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THE PROBLEM OF THIN PLATE STABILITY AT SUPERSONIC FLOW AND PRESENCE OF CONCENTRATED WEIGHT AT EDGES

The stability problem of a thin plate with finite length, streamline gas is considered. A supersonic gas and the weight of a plate are neglected. But it is considered that there are concentrated weights on the fixed edge of a plate where the critical flow speeds causing flutter instability phenomenon are found.

Keywords: plate-stability, supersonic flow, harmonic oscillations, flutter, divergence speed, concentrated weight.

Aspiration to achieve higher speeds of flight FA by their constant perfection and their use in the new constructional materials (Aluminium-Lithium alloys, composite materials), systems of active and adaptive managements promote the rigidity reduction of designs. Then these processes are accompanied by the increase of aeroelastic oscillation level of the designs and also by the increased probability of various aeroelastic phenomena display. There are numerous researches [1-6] to study the problem of plane wing stability at supersonic flow, but this present work is actually based on piston theory and devoted to the considered theory of panel flutter at supersonic gas stream which it is supposed that the inertial weights of a plate are too small, so we can neglect them, but there are concentrated weight on the fixed edge of plate. And the critical speeds of a flow which cause the flutter phenomenon are determined.

1. Let the thin plate in the Cartesian system of coordinates $oxyz$ which occupies the area $0 \le x \le l, 0 \le y \le b, -h \le z \le h$. and the plate in a direction of axis ox, one side of the plate will be exposed to a supersonic distributed flow by the speed υ. By accepting the validity of Kirchhoff hypothesis and piston theory [1] we can accept that the plate at direction of axis oy is wide enough, so it is possible to count, that the plate's oscillations have the form of a cylindrical surface, and do not depend on coordinate y. At specific assumptions the flexural vibration equation has the form [1,2]:

$$D\frac{\partial^4 w}{\partial x^4} + \alpha_0\rho_0\left(\frac{\partial w}{\partial t} + \upsilon\frac{\partial w}{\partial x}\right) + 2\rho h\frac{\partial^2 w}{\partial t^2} = 0. \qquad (1.1)$$

Here $\rho, \rho_0 -$ is the density of a plate and gas material, accordingly; α_0 is the speed of sound in gas; $w = w(x,t)$ is the function of plate deflection; D is the flexural rigidity:

$$D = \frac{2Eh^3}{3(1-\gamma^2)}. \tag{1.2}$$

The solution for equation (1.1) has been investigated by A.A.Movchan[3] under various static boundary conditions at plate's edges $x = 0$, $x = l$ and then the representation of the equation (1.1) solution will be as

$$w = f(x)e^{i\omega t}. \tag{1.3}$$

It turns out an ordinary differential equation of the fourth degree concerning the function $f(x)$, so for finding the general solution it is necessary to find the roots for characteristic equation of the fourth degree. The problem of plate stability essentially becomes simpler by applying Euler's static method. Then, instead of the equation (1.1), it is necessary to solve the equation [2],

$$\frac{\partial^4 w}{\partial x^4} + s^3 \frac{\partial w}{\partial x} = 0; \quad s^3 = \alpha_0 \rho_0 \upsilon D^{-1}. \tag{1.4}$$

In spite of the fact that for finding the general solution of equation (1.3) we find the characteristic equation which also will be in the fourth degree, however the roots for that equation are easily defined and have the form

$$p_1 = 0; \quad p_2 = -s; \quad p_{3,4} = 0.5(1 \pm i\sqrt{3}). \tag{1.5}$$

According to (1.5), the general solution of equation (1.4) will be

$$w(x) = A_1 + A_2 e^{-Sx} + (A_3 \cos\frac{\sqrt{3}}{2}sx + A_4 \sin\frac{\sqrt{3}}{2}sx)e^{\frac{Sx}{2}}. \tag{1.6}$$

