UDC 539.3 MECHANICAL ENGINEERING

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STRESS DEFORMED STATE OF PIEZOELASTIC PLATE UNDER CROSSCUT ELECTRIC FIELD

The equations of generalized plane tense condition and bending of piezoactive elastic plate made of material of class 6mm polarized in the thickness are gained on the basis of Kirchhoff hypothesis. The analogy is defined between the problems of thermoelastic and piezoactive plates in the case when the temperature field and the electric field strength broadside directed on the preface of the plate are assigned. Tense deformed state of the bending plate about a fastened edge of a plate is investigated.

Keywords: piezoelectric, elastic plate, thermoelastic, bend, analogy.

1.It is assumed that the plate in the rectangular Cartesian coordinate systems (x,y,z) occupies the field $0<x\le a$, $0\le y\le a$, $-h\le z\le h$. The plate is made of elastic piezoactive materials of class 6mm and polarized on the coordinate z. The equations of piezoelastic material are given in the articles [1], [2] and have the following form:

$$\sigma_{11}=C_{11}\varepsilon_{11}+C_{12}\varepsilon_{22}+C_{13}\varepsilon_{33}-e_{31}E_3,\sigma_{22}=C_{12}\varepsilon_{11}+C_{11}\varepsilon_{22}+C_{13}\varepsilon_{33}-e_{31}E_3,\quad \sigma_{33}=C_{13}\varepsilon_{11}+C_{13}\varepsilon_{22}+C_{33}\varepsilon_{33}-e_{33}E_3, \tag{1.1}$$

$$\sigma_{12}=1/2(C_{11}-C_{12})\,\varepsilon_{12},\ \sigma_{23}=C_{44}\varepsilon_{23}-e_{15}E_2,\sigma_{31}=C_{44}\varepsilon_{31}-e_{15}E_1, \tag{1.2}$$

$$D_1=\varepsilon_4E_1+e_{15}\varepsilon_{13},\ D_2=\varepsilon_1E_2+e_{15}\varepsilon_{23},D_3=\varepsilon_3E_3+e_{31}(\varepsilon_{11}+\varepsilon_{22})+e_{33}\varepsilon_{33}. \tag{1.3}$$

In order to make analogy with thermo elastic problems of plate theories it is con- venient to express deformation through tension. Thus it follows from equation (1.1):

$$\varepsilon_{11}=\frac{1}{E}\sigma_{11}-\frac{\nu}{E}\sigma_{22}-\frac{\nu'}{E'}\sigma_{33}+\alpha_eE_3,\varepsilon_{22}=-\frac{\nu}{E}\sigma_{11}+\frac{1}{E}\sigma_{22}-\frac{\nu'}{E'}\sigma_{33}+\alpha_eE_3,\quad \varepsilon_{33}=-\frac{\nu'}{E'}\sigma_{11}-\frac{\nu'}{E'}\sigma_{22}-\frac{1}{E'}\sigma_{33}+\beta_eE_3. \tag{1.4}$$

In case (1.4) the following symbols are accepted:

$$E=(C_{11}-C_{12})\left(1+\frac{C_{12}C_{33}-C_{13}^2}{C_{11}C_{33}-C_{13}^2}\right),\quad \nu=\frac{C_{12}C_{33}-C_{13}^2}{C_{11}C_{33}-C_{13}^2},$$

$$E'=\frac{(C_{11}+C_{12})C_{33}-2C_{13}^2}{C_{11}+C_{12}},\quad \nu'=\frac{C_{13}}{C_{11}+C_{12}},$$

$$\alpha_e=\frac{1-\nu}{E}e_{31}-\frac{\nu'}{E'}e_{33},\quad \beta_e=\frac{1}{E'}(e_{33}-2\nu'e_{31}).$$

