIMO) control systems [4, 5], subjected to external disturbances. Some essential dynamic features of that class of adaptive systems are specified and discussed. A special class of the so-called *uniform* MIMO systems [5] will be discussed in Part 2.

General-Type MIMO Systems. As a basic model of linear N-dimensional, i.e. having N inputs and N outputs, MIMO systems with constant parameters let us consider the system that can be expressed in the following standard state-space form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t)$ is an n_x-dimensional state vector; $u(t)$ and $y(t)$ are N-dimensional vectors of inputs and outputs; A, B, C are constant matrices of appropriate sizes. In what follows, we will assume that system completely controllable and observable, strictly stable (i.e. the matrix A is Hurwitz), and, maybe, with *Right Half Plane* (RHP) transmission zeros.

The MIMO system (1) can also be described in the operator form by the $N \times N$ transfer function matrix $W(s) = \{w_{ij}(s)\}$, where $w_{ij}(s)$ $(i, j = 1, 2, ..., N)$ are scalar strictly proper rational functions in complex variable *s*. The elements $w_{ii}(s)$ on the principal diagonal of $W(s)$ are the transfer functions of *separate* (or *direct*) channels, and the non-diagonal elements $w_{ij}(s)$ $(i \neq j)$ are the transfer functions of *cross-connections* from the j th channel to the i th.

Generally, the transfer matrix $W(s)$ is connected with the matrices A, B, C in (1) by the formula [4]

$$W(s) = C(sI - A)^{-1} B, \tag{2}$$

where I is an identity matrix.

Square MIMO systems can be divided into *classes* (or *types*) depending on their structural properties. In this respect, if no conditions are imposed on the form of the transfer matrix $W(s)$ (2), we will refer to that system as a *general-type* (or just *general*) MIMO system [5].

Disturbance Rejection by Means of Adaptive Control. In this section, we will adhere to the L_1 architecture with *state predictor* and low-pass matrix filter presented in [1]. Let an N -dimensional *strictly stable* MIMO system be described in state-space by the following equations:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B(u(t) + \sigma(t)), \quad x(0) = x_0, \\ y(t) &= Cx(t), \end{aligned} \tag{3}$$

where $\sigma(t)$ is an N -dimensional time-dependent vector of unknown externally bounded ($|\sigma(t)| \leq \Delta_0$) disturbances that should be rejected by adaptive control, and all other matrices and vectors have the dimensions as in (1).

The state predictor has the same structure as the system in (3):

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + B(u(t) + \hat{\sigma}(t)), \quad \hat{x}(0) = x_0, \\ \hat{y}(t) &= C\hat{x}(t), \end{aligned} \tag{4}$$

and the only difference is that the unknown disturbance vector $\sigma(t)$ is replaced by its estimate $\hat{\sigma}(t)$.

The disturbance rejection process is governed by the following adaptation law [1]

$$\dot{\hat{\sigma}}(t) = \Gamma B^T P \varepsilon(t), \tag{5}$$

where $\varepsilon(t) = x(t) - \hat{x}(t)$ is the *prediction error*, P ($P = P^T > 0$) is the solution of the Lyapunov equation

$$A^T P + PA = -Q \tag{6}$$

for an arbitrary symmetric positive definite matrix Q ($Q = Q^T > 0$), and the positive scalar Γ is called the *adaptation gain* [1].

The control signal $u(t)$ of the system is given in operator form as

$$u(s) = Q(s)(k_g r(s) - \hat{\sigma}(s)), \tag{7}$$

where $r(s)$ is an N-dimensional reference signal, k_g is an $N \times N$ static (gain) matrix, and $Q(s)$ is the transfer matrix of a low-pass filter. In the simplest case, the matrix $Q(s)$ is chosen in the form

$$Q(s) = q(s)I, \tag{8}$$

where $q(s)$ is a strictly proper scalar transfer function ,usually, satisfying the DC gain condition $q(0)$ = 1. Its state-space realization assumes zero initialization.

The block diagram of the control system with the state predictor (4), the adaptive disturbance rejection law (5), and control signal $u(s)$ (7), is shown in Figure 1.

