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COSMOLOGICAL EVOLUTION WITH NEGATIVE ENERGY DENSITIES

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For general number of spatial dimensions we investigate the cosmological dynamics driven by a cosmological constant and by a source with barotropic equation of state. It is assumed that for both those sources the energy density can be either positive or negative. Exact solutions of the cosmological equations are provided for flat models. For models with curved space and with zero cosmological constant the general solutions are expressed in terms of the hypergeometric function. The qualitative evolution is described for all values of the equation of state parameter. We specify the values of that parameter and the combinations of the signs for the cosmological constant and matter energy density for which the cosmological dynamics is nonsingular. An example is considered with positive cosmological constant and negative matter energy density induced by the polarization of the hyperbolic vacuum.

Keywords: *cosmological evolution: cosmological constant: negative energy density*

1. *Introduction*. The investigation of cosmological dynamics is carried out mainly within the framework of homogeneous and isotropic models described by Friedmann-Robertson-Walker (FRW) line element. In particular, the models containing a positive cosmological constant in addition to the matter and radiation sources of the expansion have been actively studied. This theoretical activity is motivated by the observational evidence [1,2] for accelerated expansion of the universe in recent epoch driven by a source (dark energy) with properties close to a positive cosmological constant. The cosmological model with a positive cosmological constant and cold dark matter (CDM) in addition to the usual matter (ΛCDM model) is in good agreement with observational data on the large scale structure and dynamics of the universe. Recently a problem appeared that is related to the value of the Hubble parameter H_0 at present determined by two different ways. The first one is based on direct low redshift observations [3-6] and gives the value $H_0 \approx 73$ km/s Mpc. The second way combines the Planck data on temperature anisotropies of the cosmic microwave background radiation [7] with the ΛCDM model and gives the result $H_0 \approx 67$ km/s/ Mpc. A number of models have been discussed in the literature to address this problem, also called Hubble tension (for a review see [8]). In particular, they include the models with negative cosmological constant (see [9-13]). The maximally symmetric solution of the

Einstein field equations with a negative cosmological constant as the only source of the gravitation is given by anti-de Sitter (AdS) spacetime. This geometry appears as a ground state in string theories and in supergravity. It plays an important role in braneworld models with large extra dimensions and in holographic duality models relating two theories living in different numbers of spatial dimensions. An example of the latter is the AdS/CFT correspondence (see, for example, [14]) establishing the duality between supergravity and string theories on the AdS bulk and conformal field theory on the AdS boundary.

Another example for a gravitational source with the negative energy density, that can play an important role in the expansion of the early universe, is provided by the vacuum polarization. The vacuum expectation value of the energy-momentum tensor for quantum fields may break the energy conditions of the singularity theorems in general relativity (see, e.g., [15]). This can serve as the key for solving the singularity problems in the cosmological dynamics. Here we consider the cosmological dynamics for both cases of positive and negative energy densities. Various combinations of cosmological constant and of a source with barotropic equation of state will be studied. Having in mind possible applications in higher-dimensional models, in particular, motivated by string theories, the discussion is presented for a general number of spatial dimensions. The qualitative evolution in cosmological models with scalar fields having negative potentials has been considered in [16-19]. Various cases of exact solutions to Friedmann equations in general number of spatial dimensions were discussed in [20] by using Chebyshev' theorem. Cosmological solutions in (3+1)-dimensional spacetime with a single positive and negative energy component in a flat universe and for a negative energy component in a curved universe have been described in [21].

The present paper is organized as follows. In the next section we present the cosmological equations and some qualitative features. The solutions for flat model with a cosmological constant and barotropic matter are given in section 3. They serve as past or future attractors for models with curved space and include various special cases previously considered in the literature. In section 4 we discuss models with curved space. First, the general solutions are presented in terms of the hypergeometric function for models with zero cosmological constant. Various special cases where the time-dependence of the scale factor is expressed in terms of elementary functions were discussed in the literature. Then we describe the qualitative evolution in models with curved space driven by a cosmological constant and barotropic matter source.

2. *Cosmological equations.* We consider $(D+1)$-dimensional background spacetime described by the FRW line element

$$ds^2 = N^2(t)dt^2 - a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\Omega_{D-1}^2\right), \tag{1}$$

where $d\Omega_{D-1}^2$ is the line element on a unit sphere S^{D-1} and $k = 0; \pm 1$. The choices $N(t) = 1$ and $N(t) = a(t)$ correspond to the synchronous and conformal time coordinates, respectively. Depending on the equation of state the first or the second choice of the time coordinate is convenient to present the cosmological solutions in simpler form. Assuming that the dynamics is governed by General Relativity in (D+1)-dimensional spacetime, the set of cosmological equations takes the form

$$\frac{d}{dt}\left(\frac{\dot{a}}{a}\right) + \frac{\dot{a}}{a}\left(D\frac{\dot{a}}{a} - \frac{\dot{N}}{N}\right) + (D-1)N^2\frac{k}{a^2} = \frac{8\pi G_D}{D-1}N^2(\varepsilon - p),$$
$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{N^2 k}{a^2} = \frac{16\pi G_D}{D(D-1)}N^2\varepsilon, \tag{2}$$

