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THE EFFECT OF WORK HARDENING ON THE STRESS DISTRIBUTION IN DEEP DRAWING OF CYLINDRICAL CUP

A new approach to estimate of the influence of work hardening on stress distribution in deep drawing is introduced. As a result equations which show dependence of radial, circumferential, as well as caused by a blank holder stress on intensity of work hardening are explicitly obtained. The maximal stretching stress is also obtained. Diagrams which clearly show the character of stress distribution in the flat surface of flange are presented.

Keywords: deep drawing, work hardening, blank holder, stress distribution.

The nature of stress distribution in the flange of the specimen greatly depends on work hardening in the process of deep drawing. To carry out exact count of the influence of this factor, one will encounter some mathematical difficulties. Meanwhile, using some assumptions makes possible to obtain approximate equations revealing distribution of stresses in the flange. The accuracy of the obtained results depends on the degree of the assumption validity. Many authors, particularly, P. Hill [1], L.A. Shofman [2], E.A. Popov [3], D. C. Chiang, Shiro Kobayashi [4], etc., have devoted many papers to the investigation of this problem.

In these papers the influence of work hardening on the process of drawing was determined with one or another degree of approximation. In [2] the field of stress by taking into account work-hardening by means of averaging of tangential strain value by flange width in the arbitrary instant of deformation was defined. A solution is obtained with using the power dependence of the yielding stress on the strain. In this case two terms of series decomposition of the conditional strain ε_θ equation are used in calculations which narrows the application range of the result [3]. There exists more accurate analysis of the process applying the incremental strain theory. Calculations were conducted by means of the numerical integration. A practical application of the obtained results meet some difficulties [4]. Below a new approach to solve the problem is presented.

The problem under consideration is shown in Fig.1. The circular blank of original radius R_0 is deep drawn by the flat-bottomed punch through a die opening of radius r. Here the scheme of principal stresses acting on the element of flange is shown. To avoid the appearance of wrinkles, a blank holder is applied. The changing of the specimen thickness in the process of drawing is neglected.

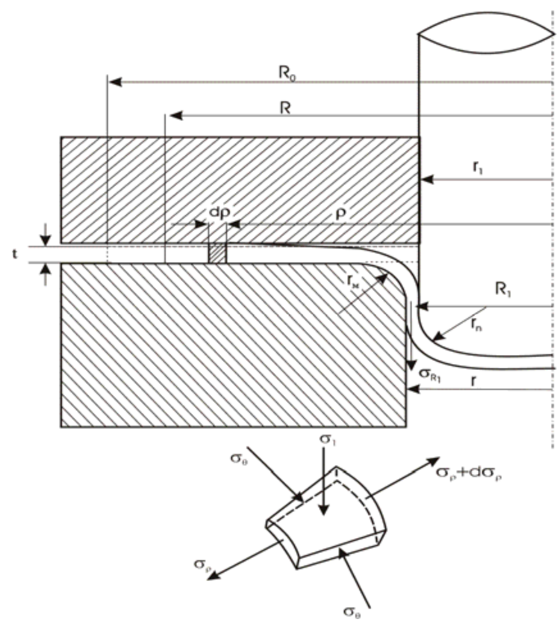

Fig. 1. Scheme of the Cylindrical Cup Deep Drawing Process

The equilibrium equation of an element of the flange is obtained in the form

$$\frac{d\sigma_\rho}{d\rho}+\frac{\sigma_\rho-\sigma_\theta}{\rho}=0\,, \tag{1}$$

where σ_ρ is the radial stress, and σ_θ is the circumferential stress. The yield condition is as follows:

$$\sigma_\rho-\sigma_\theta=\beta\sigma_s\,, \tag{2}$$

where β is the coefficient which takes into account the influence of the intermediate principal stress (in this case, stress σ_t caused by the blank holder.)