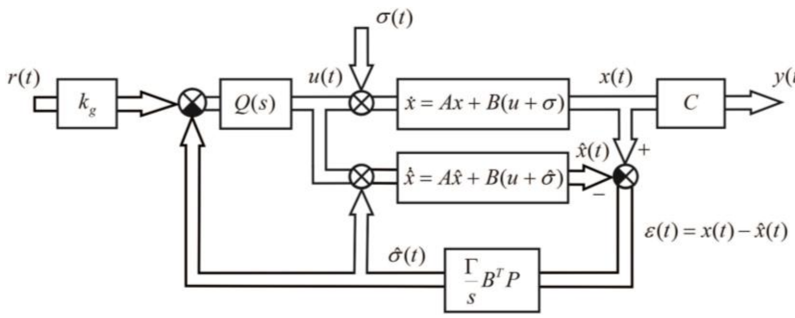


Fig. 1. Block diagram of the adaptive MIMO control system with the state predictor and the adaptive disturbance rejection law (5)

Thus, the architecture of the discussed adaptive control system represents a linear MIMO system with integral feedback and therefore can be investigated by the methods and approaches of linear multivariable feedback control [4, 5]. It should be noted that due to the adopted L_1 scheme with the state predictor, the transfer matrix $Q(s)$ (8) in the control signal $u(s)$ (7) is not present in the disturbance rejection law (5). Let us consider in more detail the structure and performance characteristics of the adaptive system in Figure 1. Toward that end, we introduce the $n_x \times N$ transfer matrix

$$W_x(s) = (sI - A)^{-1} B \tag{9}$$

relating the input to the system (1) with the state vector $x(t)$, and the corresponding (the same) matrix $\hat{W}_x(s) = W_x(s)$ for the state predictor. Then, the block diagram in Figure 1 can be recast to an equivalent form in Figure 2.

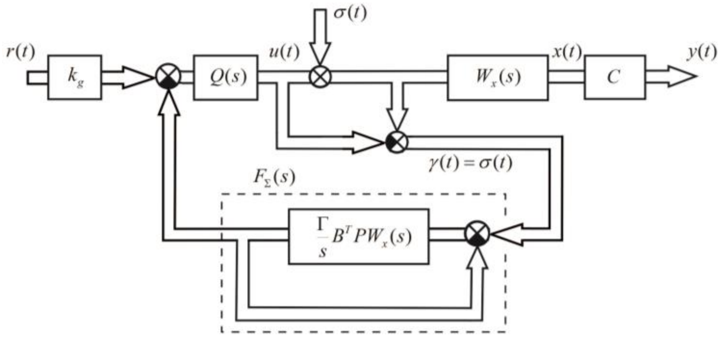


Fig. 2. Equivalent block diagram of the adaptive system in Fig. 1

Based on the block diagram in Figure 2, it is easy to derive the following matrix equations of the adaptive system with the state predictor:

$$y(s) = CW_x(s)Q(s)k_g r(s) + CW_x(s)\left[I - Q(s)\left[I + W_0(s)\right]^{-1} W_0(s)\right]\sigma(s)\,, \quad (10)$$

or, since $CW_x(s) = W(s)$ [see equations (2) and (9)],

$$y(s) = W(s)Q(s)k_g r(s) + W(s)\left[I - Q(s)\left[I + W_0(s)\right]^{-1} W_0(s)\right]\sigma(s)\,, \quad (11)$$

where

$$W_0(s) = \frac{\Gamma}{s} W_B(s)\,;\; W_B(s) = B^T P W_x(s)\,. \quad (12)$$

As can be seen from (10), the output signal of the system $y(s)$ consists of two components generated, respectively, by the input reference signal $r(s)$ and by the disturbance $\sigma(s)$. Since the adaptive MIMO system in Figures 1 and 2 is linear, the superposition principle holds and, according to the equation (10), the dynamics of the system can be represented by two independent block diagrams in Figures 3 and 4.

Let us proceed to the stability analysis of the adaptive system in Figure 1. From the block diagram in Figure 3, it is evident that it represents, since both transfer matrices $Q(s)$ and $W(s)$ are assumed stable, a stable open-loop MIMO system, which does not depend on the adaptation gain Γ. Therefore, no problem with stability can arise here.