where the dot stands for the derivative with respect to t, G_D is the gravitational constant in (D+1)-dimensional spacetime, ε is the energy density and p is the pressure for the sources driving the cosmological evolution. The latter two quantities obey the equation $\dot{\varepsilon} + D(\dot{a}/a)(\varepsilon + p) = 0$ which is obtained from the covariant conservation equation for the energy-momentum tensor. This relation can also be obtained from (2). For the second derivative of the scale factor we get

$$\frac{\ddot{a}}{a} - \frac{\dot{N}}{N}\frac{\dot{a}}{a} = -\frac{8\pi G_D}{D-1}N^2\left(p + \frac{D-2}{D}\varepsilon\right). \tag{3}$$

From this relation it follows that the accelerated expansion in terms of the synchronous time coordinate ($N(t) = 1$) is obtained under the condition $p < (2-D)\varepsilon/D$. The latter condition is satisfied by the positive cosmological constant Λ with the energy density $\varepsilon_\Lambda = \Lambda/(8\pi G_D)$ and pressure $p_\Lambda = -\varepsilon_\Lambda$.

In the discussion below we assume that the matter source contains two parts with $\varepsilon = \varepsilon_\Lambda + \varepsilon_m$ and $p = p_\Lambda + p_m$. Here, the part with the equation of state $p_\Lambda = -\varepsilon_\Lambda$ corresponds to the cosmological constant Λ with the constant energy density ε_Λ and the second contribution has an equation of state $p_m = w\varepsilon_m$ with $w = \text{const}$. The condition $p < (2-D)\varepsilon/D$ for the second source is reduced to $w < w_c \equiv 2/D - 1$ for $\varepsilon_m > 0$ and to $w > w_c$ for $\varepsilon_m < 0$. From the covariant conservation equation we get

$$\varepsilon_m = \varepsilon_{m0}(a/a_0)^{-\alpha}, \tag{4}$$

with the notation

$$\alpha = D(1+w). \tag{5}$$

We will assume that the cosmological constant Λ and the constant $\varepsilon_{m0} = \varepsilon_m\big|_{a=a_0}$

can be either positive or negative. Note that from the second equation in (2) it follows that one needs to have the condition $16\pi G_D \varepsilon \geq D(D-1)ka^{-2}$ and the total energy density ε should be nonnegative in models with $k = 0,\ 1$.

Let us consider the qualitative features of the evolution in terms of the synchronous time. Taking $N(t) = 1$, the second equation in (2) is rewritten as

$$H^2 + \frac{k}{a^2} = \frac{2\Lambda}{D(D-1)} + \frac{16\pi G_D \varepsilon_{m0}}{D(D-1)(a/a_0)^{\alpha}}, \tag{6}$$

where $H = \dot{a}/a$ is the Hubble function. From here it follows that for $w > -1$ and for a positive cosmological constant the late time evolution (large values of the scale factor) is dominated by the first term in the right hand side. In this case the de Sitter solution $a(t) \propto e^{H_\Lambda t}$, with

$$H_\Lambda = \sqrt{\frac{2|\Lambda|}{D(D-1)}} \tag{7}$$

(here we consider the case $\Lambda > 0$, the notation H_Λ for $\Lambda < 0$ is used below), is the future attractor for the general solution. For a negative cosmological constant, $\Lambda < 0$, and for $w > -1$, from (6) we see that with increasing a at some moment $t = t_m$ the Hubble function becomes zero. The corresponding value for the scale factor $a = a_m$ is determined from (6) putting $H = 0$. At that moment from the first equation (2) we get

$$\dot{H}_{t=t_m} = -Dk\frac{w - w_c}{2a_m^2} + \frac{1+w}{D-1}\Lambda. \tag{8}$$

For $k = 0,\ 1$ and $w > w_c$ the right-hand side is negative and for $t > t_m$ one obtains $H < 0$ and the initial expansion is followed by the contraction. The same is the case for $k = -1$ and $-1 < w < w_c$. For $\alpha > 2$ and $\varepsilon_{m0} > 0$, the early expansion, corresponding to small values of the scale factor, is dominated by the matter source and the solutions with flat space serve as attractors for models with $k = \pm 1$.