The work hardening characteristics of the material are assumed in the form

$$\sigma_s=A\varepsilon^n\,, \tag{3}$$

where A and n are constants which can be obtained from a simple tension test of a specimen cut from the sheet metal.

As a deformation ε we apply the circumferential strain ε_θ [1]. In the process of drawing, when original radius R_0 has moved to the intermediate radius R, the initial radius ρ_n of the element has moved to the current radius ρ. Then the logarithm deformation in circular direction is defined as

$$\varepsilon_\theta = \ln \frac{\rho_n}{\rho}.$$

To take into account the invariability of the specimen thickness, we can write:

$$R_0^2 - R^2 = \rho_n^2 - \rho^2 .$$

Hence, we have

$$\rho_n = \sqrt{R_0^2 - R^2 + \rho^2}.$$

Then

$$\varepsilon_{\theta_1} = \ln \sqrt{1 + \frac{R_0^2 - R^2}{\rho^2}} . \tag{4}$$

Combining equations (1) and (2) and using relations given by equations (3) and (4), we yield

$$d\sigma_\rho = -\beta A \left(\ln \sqrt{1 + \frac{R_0^2 - R^2}{\rho^2}} \right)^n \frac{d\rho}{\rho}. \tag{5}$$

The complexity of the obtained equation does not allow deriving formulas determining distribution of stresses in the flange. To solve the problem, it is suggested to replace the formula (4) by relation of the form

$$\varepsilon_\theta = \left(\frac{\overline{R}}{\overline{\rho}} \right)^{x\overline{R}} \ln \frac{1}{\overline{R}} , \tag{6}$$

where $\overline{R} = R / R_0$ and $\overline{\rho} = \rho / R_0$.

The value of x may be obtained by comparing ε_{θ_1} and ε_{θ_2} for different values of $\overline{R}$ and $\overline{\rho}$. In case, when $\overline{\rho} = \overline{R}$ which is equivalent to $\rho = R$, we have $\varepsilon_{\theta_1} = \varepsilon_{\theta_2} = \ln(1/\overline{R})$. The value of $x\overline{R}$ for $\overline{\rho} < R$ will look like as follows: when $\overline{R} = 0.95$ and $\overline{\rho} = 0.9;\ 0.8;\ 0.7;\ 0.6;\ 0.5,$ =0.9; 0.8; 0.7; 0.6; 0.5, then $x\overline{R}$ will be equal to 1.895; 1.89; 1.87; 1.85; 1.82, respectively. The average value of $x\overline{R}$ is 1.865. Then the value of x for $\overline{R} = 0.95$ will be 1.96. By the same way we can obtain the average value of x for $\overline{R} = 0.9$

and $\overline{R} = 0.8$ which are equal to 1.89 and 1.86, respectively. Then the average value of x for $\overline{R} = 0.95;\ 0.9;\ 0.8$ will be equal to 1.9.

Thus, equation (6) is written as

$$\varepsilon_{\theta_2} = \left(\frac{\overline{R}}{\overline{\rho}}\right)^{1.9\overline{R}} \ln\frac{1}{\overline{R}}. \tag{7}$$

The equation (7) in the range of $1 < K < 2.7$ with any value of $\overline{R}$ and $\overline{\rho}$ does not actually differ from ε_{θ_1} from the formula (4) ($K = R_0 / R_1$ is drawing ratio).