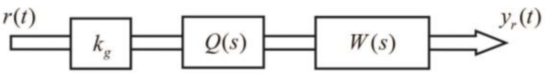


Fig. 3. Equivalent block diagram of the adaptive system with respect to the input reference signal $r(t)$

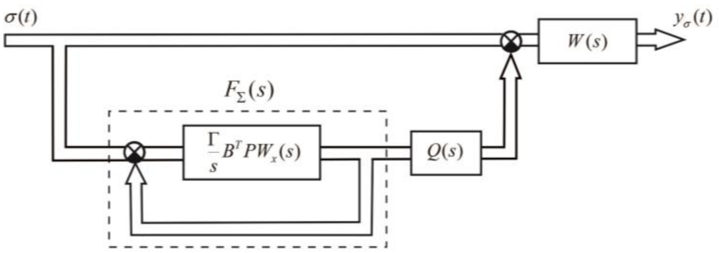


Fig. 4. Equivalent block diagram of the adaptive system with respect to the disturbance $\sigma(t)$

On the contrary, the block diagram in Figure 4 contains a negative feedback loop with the open-loop transfer matrix $W_0(s)$ (12) and the following closed-loop transfer matrix:

$$F_\Sigma(s) = \left[I + W_0(s)\right]^{-1} W_0(s) = \left[I + \frac{\Gamma}{s} W_B(s)\right]^{-1} \frac{\Gamma}{s} W_B(s)\,. \tag{13}$$

The characteristic equation of that system is

$$\det\left[I + W_0(s)\right] = \det\left[I + \frac{\Gamma}{s} W_B(s)\right] = 0\,, \tag{14}$$

and, clearly, the poles of the adaptive system depend on the adaptation gain Γ, which can be considered as the gain of the open-loop transfer matrix $W_0(s)$ (12).

Note now that, allowing for (9), the system with the $N \times N$ transfer matrix $W_B(s)$ in (12) can be written in state-space form as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y_e(t) &= C_e x(t), \end{aligned} \tag{15}$$

where

$$C_e = B^T P\,. \tag{16}$$

Taking into account the form of the matrix C_e (16) and recalling the Kalman-Yakubovich lemma [1-3, 6], we come to a conclusion that $W_B(s)$ belongs to the so-called *Positive Real* (PR) transfer matrices for which the Hermitian matrix

$$\operatorname{Re} W_B(j\omega) = \frac{1}{2}\left[W_B(j\omega) + W_B^T(-j\omega)\right] \tag{17}$$

is positive semi-definite (for brevity we shall write $\operatorname{Re} W_B(j\omega) \geq 0$) for all real ω, for which $j\omega$ is not a pole of any element of $W_B(s)$.

To get some additional insight in that issue, we invoke the characteristic transfer functions method [4, 5], on the basis of which the transfer matrix $W_B(s)$ can be represented in the following canonical form:

$$W_B(s) = L(s)\, diag\{q_i^B(s)\} L^{-1}(s)\,, \tag{18}$$

where the functions $q_i^B(s)$ $(i = 1, 2, ..., N)$ are called *Characteristic Transfer Functions* (CTF) (we will assume them *distinct*), and the *modal* matrix $L(s)$ is composed of the linearly independent eigenvectors $l_i(s)$ of the matrix $W_B(s)$. As shown in [5], the condition $\operatorname{Re} W_B(j\omega) \geq 0$ implies that all scalar CTFs $q_i^B(s)$ are also PR, that is $\operatorname{Re}[q_i^B(j\omega)] \geq 0$ for all real ω. This also means that all $q_i^B(s)$ are strictly stable and minimum-phase, have *relative degree* (the excess of number of poles over the number of zeros) 0 or 1, and the Nyquist plots of $q_i^B(j\omega)$ lie entirely in the right half complex plane or, equivalently, the phases of $q_i^B(j\omega)$ are always less or equal to $\pm 90^\circ$.