3. *Cosmological solutions in flat model.* Simple exact solutions of the cosmological equations can be found in the case of flat model, $k = 0$. In the absence of the matter source the equation (6) has solutions only for $\Lambda \geq 0$. For positive cosmological constant the de Sitter solution, $a(t) \propto e^{\pm H_\Lambda t}$, is obtained. To see the influence of the matter source, first we consider the case of positive cosmological constant and positive matter density, corresponding to $\varepsilon_{m0},\ \varepsilon_\Lambda > 0$. In the synchronous time coordinate, for the Hubble function we get

$$H = \pm H_\Lambda \sqrt{1 + \left(\frac{a_m}{a}\right)^{\alpha}}, \quad \frac{a_m}{a_0} \equiv \left|\frac{\varepsilon_{m0}}{\varepsilon_\Lambda}\right|^{1/\alpha}, \tag{9}$$

with α defined by (5). The integration of this equation leads to the following expressions for the Hubble function and the scale factor:

$$H(t) = \pm H_\Lambda \coth\left(\beta|t|\right), \quad \alpha(t) = a_m \sinh^{2/\alpha}\left(\beta|t|\right), \tag{10}$$

where

$$\beta = \frac{1}{2}|\alpha| H_\Lambda = |1+w|\sqrt{\frac{D|\Lambda|}{2(D-1)}}. \tag{11}$$

For $w > -1$ the solution (10) for the scale factor coincides with that found in [20]. In that case and for expansion models one has $0 < t < \infty$ with the upper sign in the expression for the Hubble function. At late times, $\beta t >> 1$, one has an approximately de Sitter expansion with $a(t) \propto e^{H_\Lambda t}$. Near the singularity point $t = 0$ we obtain $a(t) \propto |t|^{2/\alpha}$. The case $w < -1$ corresponds to the phantom phase (for the effective phantom phase generated by different types of sources see [22]). In this case $\alpha < 0$ and for the expansion models we have $-\infty < t < 0$. The point $t = 0$ corresponds to the Big Rip singularity. The universe starts with de Sitter expansion $a(t) \propto e^{H_\Lambda t}$, $\beta|t| >> 1$, in the infinite past and ends the evolution at Big Rip singularity at $t = 0$ with the behavior $a(t) \propto |t|^{-2/|\alpha|}$. In Fig.1 we have plotted the ratio a/a_m versus $H_\Lambda t$ for $D = 3$. The full and dashed curves correspond to the values $w = 0$ (dust matter), $w = -2/3$ and $w = -3/2$ (phantom matter). Note that under certain conditions (see [23]) the energy density for the axion field scales as $\varepsilon_{\text{axion}} \sim 1/a^3$ and the corresponding dynamics is described by the curve with $w = 0$ in Fig.1 (the cosmological dynamics with the axion field and holographic

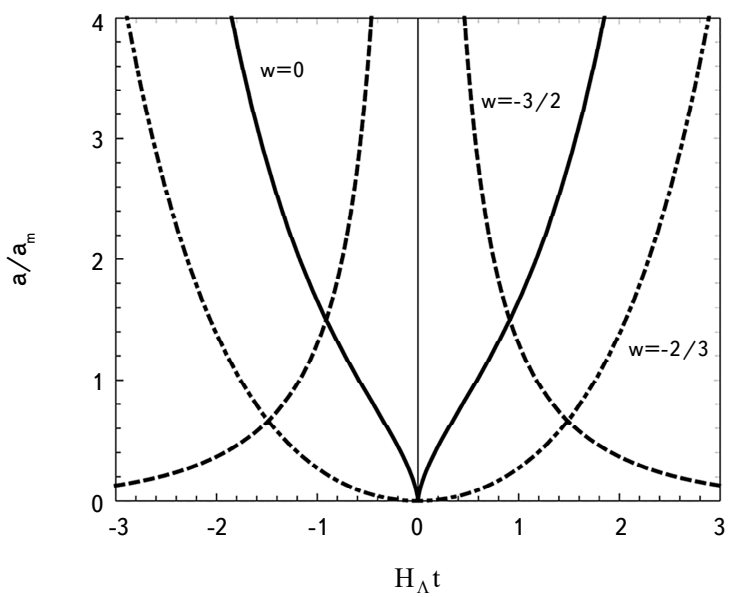

Fig.1. The time dependence of the scale factor in the model with ε_{m0}, $\varepsilon_\Lambda > 0$ for $D = 3$ and $w = 0$, -2/3, -3/2.

dark energy has been recently discussed in [24]). For expanding models we have $0 < t < +\infty$ for sources with $w > -1$ and $-\infty < t < 0$ for $w < -1$. The singular point $t = 0$ corresponds to the Big Bang in the first case and to the Big Rip in the second case. For $w > w_c$ and $w < -1$ one has $\dot{a}|_{t=0} = \infty$ and for $-1 < w < w_c$ we get $\dot{a}|_{t=0} = 0$. We see that for ε_{m0}, $\varepsilon_\Lambda > 0$ all the flat models contain singularities.

Next we consider the case $\varepsilon_\Lambda < 0 < \varepsilon_{m0}$. For the Hubble function we find

$$H = \pm H_\Lambda \sqrt{(a_m/a)^\alpha - 1}. \tag{12}$$

The time dependences for the Hubble function and for the scale factor read

$$H(t) = \pm H_\Lambda \tan(\beta|t|), \quad a(t) = a_m \cos^{2/\alpha}(\beta|t|), \tag{13}$$

with $-\pi/2\beta < t < \pi/2\beta$. For $w > -1$ this solution coincides with that presented in [20]. The authors of [20] emphasize that the solution (13) gives rise to a periodic universe. However, it should be noted that, though the function $a(t)$ in (13) is periodic with the period $t_L = \pi/\beta$, the periods are separated by singular points $|t| = \pi(l+1)/\beta$, $l = 0, 1, 2, \ldots$, and the evolution pieces separated by those points present the copies of the same universe with a finite lifetime t_L (for discussion of various types of singularities in the cosmological context see, for example, [25-27]). The dependence of the scale factor on the synchronous time coordinate, described by (13), is depicted in Fig.2 for $D = 3$ and $w = 0$, -2/3, -3/2. In models with $w > -1$ the expansion phase with $-\pi/2\beta < t < 0$ is followed by the contraction one for $0 < t < \pi/2\beta$. The maximal value of the scale factor is determined by (9).