Let's consider that one of the long plate edges has a rigid jamming slide and the other side is rigidly jammed then

$$\frac{dw}{dx} = 0; \quad \frac{d^3 w}{dx^3} = 0, \quad \text{where} \quad x = 0; \tag{1.7}$$

$$w = 0; \quad \frac{dw}{dx} = 0, \quad \text{where} \quad x = l.$$

By substituting (1.6) in (1.7) we will obtain a homogeneous algebraic system of equations which will concern the constants A_i , $i = 1,2,3,4,...$ Equality to zero determinant of the specific system gives the defined equation concerning the parameter sl

$$2\sin(\frac{\pi}{6} + \frac{\sqrt{3}}{2}sl) - e^{-\frac{3}{2}Sl} = 0. \tag{1.8}$$

Let's enter the notation

$$C(sl) = 2\sin(\frac{\pi}{6} + \frac{\sqrt{3}}{2}sl) - e^{-\frac{3}{2}sl}. \qquad (1.8a)$$

It is easy to show by the help of graphic-analytical research that:

$$C(sl) \prec 0, \quad \text{when} \quad sl \prec 0; \qquad (1.9)$$

$$C(sl) = 0, \quad \text{when} \quad sl = 0;$$

$c(sl) -$ Sign-variable function when $sl \succ 0$, zero functions are defined by using the formula

$$(sl)_0 \approx \frac{2}{\sqrt{3}}(\frac{5\pi}{6} + \pi k); \qquad k = 0,1,2,... . \qquad (1.10)$$

$$C(sl) \succ 0 \quad \text{when} \quad sl \in \frac{2}{\sqrt{3}}(2\pi k; \frac{5\pi}{6} + 2\pi k); \qquad k = 0,1,2,... . \qquad (1.10a)$$

$$C(sl) \prec 0, \quad \text{when} \quad sl \in \frac{2}{\sqrt{3}}(\frac{5\pi}{6} + 2\pi k; \frac{11\pi}{6} + 2\pi k); \quad k = 0,1,2,... . \qquad (1.10b)$$

According to (1.10) the first zero function will be:

$$(sl)_1 \approx \frac{2}{\sqrt{3}} \times \frac{5\pi}{6} \approx 3,012 . \qquad (1.11)$$

It is possible to find υ_{cr} for the stream from expression (1.4), by substituting (1.11) in (1.4):

$$\upsilon_{cr} \approx \frac{27\, D}{\alpha_0 \rho_0 l^3}. \qquad (1.12)$$

At this undistributed speed form the plate balance stability ceases and divergence conditions take place, then it will be obvious that from equations (1.9), (1.10) the divergence conditions arise only in the case when the gas stream is directed from rigid jamming slide edge to rigidly jammed edge.

2. As we see the equation (1.4) possesses remarkable property which corresponds to the characteristic equation and has simple roots (1.5), so there is a dynamic research idea for the (flutter instability) problem in which dynamic members are taken into account in the boundary conditions. Such an idea has been used by B.B.Bolotin in his method where the solution of plate stability problem was studied by keeping up the force system [1]. According to that method instead of the equation (1.1) we can use the considered equation

$$\frac{\partial^4 w}{\partial x^4} + s^3 \frac{\partial w}{\partial x} = 0; \qquad s^3 = \alpha_0 \rho_0 \upsilon D^{-1}. \qquad (2.1)$$

We have to neglect the dynamic forces in the equation (1.1), as well as in the static approach (1.4). It is supposed that for edge $x = 0$ the rigid jamming slide is a concentrated weight m which creates inertial force $(-m\frac{\partial^2 w}{\partial t^2})$. So the boundary conditions (1.7) are replaced by the following conditions [4]:

$$\frac{\partial w}{\partial x}=0; \qquad D\frac{\partial^3 w}{\partial x^3}=-m\frac{\partial^2 w}{\partial t^2}, \quad \text{when } x=0, \qquad (2.2)$$

$$w=0; \ \frac{\partial w}{\partial x}=0, \text{ when } \ x=l.$$

By substituting the equation (2.1) the solution will be as the harmonic oscillation (1.3), and we will obtain an equation concerning the function $f(x)$ which the general solution is related to (1.6) and by substituting (1.3) in the boundary conditions (2.2) we obtain the conditions concerning the function $f(x)$:

$$f'=0; \quad f'''-\beta\omega^2 f=0, \quad \text{when } \ x=0 \qquad (2.3)$$

$$f=0; \ f'=0, \text{ when } \ x=l,$$

where

$$\beta=\frac{m}{D}; \qquad (2.4)$$

where $\beta\geq 0$ is the parameter, describing the concenrated weight which is applied on edge x=0.