Inspection of equations (12) shows that the matrices $W_0(s)$ and $W_B(s)$ differ by a scalar multiplier Γ / s. Since the multiplication of transfer matrices (for $s = const$) by a scalar multiplier does not change the eigenvectors and results in multiplication of all eigenvalues by the same multiplier [5], the canonical representation of the matrix $W_0(s)$ (12) will have, allowing for (18), the form

$$W_0(s) = L(s)\, diag\{q_i^0(s)\} L^{-1}(s)\,, \tag{19}$$

where

$$q_i^0(s) = \frac{\Gamma}{s} q_i^B(s) \quad (i = 1, 2, ..., N) \tag{20}$$

are the CTFs of the transfer matrix $W_0(s)$ (12), which differ from the CTFs $q_i^B(s)$ of $W_B(s)$ by the multiplier Γ / s. Therefore, the CTFs $q_i^0(s)$ have relative degree 1 or 2 and are minimum-phase, even if the initial system $W(s)$ (2) has RHP transmission zeros. Besides, the phases of $q_i^0(j\omega)$ are always less or equal to $\pm 180^\circ$ (since the phase shift of the pure integrator in $q_i^0(s)$ is constant and equal to $\pm 90^\circ$), which implies that the Nyquist plots of $q_i^0(j\omega)$ cannot encircle the critical point $(-1, j0)$, irrespectively of the value of the gain Γ. In turn, the root loci of the CTFs $q_i^0(s)$ (20) will tend to infinity, as $\Gamma \to \infty$, along the negative real semi-axis or along the asymptotes that are parallel to the imaginary axis and lie in the left half plane.

Summarizing, we have shown that the adaptive system in Figure 1 is stable for any strictly stable transfer matrix $W(s)$ and any value of the adaptation gain Γ. That feature ensues from the fact that the transfer matrix $W_B(s)$ (12) of the equivalent MIMO system in Figures 2 and 3 always belongs to the class of PR matrices.

Example. Consider a two-dimensional ($N = 2$) MIMO system with the transfer matrix

$$W(s) = \begin{bmatrix} \dfrac{500}{(s+2)(s+5)} & \dfrac{18.75}{(s+0.5)(s+10)} \\ \dfrac{25}{s+4} & \dfrac{112.5}{(s+1)(s+2)} \end{bmatrix} \tag{21}$$

and the matrix $Q(s) = [1/(0.1s+1)]I$.

The matrices $A, B,$ and C of the state-space representation of that system are

$$A = \begin{bmatrix} -2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 16 & 0 \\ 0 & 0 \\ 0 & 4 \\ 4 & 0 \\ 0 & 0 \\ 0 & 8 \end{bmatrix}, \tag{22}$$

$$C = \begin{bmatrix} 31.25 & 0 & 4.6875 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6.25 & 14.0625 & 0 \end{bmatrix}. \tag{23}$$

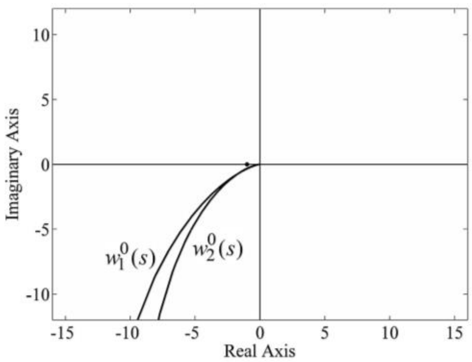
The solution P of the Lyapunov equation (6) for $Q = I$ is

$$P = \begin{bmatrix} 0.25 & 0.0357 & 0 & 0 & 0 & 0 & 0 \\ 0.0357 & 0.1071 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0.0952 & 0 & 0 & 0 \\ 0 & 0 & 0.0952 & 0.0595 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.125 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5000 & 0.1667 \\ 0 & 0 & 0 & 0 & 0 & 0.1667 & 0.3333 \end{bmatrix}. \tag{24}$$

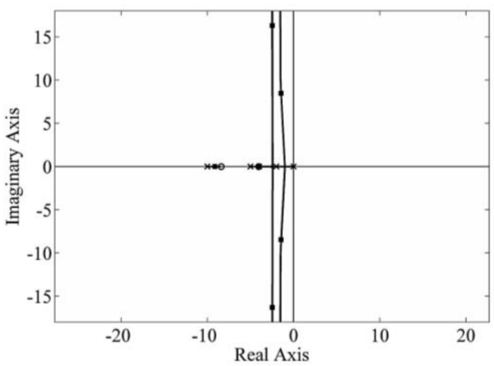
The transfer matrix $W_0(s)$ (12) for the system in (21)-(23) and the matrix P (24) is diagonal (see Appendix A) and equal to

$$W_0(s) = \frac{\Gamma}{s}\begin{bmatrix} \frac{272(s+4.059)}{(s+5)(s+4)} & 0 \\ 0 & \frac{80(s+8.4)}{(s+10)(s+2)} \end{bmatrix}. \quad (25)$$