From (1.2) and (1.3) it goes the following:

$$\varepsilon_{12}=\frac{2}{C_{11}-C_{12}}\sigma_{12},\quad \varepsilon_{23}=\frac{1}{C_{44}}(\sigma_{23}+e_{15}E_2),\varepsilon_{31}=\frac{1}{C_{44}}(\sigma_{31}+e_{15}E_1), \tag{1.5}$$

$$D_1 = \varepsilon_1(1+\lambda)E_1 + \frac{e_{15}}{C_{44}}\sigma_{31}, D_2 = \varepsilon_1(1+\lambda)E_2 + \frac{e_{15}}{C_{44}}\sigma_{32},$$
$$D_3 = \varepsilon_3' E_3 + \alpha_e(\sigma_{11}+\sigma_{22}) + \beta_e\sigma_{33}, \tag{1.6}$$

where

$$\lambda = \frac{e_{15}^2}{\varepsilon_1 C_{44}}, \quad \varepsilon_3' = \varepsilon_3 + 2\frac{1-\nu}{E}e_{31}^2 - \frac{4\nu'}{E'}e_{31}e_{33}.$$

The above listed forms of substance equations are convenient for problem solving of plate bending [3]. In accordance with Kirchhoff theory of plates the tension of σ_{33} is ignored and $\sigma_{31} = \sigma_{32} = 0$ accepted in correlations (1.4), (1.5), (1.6). New correlations will have the following form:

$$\varepsilon_{11} = \frac{1}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} + \alpha_e E_3, \varepsilon_{22} = -\frac{\nu}{E}\sigma_{11} + \frac{1}{E}\sigma_{22} + \alpha_e E_3,$$

$$\varepsilon_{33} = -\frac{\nu'}{E'}\sigma_{11} - \frac{\nu'}{E'}\sigma_{22} + \beta_e E_e,$$

$$\varepsilon_{12} = \frac{2}{C_{11}-C_{12}}\sigma_{12}, \ \varepsilon_{23} = \frac{e_{15}}{C_{44}}E_2, \ \varepsilon_{31} = \frac{e_{15}}{C_{44}}E_1,$$

$$D_1 = \varepsilon_1(1+\lambda)E_1, \ D_2 = \varepsilon_1(1+\lambda)E_2, D_3 = \varepsilon_3' E_3 + \alpha_e(\sigma_{11}+\sigma_{22}).$$

Averaged equations of equilibrium have the following form [3]:

$$\frac{\partial T_1}{\partial x} + \frac{\partial S}{\partial y} = 0, \ \frac{\partial S'}{\partial x} + \frac{\partial T_2}{\partial y} = 0, \tag{1.7}$$

$$\frac{\partial M_1}{\partial x} + \frac{\partial H}{\partial y} - N_1 = 0, \ \frac{\partial H}{\partial x} + \frac{\partial M_2}{\partial y} - N_2 = 0, \frac{\partial N_1}{\partial x} + \frac{\partial N_2}{\partial y} = 0, \tag{1.8}$$

where

$$T_k = \int_{-h}^{h}\sigma_{kk}dz, \ S = \int_{-h}^{h}\sigma_{12}dz, \ k=1,2, \tag{1.9}$$

$$M_k = \int_{-h}^{h} z\sigma_{kk}dz, \ H = \int_{-h}^{h} z\sigma_{12}dz, \ N_k = \int_{-h}^{h}\sigma_{k3}dz.$$

Defining the main tensions from (1.4) and using the links

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

and substituting into (1.9), (1,4) we will define forces and moments which will have the following form:

$$T_1 = C\left(\frac{\partial u}{\partial x} + \nu\frac{\partial v}{\partial y} - Q_e\right), \ S = \frac{1-\nu}{2}C\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right), T_2 = C\left(\frac{\partial v}{\partial y} + \nu\frac{\partial u}{\partial x} - Q_e\right), \tag{1.10}$$

$$M_1 = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2} + R_e\right), H = -(1-\nu)D\frac{\partial^2 w}{\partial y\partial x}, M_2 = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2} + R_e\right). \tag{1.11}$$

In formulas (1.10), (1.11) longitudinal and flexural rigidity is accepted in the following form:

$$C = \frac{2Eh}{1-\nu^2}, \quad D = \frac{2Eh^3}{3\left(1-\nu^2\right)}.$$

Q_e, R_e are the volumetric forces and moments caused by the electric field

$$Q_e = \frac{1+\nu}{2h}\alpha_e \int_{-h}^{h} E_3 dz, \quad R_e = \frac{3(1+\nu)}{2h^3}\alpha_e \int_{-h}^{h} zE_3 dz.$$