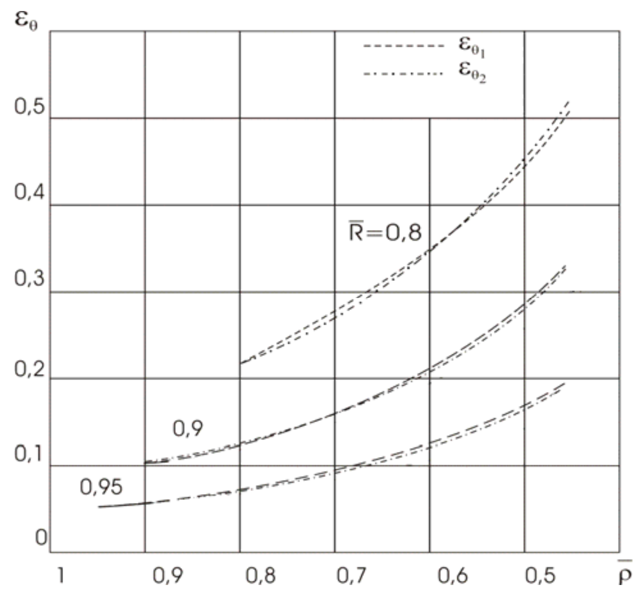

Fig. 2. Circumferential strain ε_{θ_1} and relation ε_{θ_2} as functions of $\overline{\rho}$ for several values of $\overline{R}$

In Fig. 2 are shown ε_{θ_1} and ε_{θ_2} as functions of $\overline{\rho}$ for various values of $\overline{R}$. When we replace ε_{θ_1} in the equation (5) by relation (6) we get the following result

$$d\sigma_\rho = -\beta A\left(\frac{\overline{R}}{\overline{\rho}}\right)^{\varphi\overline{R}} \ln^n \frac{1}{R}\frac{d\rho}{\rho},$$

where $\varphi = 1.9n$.

After integrating and using the boundary condition $\sigma_\rho = 0$, when $\overline{\rho} = \overline{R}$, we have

$$\frac{\sigma_\rho}{A} = \frac{\beta}{\varphi \bar{R}} \left[\left(\frac{\bar{R}}{\bar{\rho}} \right)^{\varphi \bar{R}} - 1 \right] \ln^n \frac{1}{\bar{R}} \tag{8}$$

and the yield condition is

$$\frac{\sigma_\theta}{A} = \frac{\beta}{\varphi \bar{R}} \left[\left(\frac{\bar{R}}{\bar{\rho}} \right)^{\varphi \bar{R}} (1 - \varphi \bar{R}) - 1 \right] \ln^n \frac{1}{\bar{R}} . \tag{9}$$

The obtained equations are good approximation to the problem solving when the hard blank holder is applied; clearance between the blank holder and surface of the die is equal to the thickness of the material and it is considered that the process of drawing take place in the condition of plane strain. In this case $\beta = 2/\sqrt{3}$.

In Fig. 3 and 4 the radial and circumferential stress distributions are shown for various values of *n* at several stages of drawing.

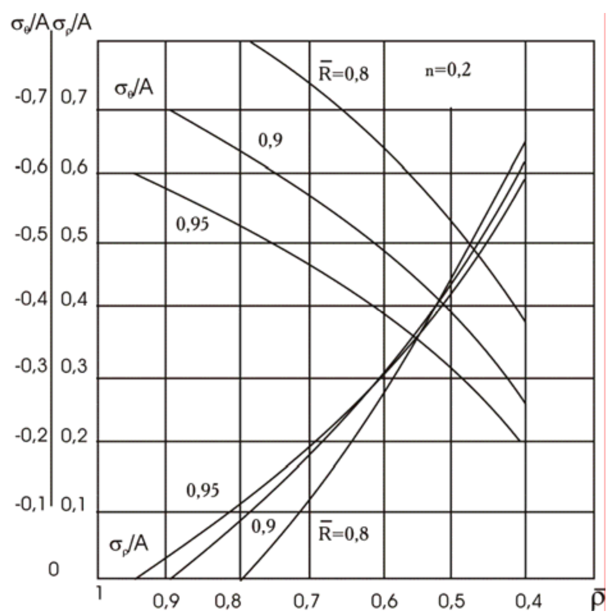

Fig. 3. Distribution of radial and circumferential stresses at several stages of drawing for $n = 0.2$