The Nyquist plots, as well as the root loci of the diagonal elements $w_{11}^0(s)$ and $w_{22}^0(s)$ of the matrix $W_0(s)$ (25) are shown in Figure 5. As can be seen from the graphs in Figure 5, the two-dimensional adaptive system with the transfer matrix $W(s)$ (21) is stable for any value of the adaptation gain $\Gamma > 0$.



(a)



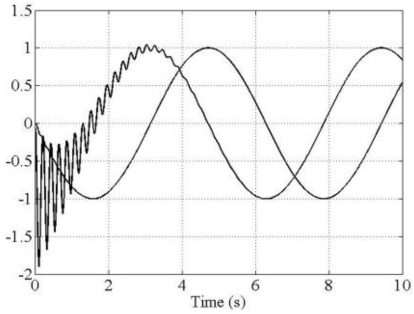
(b)

Fig. 5. Nyquist plots (a) and root loci (b) of $w_{11}^0(s)$ and $w_{22}^0(s)$

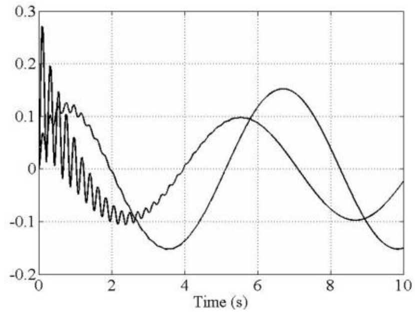
The results of simulation of the two-dimensional adaptive system with the help of *Simulink* for $r(t) = [0,0]^T$, sinusoidal disturbances with unit amplitudes in both channels and the period $T = 6.28\,s$, where the oscillations in the second channel are shifted by $+90$ degrees, and three different values of the adaptation gain Γ ($\Gamma = 50$, $\Gamma = 250$, and $\Gamma = 1000$), are shown in Figure 6. As can be seen from the transient responses, the behavior of the adaptive system agrees with the frequency and root characteristics of $W_0(s)$ (25).

The examination of the root loci in Figure 5(b) and of the graphs in Figure 6 allows one to reveal another specific feature of adaptive systems designed for rejection of external disturbances. From Figure 5(b), it is clear that if the relative degree of any of the CTFs $q_i^0(s)$ of the matrix $W_0(s)$ is 2, then, as the value of the adaptation gain Γ

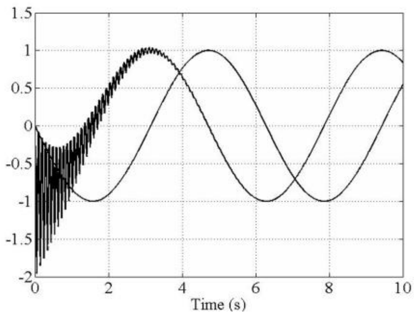
increases indefinitely, the corresponding root loci tend to infinity along the asymptotes that are parallel to the imaginary axis and lie in the left half plane. In other words, the imaginary parts and, consequently, the natural frequencies of the poles of the closed-loop system with the transfer matrix $F_{\Sigma}(s)$ in Figure 4 increase as the gain Γ increases. However, these poles do not have the opportunity to cross the imaginary axis to the right-half plane. Because the estimation loop is decoupled from the control loop, the external disturbance and noise cannot affect this pattern of the poles and have no opportunity to lead to instability.