Substitution (1.10) into (1.7) gives the equations of generalized plane tense state of the plate in the displacements.

$$\frac{\partial^2 u}{\partial x^2} + \frac{1+\nu}{2}\frac{\partial^2 v}{\partial x \partial y} + \frac{1-\nu}{2}\frac{\partial^2 u}{\partial y^2} - \frac{\partial Q_e}{\partial x} = 0, \frac{\partial^2 v}{\partial y^2} + \frac{1+\nu}{2}\frac{\partial^2 u}{\partial x \partial y} + \frac{1-\nu}{2}\frac{\partial^2 v}{\partial x^2} - \frac{\partial Q_e}{\partial y} = 0.$$

Equations of plate bending problem corresponding to equations (1.8), (1.11) have the following form:

$$\frac{\partial N_1}{\partial x} + \frac{\partial N_2}{\partial y} = 0, D\frac{\partial}{\partial x}\left(\Delta w + R_e\right) + N_1 = 0, D\frac{\partial}{\partial y}\left(\Delta w + R_e\right) + N_2 = 0. \quad (1.12)$$

The second and third equations in the system (1.12) define forces N_1, N_2. Excluding N_1 and N_2 from (1.12) we get the formula which defines deflection of a plate

$$\Delta^2 w + \Delta R_e = 0. \quad (1.13)$$

The correlations and equations listed herein have the same forms as the correspon- ding expressions of problems of thermo elastic plates [4]. Hence the problem of thermo elastic plates is got if force Q_e and moment R_e are replaced by Q_t and R_t

$$Q_t = \frac{(1+\nu)}{2h}\alpha_t \int_{-h}^{h} T dz, R_t = \frac{3(1+\nu)}{2h^3}\alpha_t \int_{-h}^{h} zT dz. \quad (1.14)$$

In the formula (1.14) the function T=T(x,y,z) defines thermo field, where α_t is the coefficient of heat expansion. Hence if the temperature field is known T=T(x,y,z) and the electric field strength broadside are directed to the preface of the plate $E_3(x,y,z)$, then the problem of thermo elastic and piezoelastic plates are solved through the same equations and boundary conditions.

2. If one of the sides of rectangular plate is much bigger than the other (a>>b), it is ordinary to assume that the conditions of fastened edge x=a will not influence the tense deformed condition of piezoelastic plate about x=0. Taking into consideration this circumstance Nadai reviewed the problems of bending of the plate-line under the influence of uniform distributed load. It is assumed that when removing from the edge of a plate the function of plate bending approximates to the function of bending by cylinder surface. That is function of bending does not depend on coordinates x. Solving of Nadai's problem is given in monograph [6].

Let us assume that piezoactive (or thermo elastic) plate occupies the field of $0 \le x < \infty$, $0 \le y \le b$, $-h \le z \le h$. It is assumed that the edges of plate y=0, b are hingedly supported

$$w=0, \quad M_y=0$$

or

$$w = 0, \; \partial^2 w / \partial y^2 + R_i = 0. \quad (2.1)$$

If we assume that the form of bending surface of plate does not depend on coordinate x then solving of equation (1.13), satisfying conditions (2.1) will have the view

$$w_0 = 0.5y(b-y). \qquad (2.2)$$

Following Nadai's idea for semi-infinite strip under consideration the following requirement is set

$$\lim_{x\to 0} w(x,y) = w_0(x,y).$$

Problem solving (1.13) is introduced in the form

$$w = F(x,y) + w_0(x,y).$$

According to (1.13), (2.1) and (2.2) function F should satisfy the equation

$$\Delta^2 F = 0 \qquad (2.3)$$

and boundary conditions

$$F = 0,\ \partial^2 F/\partial y^2 = 0,\ \ y = 0, b \qquad (2.4)$$

and

$$\lim_{x\to 0} F = 0. \qquad (2.5)$$

Solving of equation (2.3) satisfying boundary conditions (2.4) is presented in the form

$$F = \sum_{n=0}^{\infty} f_n(x)\sin\lambda_n y,\ \lambda_n = \pi n/b. \qquad (2.6)$$