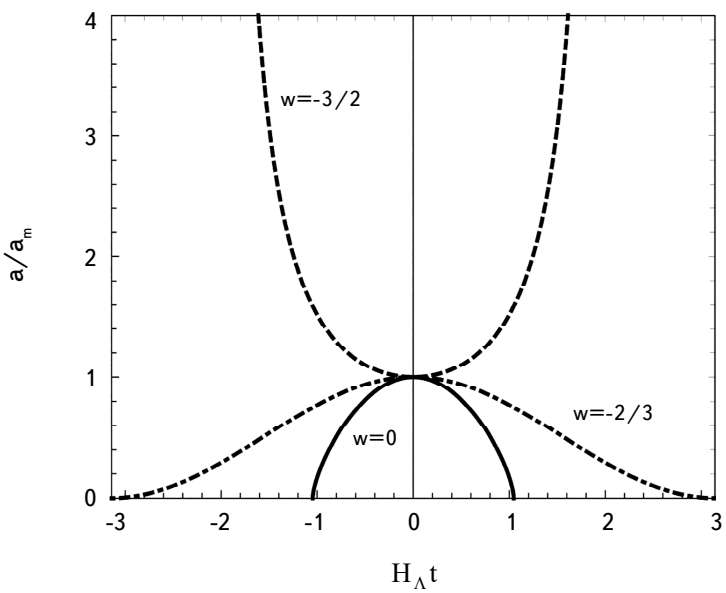

Fig.2. The same as in Fig.1 for the model with $\varepsilon_\Lambda < 0 < \varepsilon_{m0}$.

For sources with $w < -1$ the same relation determines the minimal value of the scale factor. Similar to the previous case, the flat models contain singularities for all values of the parameters.

Now we turn to the case $\varepsilon_{m0} < 0 < \varepsilon_\Lambda$. The Hubble function is expressed as

$$H = \pm H_\Lambda \sqrt{1 - (a_m/a)^\alpha}\,, \tag{14}$$

where a_m is the minimal (maximal) value of the scale factor for $w > -1$ ($w < -1$). The time dependence is given by the formulas

$$H = \pm H_\Lambda \tanh(\beta|t|), \quad a = a_m \cosh^{2/\alpha}(\beta|t|), \tag{15}$$

with $-\infty < t < +\infty$. The time dependence of the scale factor given by (15) is plotted in Fig.3 for the values of the parameters $D = 3$ and $w = 1/3,\ 0,\ -2/3,\ -3/2$. The models in this case have no singularities. The value $a = a_m$ determines the minimum/maximum value of the scale factor. Note that flat cosmological models with $\varepsilon_{m0},\ \varepsilon_\Lambda < 0$ are not allowed by the equation (6). The corresponding models with curved space will be discussed in the next section.

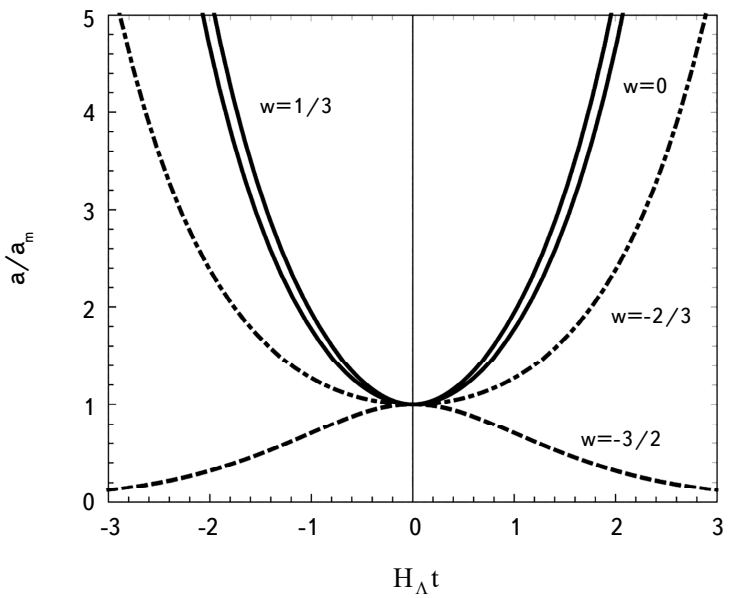

Fig.3. The scale factor versus the time coordinate in the model with $\varepsilon_{m0} < 0 < \varepsilon_\Lambda$ for $D = 3$ and $w = 1/3,\ 0,\ -2/3,\ -3/2$.