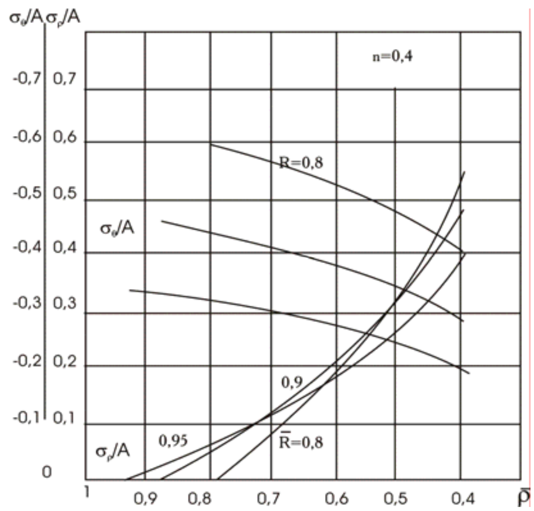

Fig. 4. Distribution of radial and circumferential stresses at several stages of drawing for $n = 0.4$

The stress σ_t may be obtained from the condition $\sigma_t = 1/2(\sigma_\rho + \sigma_\theta)$:

$$\frac{\sigma_t}{A} = \frac{\beta}{\varphi\bar{R}}\left[\left(\frac{\bar{R}}{\bar{\rho}}\right)^{\varphi\bar{R}}(1-0.5\varphi\bar{R})-1\right]\ln^n\frac{1}{\bar{R}}. \tag{10}$$

Thus, equations (8), (9), and (10) may be recommended for determining the field of stresses in the deep drawing.

The radial stress σ_ρ reaches its greatest value in the edge of the die, when $\bar{\rho} = \bar{R}_1$ ($\bar{R}_1 = R_1 / R_0$). Substitution of ρ in the equation (8) by R_1 results in

$$\frac{\sigma_{R_1}}{A} = \frac{\beta}{\varphi\bar{R}}\left[\left(\frac{\bar{R}}{\bar{R}_1}\right)^{\varphi\bar{R}}-1\right]\ln^n\frac{1}{\bar{R}}$$

or

$$\frac{\sigma_{R_1}}{A} = \frac{\beta}{\varphi\bar{R}}\left[\left(\bar{R}K\right)^{\varphi\bar{R}}-1\right]\ln^n\frac{1}{\bar{R}}. \tag{11}$$

As it is seen from the equation (11), σ_{R_1} increases from zero at the beginning of the process when $\bar{R} = 1$ reaching the maximum somewhere for $1 > \bar{R} > 1/K$, becomes zero when $\bar{R} = 1/K$.

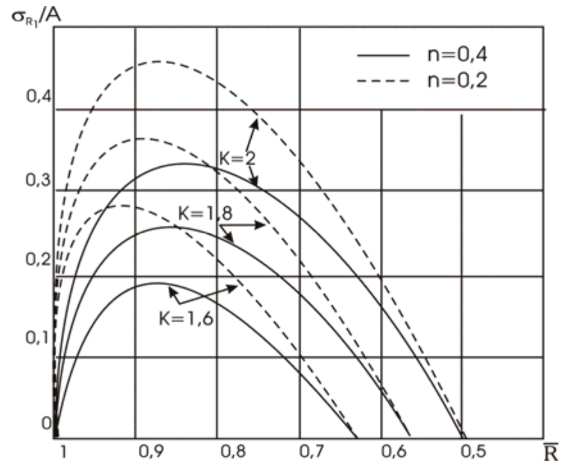

Fig. 5. Diagrams of σ_{R_1} as a function of $\overline{R}$ for several values of n and K

In the Fig. 5 are shown diagrams of σ_{R_1} as a function of $\overline{R}$ for several values of n and K. One can see in the Figure that σ_{R_1} is higher for smaller value of n for the same values of K. The point where σ_{R_1} reaches its maximum moves toward the end of the process with increasing of the work hardening coefficient n and drawing ratio K. In [4] diagrams identical to those shown in Fig. 3, 4 and 5 are constructed.