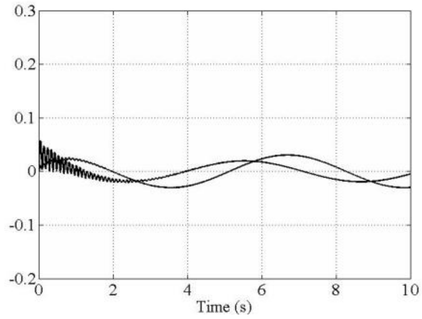
(a) *Control signal* $u(t)$



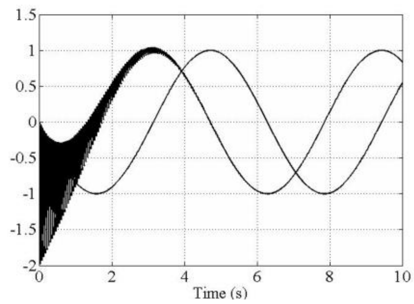
(b) *Output signal* $y(t)$



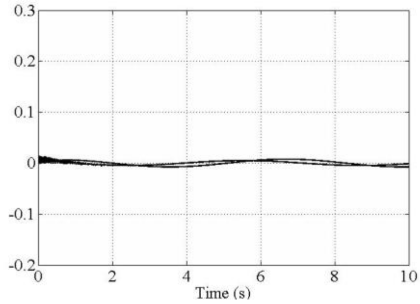
(c) *Control signal* $u(t)$



(d) *Output signal* $y(t)$



(e) *Control signal* $u(t)$



(f) *Output signal* $y(t)$

Fig. 6. Simulation results: (a),(b) $\Gamma = 50$ *; (c),(d)* $\Gamma = 250$ *; (e),(f)* $\Gamma = 1000$

Note now that the output signal $y(t)$ of the adaptive system represents, since $r(t)=0$, an undesirable (or error) motion, which is due to external disturbances. In this respect, as can be seen from Figure 6, the increase in Γ brings to smaller deviations of $y(t)$ from zero, i.e. to the higher performance of the L_1 adaptive system (the absolute maximum deviation of the output signals for $\Gamma=1000$ is more than 20 times as small as for $\Gamma=50$). The further increase in Γ will result in smaller errors tending to zero as $\Gamma\to\infty$.

Appendix A. The state vector $x(t)$ in (1) can be chosen as a combination of state spaces $x_{ij}(t)$ of all transfer functions $w_{ij}(s)$ forming the transfer matrix $W(s)=\{w_{ij}(s)\}$ (2). Let us assume that each scalar transfer function $w_{ij}(s)$ has the state-space representation

$$\dot{x}_{ij}(t)=A_{ij}x_{ij}(t)+b_{ij}u_j(t),\quad y_{ij}(t)=c_{ij}^T x_{ij}(t),\quad (i,j=1,2,...,N), \tag{A.1}$$

where the orders of the vectors $x_{ij}(t)$ are n_{ij}, the matrices A_{ij} are of size $n_{ij}\times n_{ij}$, and the sizes of column vectors b_{ij} and c_{ij} are $n_{ij}\times 1$**.** The system matrix A in that case has a block-diagonal structure with the diagonal matrix blocks A_{ij}, i.e. $A=diag\{A_{ij}\}$, and the positive definite matrix Q ($Q=Q^T>0$) in the Lyapunov equation (6) can be chosen block-diagonal and matching the structure of the matrix A, i.e. $Q=diag\{Q_{ij}\}$, where $Q_{ij}=Q_{ij}^T>0$ ($i,j=1,2,...,N$).

Under such conditions, the Lyapunov equation (6) reduces to the following set of N^2 equations

$$A_{ij}^T P_{ij}+P_{ij}A_{ij}=-Q_{ij}\quad (i,j=1,2,...,N). \tag{A.2}$$

It can be shown that, in such a case, the matrix $W_0(s)$ (12) is diagonal:

$$W_0(s)=diag\left\{w_{kk}^0(s)\right\}, \tag{A3}$$

with the diagonal elements

$$w_{kk}^0(s)=\frac{\Gamma}{s}\sum_{i=1}^{N} b_{ik}^T P_{ik}(sI-A_{ik})^{-1}b_{ik}\qquad (k=1,2,...,N). \tag{A.4}$$