In [28,29] it has been shown that in de Sitter spacetime the vacuum expectation value of the energy-momentum tensor for a conformally coupled massless scalar field in the hyperbolic vacuum has the form

$$\left\langle T_i^k \right\rangle = \varepsilon_\Lambda \mathrm{diag}(1,1,...,1) + \frac{C_D}{\alpha^{D+1}} \mathrm{diag}\left(1, -\frac{1}{D}, ..., -\frac{1}{D}\right), \tag{16}$$

where the first term in the right-hand side corresponds to a cosmological constant and the constant C_D is negative. The second term can be identified with the source we have considered above having the equation of state $p_m = \varepsilon_m / D$. Hence, for this source one gets $w = 1/D$ and $\varepsilon_{m0} < 0$. The corresponding cosmological solution is given by (15) with $\alpha = D+1$ and $\beta = (D+1)H_\Lambda/2$. The time dependence of the scale factor for $D = 3$ is presented in Fig.3 by the curve with $w = 1/3$. The corresponding models are nonsingular.

4. *Cosmological dynamics in models with curved space.* Passing to the models with curved space, first let us recall the well-known solutions in the absence of matter sources. The models with $k = 1$ are allowed only in the case $\Lambda > 0$ and the corresponding solution for the scale factor is given by $a = \cosh(H_\Lambda t)/H_\Lambda$. For $k = -1$ and $\Lambda > 0$ the solution reads $a = \sinh(H_\Lambda |t|)/H_\Lambda$. For $k = -1$ and negative cosmological constant we have the solution $a = \sinh(H_\Lambda |t|)/H_\Lambda$. Note that in models with $k = -1$ and $H_\Lambda |t| << 1$ the evolution is approximated by linear scale factor $a(t) = |t|$. The latter describes a flat spacetime and corresponds to the Milne universe.

Another special case corresponds to the absence of cosmological constant. From the equation (6) we get

$$\frac{dy}{d(t/a_0)} = \pm\sqrt{\gamma_0 y^\gamma - k}\,, \tag{17}$$

with the notations

$$y = \frac{a}{a_0}, \quad \gamma_0 = \frac{16\pi G_D \varepsilon_{m0} a_0^2}{D(D-1)}, \quad \gamma = D(w_c - w). \tag{18}$$

Separating the variables, the integrals in (17) can be expressed in terms of the incomplete beta function $B_z(u,v)$. Presenting the latter through the hypergeometric function $F(a,b;c;z)$ (see, for example, [30]), for the models with $k = -1$ we find

$$t = \frac{a}{\sqrt{\gamma_0 y^\gamma + 1}} F\left(\frac{1}{2}, 1; 1 + \frac{1}{\gamma}, \frac{\gamma_0 y^\gamma}{\gamma_0 y^\gamma + 1}\right). \tag{19}$$

In a similar way, for the models with $k = 1$ the integration gives

$$t = \frac{2a_0}{\gamma \gamma_0^{1/\gamma}} \left(1 - \frac{1}{\gamma_0 y^\gamma}\right)^{1/2} F\left(\frac{1}{2}, \frac{1}{2} + \frac{1}{\gamma}; \frac{3}{2}, 1 - \frac{1}{\gamma_0 y^\gamma}\right). \tag{20}$$

The various special cases of these general formulas have been considered in the

literature. In particular, the examples when for general number of spatial dimension the solutions are expressed in terms of elementary functions have been discussed in [20].

Now we turn to the general case of models with curved space in the presence of a cosmological constant and barotropic matter. The equation (6) is rewritten as

$$\frac{dx}{dt} = \pm H_\Lambda \sqrt{s_\Lambda x^2 + b x^\gamma - k}\,, \tag{21}$$

where

$$x = H_\Lambda a\,, \quad s_\Lambda = \mathrm{sgn}(\Lambda), \tag{22}$$

and

$$b = \frac{16\pi G_D\, \varepsilon_{m0}\, a_0^2}{D(D-1)} (H_\Lambda a_0)^\gamma\,. \tag{23}$$

Simple solutions are found for the special case of the source with $w = w_c$. For $\Lambda > 0$ and $\gamma_0 - k > 0$ the solution has the form $x = \sqrt{\gamma_0 - k}\sinh(H_\Lambda |t|)$. In the case $\Lambda > 0$ and $\gamma_0 - k < 0$, the solution reads $x = \sqrt{k - \gamma_0}\cosh(H_\Lambda t)$. For $\Lambda < 0$ one needs to have $\gamma_0 - k > 0$ and the corresponding solution is given by $x = \sqrt{\gamma_0 - k}\sinh(H_\Lambda t)$, $0 < t < \pi/H_\Lambda$. For $\gamma_0 = 0$ the first two solutions are reduced to the de Sitter solutions.