Furthermore, substituting the general solution of the form (1.6) in boundary conditions (2.3) we obtain a homogeneous algebraic system of equations concerning constants $A_i, i=1,2,3,4$:

$$\begin{cases} A_2-\frac{1}{2}A_3-\frac{\sqrt{3}}{2}A_4=0; \\ \beta\omega^2 A_1+(s^3+\beta\omega^2)A_2+(s^3+\beta\omega^2)A_3=0; \\ A_1+e^{-Sl}A_2+(e^{\frac{Sl}{2}}\cos\frac{\sqrt{3}}{2}sl)A_3+(e^{\frac{Sl}{2}}\sin\frac{\sqrt{3}}{2}sl)A_4=0; \\ e^{-Sl}A_2-(e^{\frac{Sl}{2}}\sin(\frac{\pi}{6}-\frac{\sqrt{3}}{2}sl))A_3-(e^{\frac{Sl}{2}}\cos(\frac{\pi}{6}-\frac{\sqrt{3}}{2}sl))A_4=0. \end{cases} \qquad (2.5)$$

Equating to zero system determinant (2.5), we obtain α_n expression defending own frequencies square of plate oscillations:

$$\omega^2=\frac{s^3}{\beta}\times\frac{B(sl)}{A(sl)}; \ \beta\neq 0; \qquad (2.6)$$

$$A(sl)=(ch(sl)-ch\frac{sl}{2}\cos\frac{\sqrt{3}}{2}sl-\sqrt{3}sh\frac{sl}{2}\sin\frac{\sqrt{3}}{2}sl); \qquad (2.7)$$

$$B(sl) = e^{\frac{sl}{2}}(\sin(\frac{\pi}{6} + \frac{\sqrt{3}}{2}sl) - \frac{1}{2}e^{-\frac{3}{2}sl}). \qquad (2.8)$$

It is obvious that from the comparison of expressions (2.8) and (1.8a), and the behavior of function B(sl) as well as at function C(sl) according to (1.9) and (1.10б) we have

$$B(sl) \prec 0 \ \text{At} \ sl \prec 0, \text{ when } sl \in \left(\frac{5\pi}{6} + 2\pi k;\ \frac{11\pi}{6} + 2\pi k\right);\ k = 0,1,2,.... \qquad (2.9)$$

$$B(sl)=0 \ \text{At} \ sl=0, \text{ when } (sl_0) \approx \frac{2}{\sqrt{3}}\left(\frac{5\pi}{6} + \pi k\right);\ k = 0,1,2,....;$$

$$B(sl) \succ 0 \ \text{At} \quad sl \in \frac{2}{\sqrt{3}}(2\pi k;\ \frac{5\pi}{6} + 2\pi k);\ k = 0,1,2,.....$$

It is easy to show by the help of graphic-analytical researches that

$$A(sl) \succ 0 \ \text{At} \ s \neq 0 \text{ and } A(sl)=0 \ \text{at} \ s=0. \qquad (2.10)$$

Let's notice that by accepting in the equation (2.6) the limit at $s \to 0$ $(\upsilon \to 0)$, we obtain the accepted value approach of frequency square for free plate oscillations with the concentrated weight m on the edge when $x = 0$

$$\omega_{fr}^2 = \lim_{S \to 0} \frac{s^3}{\beta} \times \frac{B(sl)}{A(sl)} = \frac{12}{\beta l^3}. \qquad (2.11)$$

It is clear that $\omega_{fr}^2 \succ 0$ at $\beta \succ 0$.