Correspondingly, the transfer matrix $F_\Sigma(s)$ (13) also takes on the diagonal form:

$$F_\Sigma(s)=diag\left\{\frac{w_{kk}^0(s)}{1+w_{kk}^0(s)}\right\}. \tag{A.5}$$

and the characteristic equation (14)

$$\det\left[I + diag\left\{w_{kk}^0(s)\right\}\right] = \prod_{k=1}^{N}\left[1 + w_{kk}^0(s)\right] = 0 \tag{A.6}$$

splits up into N “one-dimensional” equations:

$$1 + \frac{\Gamma}{s}\sum_{i=1}^{N} b_{ik}^T P_{ik}(sI - A_{ik})^{-1} b_{ik} = 0 \quad (k = 1, 2, ..., N). \tag{A.7}$$

These equations allow to investigate the stability of the N-dimensional adaptive system in Figure 1 on the basis of standard methods of classical feedback control.

REFERENCES

1. **Hovakimyan N. and Cao. C.** L_1 Adaptive Control Theory - Guaranteed Robustness with Fast Adaptation. - SIAM, Philadelfia, 2010. – 340 p.
2. **Astrom K. and Wittenmark B.** Adaptive Control. - Addison-Wesley Longman Publishing Co., Inc., Boston, MA, 1994. -575 p.
3. **Ioannou P. and Sun J.** Robust Adaptive Control. - Prentice Hall, 1996. – 834 p.
4. **Skogestad S. and Postlethwaite I.** Multivariable Feedback Control. Analysis and Design. -John Wiley and Sons Ltd., Chichester, Sussex, UK, 2005. - 595 p.
5. **Gasparyan O. N.** Linear and Nonlinear Multivariable Feedback Control: A Classical Approach. - John Wiley & Sons, UK, 2008. - 354 p.
6. **Slotine J.-J. E. and Li W.** Applied Nonlinear Control. - Prentice Hall, Englewood Cliffs, USA, 1991. - 476 p.

SEUA (Polytechnic). The material is received 21.08.2014.

Տ.Ն. ՀՈՎՀԱՆՆԻՍՅԱՆ, Ն.Հ. ՎԱՐԴԱՆՅԱՆ, Ե.Ռ. ԽԱՐԻՍՈՎ, Օ.Ն. ԳԱՍՊԱՐՅԱՆ, Ն.Գ. ՀՈՎԱԿԻՄՅԱՆ

ԲԱԶՄԱՉԱՓ ԿԱՌԱՎԱՐՄԱՆ ՀԱՄԱԿԱՐԳԵՐՈՒՄ L1 ՀԱՐՄԱՐՎՈՂ ԿԱՌԱՎԱՐՄԱՆ ԿԻՐԱՌՈՒԹՅԱՆ ՎԵՐԱԲԵՐՅԱԼ

Մաս 1. Ընդհանուր տեսքի բազմաչափ համակարգեր

Դիտարկված են որոշ հարցեր, որոնք առնչվում են արտաքին վրդովմունքները չեզոքացնելու համար նախատեսված հարմարվող քառակուսի (այսինքն՝ մուտքերի և ելքերի միևնույն քանակն ունեցող) բազմաչափ կառավարման համակարգերի կայունությանը: Հիմնվելով դրական իրական փոխանցման մատրիցների հատկությունների վրա՝ ցույց է տրված, որ այդպիսի համակարգերը կայուն են հարմարման գործակցի կամայական մեծ արժեքների, նույնիսկ աջակողմյան զրոներով համակարգերի դեպքում:

Առանցքային բառեր. կառավարման բազմաչափ համակարգ, հարմարվող կառավարում, չափանմուշային մոդել, կայունություն, դրական իրական համակարգ:

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О ПРИМЕНЕНИИ L1 АДАПТИВНОГО УПРАВЛЕНИЯ В МНОГОМЕРНЫХ СИСТЕМАХ УПРАВЛЕНИЯ

Часть 1. Многомерные системы общего вида

Рассмотрены некоторые вопросы, связанные с устойчивостью адаптивных квадратных (т.е. имеющих равное число входов и выходов) многомерных систем управления, предназначенных для компенсации внешних возмущений. Основываясь на свойствах положительных действительных передаточных матриц, показано, что подобные системы устойчивы при произвольно больших значениях коэффициента адаптации, даже в случае систем с правосторонними нулями.

Ключевые слова: многомерная система управления, адаптивное управление, эталонная модель, устойчивость, положительная действительная система.