We will denote by $x = x_m > 0$ the value of the function $x(t)$ at its possible extremum, $dx/dt\big|_{x=x_m} = 0$. The extrema are zeros of the expression under the square root in (21). Taking the corresponding value of the time coordinate as $t = 0$ and expanding near the extremum we get

$$\frac{a(t)}{a_m} \approx 1 + \frac{D}{4}\left[s_\Lambda (1+w) + \frac{w_c - w}{x_m^2} k \right] (H_\Lambda t)^2\,, \tag{24}$$

where $a_m = x_m / H_\Lambda$. The nature of the extremum (minimum or maximum) is determined by the sign of the expression in the square brackets. Note that for the extremum we have $b x_m^{-\alpha} = k/x_m^2 - s_\Lambda$. In the definition of the constant b we have taken $a_0 = a(t_0)$ and $\varepsilon_{m0} = \varepsilon_m(t_0)$ for a fixed time $t = t_0$. Taking $t_0 = t_m$, where t_m corresponds to the extremal value x_m, $x(t_m) = x_m$, from (6) we get the following relation

$$x_m^2 = \frac{s_\Lambda k}{1 + \varepsilon_{(m)}/\varepsilon_\Lambda}\,, \tag{25}$$

where $\varepsilon_{(m)} = \varepsilon(t_m)$ is the matter energy density at the extremum point. Note that, assuming the presence of the extremum $x = x_m$, the equation (21) is written as

$$\frac{dy}{dt} = \pm H_\Lambda \sqrt{s_\Lambda \left(y^2 - 1\right) + \frac{\varepsilon_{(m)}}{|\varepsilon_\Lambda|}\left(y^\gamma - 1\right)}, \tag{26}$$

with $y = x/x_m = a/a_m$.

Let us consider different combinations of the signs for the energy densities. For ε_{m0}, $\varepsilon_\Lambda > 0$ and $w > w_c$, the early dynamics, corresponding to small values of x, is dominated by the source with the energy density ε_m and the expansion law is close to the one for the flat model. At late times, corresponding to $x \gg 1$, the expansion is dominated by the cosmological constant and, again, the curvature term is subdominant. The solution corresponding to the flat model is the future attractor for models with curved space. The dependence of the scale factor on time coordinate is qualitatively similar to that depicted in Fig.1 for $w = 0$.

For ε_{m0}, $\varepsilon_\Lambda > 0$, $w < w_c$, and $k = -1$, the early dynamics ($x \ll 1$) for expanding models is dominated by the curvature term and $a(t) \approx t$, $t \to 0$. As it has been mentioned above, the spacetime with $k = -1$ and $a(t) = t$ is flat and corresponds to the Milne universe. The matter energy density behaves as $\varepsilon_m \propto x^{-\alpha}$ and for $w > -1$ it diverges at $t = 0$ like $\varepsilon_m \sim t^{-\alpha}$. In the model with ε_{m0}, $\varepsilon_\Lambda > 0$, $w < w_c$, and $k = 1$ the scale factor has a minimal value that corresponds to the zero $x = x_m$ of the expression in the right-hand side of (21). At this point the Hubble function becomes zero. The time-dependence of the scale factor near the minimum, $H_\Lambda t << 1$, is given by (24) with $s_\Lambda = 1$ and $a_m = a_{min}$. At late times of the expansion, $x >> 1$, the curvature term in (21) can be ignored and the cosmological dynamics is well approximated by the solutions for flat model (see the graphs with $w = -2/3, -3/2$ in Fig.1). We conclude that the models with ε_{m0}, $\varepsilon_\Lambda > 0$, $w < w_c$, and $k = 1$ are nonsingular.

Let us turn to the models with $\varepsilon_\Lambda < 0 < \varepsilon_{m0}$. For $w > w_c$ the maximum allowed value for x is determined by the zero $x = x_m$ of the right-hand side in (21). The asymptotic behavior near the maximum is described by (24) with $s_\Lambda = 1$ and $a_m = a_{max}$. For $x << 1$, in the right-hand side of (21) we can omit the curvature term and x^2. The scale factor is approximated by the solution for the flat model and near the Big Bang, corresponding to $t = -t_1$, $t_1 > 0$, one has $a(t) \propto (t + t_1)^{2/\alpha}$. The model has finite lifetime $2t_1$ and the corresponding time-dependence of the scale factor is qualitatively similar to that for the flat model presented by the graph with $w = 0$ in Fig.2.