The value $\overline{R} = \overline{R}_e$ where σ_{R_1} reaches its extremum can be calculated if derivative of σ_{R_1} by $\overline{R}$ is equalized to zero and the obtained equation is solved in respect to $\overline{R}$. However, the availability of this parameter in the exponent of the formula (11) makes it difficult to get acceptable equation for $\overline{R}_e$. Therefore, remaining technological parameters – extremal value of the greatest stretching stress, drawing force, limiting drawing ratio – are not here defined. They are presented in [5], where the formula (6) with fixed exponent is applied.

If in formulas (8), (9), (10) and (11) we take $n = 0$ (lack of work hardening), then after opening of uncertainty, they are transformed as follows:

$$\frac{\sigma_\rho}{\sigma_s} = \beta \ln \frac{R}{\rho}, \quad \frac{\sigma_\theta}{\sigma_s} = \beta\left(\ln \frac{R}{\rho} - 1\right), \quad \frac{\sigma_t}{\sigma_s} = \beta\left(\ln \frac{R}{\rho} - \frac{1}{2}\right) \text{ and } \frac{\sigma_{R_1}}{\sigma_s} = \beta \ln\left(\overline{R}K\right).$$

We represent the equation (8) in the following way

$$\frac{\sigma_\rho}{A} = \frac{\beta}{\varphi \overline{R}}\left[\left(\frac{\overline{R}}{\overline{\rho}}\right)^{\varphi \overline{R}} \ln^n \frac{1}{\overline{R}} - \ln^n \frac{1}{\overline{R}}\right]. \tag{12}$$

The left side of the expression in brackets represents deformation ε_{θ_2} expressed by the formula (7) raised to the *n*-th power. It replaces the deformation ε_{θ_1} in the differential equation (5). Let us make an inverse replacement in formula (12), i.e. replace ε_{θ_2} by ε_{θ_1}. Then it will be written as

$$\frac{\sigma_\rho}{A} = \frac{\beta}{\varphi \bar{R}} \left[\ln^n \sqrt{1 + \frac{1 - \bar{R}^2}{\bar{\rho}^2}} - \ln^n \frac{1}{\bar{R}} \right]. \tag{13}$$

Making the same replacements in formulas (9) and (10) we obtain

$$\frac{\sigma_\theta}{A} = \frac{\beta}{\varphi \bar{R}} \left[\ln^n \sqrt{1 + \frac{1 - \bar{R}^2}{\bar{\rho}^2}} \left(1 - \varphi \bar{R}\right) - \ln^n \frac{1}{\bar{R}} \right], \tag{14}$$

$$\frac{\sigma_t}{A} = \frac{\beta}{\varphi \bar{R}} \left[\ln^n \sqrt{1 + \frac{1 - \bar{R}^2}{\bar{\rho}^2}} \left(1 - 0.5\varphi \bar{R}\right) - \ln^n \frac{1}{\bar{R}} \right]. \tag{15}$$

The greatest stress σ_{R_1} will be obtained if we substitute $\bar{\rho}$ by $\bar{R}_1$ in the expression (13):

$$\frac{\sigma_{R_1}}{A} = \frac{\beta}{\varphi \bar{R}} \left[\ln^n \sqrt{1 + K^2 \left(1 - \bar{R}^2\right)} - \ln^n \frac{1}{\bar{R}} \right]. \tag{16}$$

Conducted transformations reduce to minimum the error obtained by problem solving with changing ε_{θ_1} by ε_{θ_2} in the differential equation (5).