According to (2.9) and (2.10), from (2.6) we will obtain

$$\omega^2 = 0 \quad \text{when} \quad (sl) \approx \frac{2}{\sqrt{3}}(\frac{5\pi}{6} + \pi k);\ k = 0,1,2,... \qquad (2.12)$$

$$\omega^2 \prec 0 \ \text{when} \ (sl) \in \frac{2}{\sqrt{3}}(\frac{5\pi}{6} + 2\pi k; \frac{11\pi}{6} + 2\pi k);\ \ k = 0,1,2,... \qquad (2.13)$$

$$\omega^2 \succ 0 \ \text{when} \ (sl) \prec 0 \ \text{и} \ (sl) \in \frac{2}{\sqrt{3}}(2\pi k; \frac{5\pi}{6} + 2\pi k);\ \ k = 0,1,2,.... \qquad (2.14)$$

It means that from intervals (2.13) the movement becomes unstable and changes to oscillatory movement; from intervals (2.14) the movement will be steady and oscillatory movement is fading. Thus the change of ω^2 from positive values to negative ones (or on the contrary) through ($\omega^2 = 0$) zero we have a small change from fading oscillations to increasing oscillations (or on the contrary) – type of flutter oscillations.

From (2.14) we can see if the stream is directed from rigidly jammed edge $x = l$ to an edge which has rigid jamming slide $x = 0$, the plate movement will be steady at all values of speed for gas stream of flow. For corresponding to zero functions C(sl) and B(sl) the value of critical divergence speeds are equal so according to considered equations (2.12) and (2.1) we can write the expression

$$\upsilon_{cr} = \left[\frac{2}{\sqrt{3}} \left(\frac{5\pi}{6} + \pi\kappa \right) \right]^3 \alpha_0^{-1} \rho_0^{-1} l^{-3}; \qquad k = 0,1,2,\dots \tag{2.15}$$

According to the first zero $(sl)_0 = \frac{2}{\sqrt{3}} \frac{5\pi}{6}$ and by taking into account the equation (1.12) we have

$$\upsilon_{cr\,0} = 27\, D\, \alpha_0^{-1} \rho_0^{-1} l^{-3}.$$

Let's note that if the jammed edge of plate and the other edge is free, and the streamline of gas stream with concentrated weight is at free end, the value of critical gas stream speed takes place by the type of flutter oscillations and equals three times of value of considered equation (2.15), approximately [5].

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Գ.Ա. ՍԱՄՐԱ

ԳԵՐՁԱՅՆԱՅԻՆ ՇՐՋՀՈՍՄԱՆ ԵՎ ԵԶՐԵՐԻՆ ԿԵՆՏՐՈՆԱՑԱԾ ԶԱՆԳՎԱԾՆԵՐԻ ԱՌԿԱՅՈՒԹՅԱՆ ԴԵՊՔՈՒՄ ՍԱԼԻ ԿԱՅՈՒՆՈՒԹՅԱՆ ԽՆԴԻՐԸ

Դիտարկվում է գազի գերձայնային հոսքով շրջհոսվող վերջավոր երկարության բարակ սալի կայունության խնդիրը, սալի զանգվածը արհամարհվում է, սակայն համարվում է, որ սալի ամրակցված եզրերին կան նաև կենտրոնացած զանգվածներ, որոշված են ֆլաթերային անկայունության հանգեցնող շրջհոսման կրիտիկական արագությունները:

Առանցքային բառեր. սալեր, կայունություն, գերձայնային հոսք, ներդաշնակ տատանումներ, թրթռում, արագության տարամիտում, կենտրոնացված կշիռ:

Г.А. САМРА

ЗАДАЧА УСТОЙЧИВОСТИ ТОНКОЙ ПЛАСТИНКИ ПРИ СВЕРХЗВУКОВОМ ОБТЕКАНИИ И НАЛИЧИИ СОСРЕДОТОЧЕННОЙ МАССЫ НА КРОМКАХ

Рассматривается задача устойчивости тонкой пластинки конечной длины, обтекаемой сверхзвуковым потоком газа. Масса пластинки пренебрегается, но считается, что на ее закрепленных кромках имеются сосредоточенные массы. Найдены критические скорости обтекания, приводящие к флаттерной неустойчивости.

Ключевые слова: пластинка, стабильность, сверзвуковой поток, гармонические колебания, вибрация, скорость расхождения, сконцентрированный вес.