For $\varepsilon_\Lambda < 0 < \varepsilon_{m0}$ and $-1 < w < w_c$ the function $x(t)$ has a maximal allowed value $x = x_m$ determined by the zero of the right-hand side in (21). Taking $x(0) = x_m$, near the maximum point we have the approximation (24) with $s_\Lambda = 1$ and $a_m = a_{max}$. For $k = -1$, the models start the expansion at $t = -t_1$ with the scale factor $a(t) \approx t + t_1$ and the behavior of the scale factor is close to the one for the Milne universe. The expansion is stopped at $t = 0$ and for $t > 0$ the model enters the

contraction phase. The latter is ended at $t = t_1$ with $a(t) \approx t_1 - t$. Hence, the $k = -1$ models have lifetime $2t_1$ and the Milne universe is the past and future attractor for the corresponding dynamics. Note that, though the first derivative of the scale factor is finite at the points $t = \pm t_1$ ($\left|\dot{a}\right|_{t=\pm t_1} = 1$), the matter energy density diverges at those points as $\varepsilon_m \sim 1/\left|t \pm t_1\right|^{D(1+w)}$. The models with $k = 1$ start the expansion from the finite value of the scale factor a_{min} at $t = -t_{min}$. At that point $\dot{a}(-t_{min}) = 0$. At $t = 0$ the scale factor takes its maximal value $a_{max} = x_m / H_\Lambda$ and then it enters into the contraction phase. Near the maximum we have the approximation (24). The evolution is ended at $t = t_{min}$ with $a = a_{min}$ and $\dot{a}(t_{min}) = 0$. Hence, in this case we have nonsingular evolution for $-t_{min} \le t \le t_{min}$. Joining the evolutionary pieces with duration $2t_{min}$, we obtain a model with periodically oscillating scale factor in the limits $a_{min} \le a \le a_{max}$ for $-\infty < t < +\infty$.

In models with $\varepsilon_\Lambda < 0 < \varepsilon_{m0}$, $w < -1$, and for large values of x the expansion law is close to the one for the flat model and the corresponding behavior is qualitatively close to the one given by the curve with $w = -3/2$ in Fig.2. For small values of x and for models with $k = -1$ the expansion/contraction law is approximated by $a(t) \approx |t|$. At $t = 0$ the matter energy density vanishes as $\varepsilon_m \sim |t|^{D(1+w)}$. In models with $k = 1$ the scale factor has a minimum value $a = a_{min}$ determined by the zero of the right-hand side in (21) and the evolution for all values of $x \ge x_{min}$ is qualitatively similar to that described by the curve with $w = -3/2$ in Fig.2. The expansion models have Big Rip singularity.

Now let us consider models with the energy densities in the range $\varepsilon_{m0} < 0 < \varepsilon_\Lambda$. For $w > w_c$ the scale factor has a minimal value $a = a_{min}$ which is determined by the zero of the righthand side in (21). Taking $t = 0$ for the corresponding value of the time coordinate, near the minimum one has the approximation (24) with $a_m = a_{min}$ and $x_m = x_{min}$. For $w > w_c$ and for large values of x the evolution is approximated by de Sitter spacetime with the Hubble constant H_Λ. The behavior of the scale factor is qualitatively similar to that depicted in Fig.3 by the curves with $w = 0$, $1/3$ and the corresponding models have no singularities. An example with positive cosmological constant, negative matter energy density and the equation of state parameter $w = 1/D > w_c$ is provided by (16). In the range $-1 < w < w_c$ and for large values of x the evolution is again dominated by the cosmological constant with de Sitter spacetime being the past or future attractor. In the same range for w and for $k = -1$ one gets the approximate solution $a(t) \approx |t|$ for $H_\Lambda |t| << 1$, corresponding to the Milne universe. The matter energy density diverges at $t = 0$. For $-1 < w < w_c$ and $k = 1$ the scale factor has a minimal value determined by the zero of the right-hand side of (21). Near that minimum the scale factor is approximated by (24) and the model is nonsingular. In the range $w < -1$ the scale factor has the maximal value a_{max} which is given by the zero

of the right-hand side in (21). For models with $k=-1$ the expansion starts at $t=-t_1$ with the asymptotic $a(t)\approx t+t_1$ (curvature dominated expansion) and ends at $t=0$ with the asymptotic given by (24). The expansion phase is followed by the contraction for $0<t<t_1$ with $a(t)\approx t_1-t$ near $t=t_1$. For models with $k=1$ the scale factor varies between two nonzero values $0<a_{min}\leq a\leq a_{max}<\infty$. The corresponding models are nonsingular and can be extended for $t\in(-\infty,+\infty)$. The qualitative dynamics is similar to that we have described above for the case $k=1$, $\varepsilon_\Lambda<0<\varepsilon_{m0}$, $-1<w<w_c$.

Finally, for ε_{m0}, $\varepsilon_\Lambda<0$, in accordance with (6), the models with $k=0$ and $k=1$ are not allowed. Let us consider the features of the cosmological dynamics in this case for $k=-1$. For $w>w_c$, from the condition for the positivity of the expression under the square root in (21), we can see that the model is allowed under the constraint

$$|b|<\frac{2}{D(w-w_c)}\left(\frac{w-w_c}{w+1}\right)^{\alpha/2}. \tag{27}$$

This condition restricts the allowed values for the negative energy density ε_{m0}. In the range determined by (27), the right-hand side of (21) has two zeros and they determine the minimal and maximal values for the scale factor, $0<a_{min}\leq a(t)\leq a_{max}$. At those points $\dot{a}=0$ and $H=0$. Near the extrema the scale factor is approximated by (24) with $s_\Lambda=-1$ and $k=-1$. From (24) it follows that