Substitution of (2.6) into (2.3) derives the equation of the fourth order relative to $f_n(x)$, general solving of which satisfying condition (2.5) has the form:

$$f_n(x) = C_1 e^{-\lambda_n x} + C_2 x e^{-\lambda_n x}.$$

Final solving of equation (1.13) satisfying boundary conditions (2.1) and conditionally (2.2) will be:

$$w = \sum_{n=0}^{\infty} (C_1 + C_2) e^{-\lambda_n x} \sin\lambda_n y + 0.5R_i y(b-y). \qquad (2.7)$$

3. Let the edge of a plate x=0 is fastened hard (clumped):

$$w = 0, \partial w/\partial x = 0. \qquad (3.1)$$

Using decomposition

$$y(y-b) = \sum_{n=1}^{n} b_n \sin\lambda_n y \qquad (3.2)$$

and substituting (2.11) into conditions (3.1) the following will be derived:

$$C_3 = -0.5R_i b_n,\ C_4 = -0.5R_i \lambda_n b_n.$$

Expression for function of bending is got in the form:

$$w = 0.5\sum_{n=1}^{\infty} b_n [1 - (1+\lambda_n x)e^{-\lambda_n x}]\sin\lambda_n y. \qquad (3.3)$$

Having the function of bending (3.3) it is not difficult to calculate moments and forces.
In particular

$$M_1(0,y) = -(1-\nu)R_i D\left[1 - \frac{1+\nu}{2(1-\nu)}\sum_{n=1}^{\infty}\lambda_n^2 b_n \sin\lambda_n y\right]. \qquad (3.4)$$

If in (3.4) the second derivative equation (3.2) is used

$$[y(y-b)]'' = -\sum_{n=1}^{n} \lambda_n^2 b_n \sin\lambda_n y ,$$

then formula (3.5) will be registered in the following form:

$$M_1(0, y) = 2\nu R_i D .$$

Let us also present expressions of generalized transversal forces on the edges of a plate (support reactions)

$$\tilde{N}_1(0, y) = -\nu D \sum_{n=1}^{\infty} \lambda_n^3 b_n \sin\lambda_n y,$$

$$\tilde{N}_2(x,0) = -\tilde{N}_y(x, b) = 0.5 R_i D \sum_{n=1}^{\infty} \lambda_n^3 b_n [1-(1+\lambda_n x)] e^{-\lambda_n x} .$$

From formula (3.8) it follows

$$\tilde{N}_1(0,0) = \tilde{N}_1(0,b) = \tilde{N}_2(0,0) = \tilde{N}_2(0,b) = 0 . \qquad .$$

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ՊԻԵԶՈԱՌԱՁԳԱԿԱՆ ՍԱԼԻ ԼԱՐՎԱԾԱԴԵՖՈՐՄԱՑՎԱԾ ՎԻՃԱԿԸ ԼԱՅՆԱԿԱՆ ԷԼԵԿՏՐԱԿԱՆ ԴԱՇՏՈՒՄ

Կիրխհոֆի վարկածի հիման վրա ստացված են 6 *մմ* դասի պյիեզոակտիվ նյութից պատրաստված, լայնական կոորդինատով բևեռացված առաձգական սալի ընդհանրացված հարթ լարվածային վիճակի և ծռման հավասարումները: Սահմանված է թերմոառաձգական և պիյեզոակտիվ սալերի խնդիրների նմանակությունը:

Առանցքային բառեր. պիզոէլեկտրիկ, առաձգական սալ, ջերմաառաձգականություն, ծռում, նմանակություն:

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НАПРЯЖЕННО-ДЕФОРМИРОВАННОЕ СОСТОЯНИЕ ПЬЕЗОУПРУГОЙ ПЛАСТИНЫ В ПОПЕРЕЧНОМ ЭЛЕКТРИЧЕСКОМ ПОЛЕ

На основе гипотез Кирхгофа получены уравнения обобщённого плоского напряжённого состояния и изгиба пьезоактивной упругой пластинки из материала класса 6 *мм*, поляризованной по толщинной координате. Установлена аналогия между задачами термоупругой и пьезоактивной пластин.

Ключевые слова: пьезоэлектрик, упругая пластина, термоупругость, изгиб, аналогия.