Equations (8), (9), (10), as well as equations (13), (14), (15) can be written in more simple way by the following formulas

$$\frac{\sigma_\rho}{A} = \frac{\beta}{\varphi \bar{R}} \left(\varepsilon^n{}_\theta - \ln^n \frac{1}{\bar{R}} \right), \tag{17}$$

$$\frac{\sigma_\theta}{A} = \frac{\beta}{\varphi \bar{R}} \left[\varepsilon^n{}_\theta \left(1 - \varphi \bar{R}\right) - \ln^n \frac{1}{\bar{R}} \right], \tag{18}$$

$$\frac{\sigma_t}{A} = \frac{\beta}{\varphi \bar{R}} \left[\varepsilon^n{}_\theta \left(1 - 0.5\varphi \bar{R}\right) - \ln^n \frac{1}{\bar{R}} \right]. \tag{19}$$

If we replace in the equations (17), (18), (19) ε_θ by ε_{θ_2} defined by the formula (7), then we get formulas (8), (9), (10). If we substitute ε_{θ_1} defined by (4), then we get expressions (13), (14), (15). These equations are practically identical. Some divergence is occurred by $\bar{\rho} \leq 0.45$. The difference between formulas (11) and (14) is marked when $K \geq 2.2$. In other cases they are equivalent.

REFERENCES

1. **Hill R.** Mathematical Theory of Plasticity. - Oxford, England: the Clarendon press, 1950.
2. **Shofman L.A.** The Theory and Calculation of Cold Stamping Processes. - Moskow: Mashinostroenie, 1964.
3. **Popov E.A.** The Principles of the Theory of the Sheet Metal Stamping. - Moskow: Mashgiz, 1977.
4. **Chiang D.C., Kobayashi Shiro.** The Effect of Anisotropy and Work-hardening Characteristics on the Stress and Strain Distribution in Deep Drawing // Journal of Engineering for Industry, Transactions of the ASME.-1966. - Vol. 99, No 4 - P. 443-448.
5. **Stepanyan R.L.** The Deep Drawing of Sheet Metal. – Yerevan: Publishing House "Gitutyun", NAS of the RA, 2001- P. 26-31.

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Ռ.Լ. ՍՏԵՓԱՆՅԱՆ

ԱՄՐԱՑՄԱՆ ԱԶԴԵՑՈՒԹՅՈՒՆԸ ԼԱՐՈՒՄՆԵՐԻ ԲԱՇԽՄԱՆ ՎՐԱ ԳԼԱՆԱՁԵՎ ԲԱԺԱԿԻ ԽՈՐ ԱՐՏԱՁԳՄԱՆ ԴԵՊՔՈՒՄ

Ներկայացված է նոր մոտեցում խոր արտաձգման գործընթացում լարումների բաշխման վրա ամրացման ազդեցությունը բացահայտելու համար: Արդյունքում ստացվել են բացահայտ տեսքով բանաձևեր, որոնք տալիս են շառավղային, շրջանային և սեղմիչի ճնշումից առաջացած լարման կախվածությունը ամրացման ինտենսիվությունից: Ստացվել է նաև մաքսիմալ ձգող լարման բանաձևը: Ներկայացված են դիագրամներ, որոնք պարզորեն ցույց են տալիս կցաշուրթի հարթ մասում լարումների բաշխման բնույթը:

Առանցքային բառեր. խոր արտաձգում, ամրացում, սեղմիչ, լարումների բաշխում:

Р.Л. СТЕПАНЯН

ВЛИЯНИЕ УПРОЧНЕНИЯ НА РАСПРЕДЕЛЕНИЕ НАПРЯЖЕНИЙ ПРИ ГЛУБОКОЙ ВЫТЯЖКЕ ЦИЛИНДРИЧЕСКОГО СТАКАНА

Представлен новый подход к определению влияния упрочнения на распределение напряжений в процессе глубокой вытяжки. Получены в явном виде формулы, показывающие зависимость радиального, окружного, а также вызванного прижимом напряжения от интенсивности упрочнения. Выведены также формулы для определения максимального растягивающего напряжения. Представлены диаграммы, показывающие характер распределения напряжения в плоской части фланца.

Ключевые слова: глубокая вытяжка, упрочнение, прижим, распределение напряжения.