$$a_{min}<\frac{1}{H_\Lambda}\sqrt{\frac{w-w_c}{1+w}}<a_{max}. \tag{28}$$

For ε_{m0}, $\varepsilon_\Lambda<0$, $w<w_c$ the right hand side of (21) has a single zero that determines the maximal value of the scale factor $a_{max}=a(0)$. Near the maximum the scale factor behaves like (24) with $s_\Lambda=-1$ and $k=-1$. For small values of x the dynamics is dominated by the curvature term with the Milne universe as the asymptotic. The expansion starts at $t=-t_1$ with $a(t)\approx t+t_1$ and stops at $t=0$ with the maximal value of the scale factor. The evolution for $0<t<t_1$ corresponds to the contraction phase with the future attractor $a(t)\approx t_1-t$. At the points $t=\pm t_1$ the matter energy density vanishes for $w<-1$ and diverges for $-1<w<w_c$.

5. *Conclusion*. We have considered the dynamics of $(D+1)$-dimensional FRW cosmological models driven by the cosmological constant and the matter source with barotropic equation of state assuming that the energy densities for those sources can be either positive or negative. Exact solutions are provided for models with flat space which include various special cases previously considered in the literature. In particular, it has been demonstrated that nonsingular solutions are obtained only for negative energy density of the matter, regardless the sign of the

cosmological constant. The corresponding scale factor is given by (15). Another classes of exact solutions, expressed in terms of the hypergeometric function (see (19) and (20)), are obtained for models with curved space in the absence of cosmological constant. A number of special cases of those solutions, when they are expressed in terms of elementary function, have been discussed in the literature (see, for example, [20]). The qualitative evaluation for models with curved spaces and with a cosmological constant and matter source has been described in the second part of section 4 for all the values of the equation of state parameter w and for all combinations of the signs of the energy densities. Depending on the values of w one can have Big Bang or Big Rip type singularities. We have also specified nonsingular models with curved space. For k = 1, nonsingular modelas are obtained for the following combinations of conditions: (i) ($\varepsilon_{0m} > 0$, $\varepsilon_\Lambda > 0$, $w < w_c$), (ii) ($\varepsilon_{0m} > 0$, $\varepsilon_\Lambda < 0$, $-1 < w < w_c$), (iii) ($\varepsilon_{0m} < 0$, $\varepsilon_\Lambda > 0$). In models (ii) and ($\varepsilon_{0m} < 0$, $\varepsilon_\Lambda > 0$, $w > -1$) the evolution of the scale factor, as a function of time coordinate, is periodically oscillatory in the limits $a_{min} \le a(t) \le a_{max}$. In the remaining cases, the qualitative evolution of $k = 1$ nonsingular models is similar to that depicted in Fig.3 for $w = -2/3$, 0, 1/3. For models with negative curvature space there exists at least one point on the time axis where the scale factor becomes zero. Near those points the evolution is dominated by the matter source for $w > w_c$ and by the curvature term for $w < w_c$. In the second case the scale factor is approximated by a linear expansion/contraction as a function of the time coordinate. At the point with zero scale factor the matter energy density diverges for $-1 < w < w_c$ and vanishes for $w < -1$.

We have seen that the negative energy densities for both the cosmological constant and matter source enlarge the possible scenarios of cosmological dynamics. Bearing in mind applications in higher-dimensional models, it would be interesting to generalize the corresponding results for models with extra compact dimensions. The compactification leads to additional contributions to the vacuum expectation value of the energy-momentum tensor. In general, the effective pressures along compact dimensions differ and for massless conformally coupled fields the topological contributions are equivalent to barotropic perfect fluid with anisotropic pressures. In particular, the coefficients w in the respective equations of state may have different signs. In the corresponding anisotropic cosmological models one can have an expansion for a part of dimensions and a contraction for the remaining ones. The analysis of different cosmological scenarios can be done in a way similar to that we have described above. We can also use the methods of qualitative analysis of dynamical systems to classify qualitatively different cosmological models. The corresponding results for a toroidal compactification will be presented elsewhere.

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КОСМОЛОГИЧЕСКАЯ ЭВОЛЮЦИЯ С ОТРИЦАТЕЛЬНОЙ ПЛОТНОСТЬЮ ЭНЕРГИИ

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Для произвольного числа пространственных измерений исследована космологическая динамика, управляемая космологической постоянной и источником с баротропным уравнением состояния. Предполагается, что для обоих источников плотность энергии может быть как положительной, так и отрицательной. Для плоских моделей приведены точные решения космологических уравнений. Для моделей с искривленным пространством и с нулевой космологической постоянной общие решения выражаются через гипергеометрическую функцию. Качественная эволюция описана для всех значений параметра уравнения состояния. Выделены значения этого параметра и комбинации знаков для космологической постоянной и плотности энергии материи, для которых космологическая динамика несингулярна. Рассмотрен пример с положительной космологической постоянной и отрицательной плотностью энергии вещества, индуцированной поляризацией гиперболического вакуума.

Ключевые слова: *космологическая эволюция: космологическая постоянная: отрицательная плотность энергии*

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