ISSN 0002-306X. Изв. НАН РА и ГИУА. Сер. ТН. 2005. Т. LVIII, № 3.

UDC 539.374 MACHINEBUILDING

G. PETROSYAN, P. BEISS*

PARTICULIARITY OF CONTACT DEFORMATION PROCESS OF SINTERED PM GEARS

The problems of the investigation of elastic deformation and first plastic deformation appear in the processes of the contact PM gear with the die gear. It is continuation of [1]. The mechanical and geometrical parameters of gear tooth contact are determined. To define the maximum pressure values and other parameters of the contact area when in the PM gear tooth the first plastic deformations arise, the plasticity condition of real porous materials is used. The numerical examples are given.

Keywords: PM gear, surface contact, densification process, real properties, strength, plastic deformation, maximum pressure.

The problems of the powder material (PM) gear surface contact is of great importance for solving densification of technological processes, when they change hardness and density of PM. Therefore, taking into account the PM real properties it is necessary to define the existence of the porosity influence. That is the main difference between powder and ordinary materials. It is shown in [1] that the process of two cylinder contact has the mechanical and geometrical particularities. The analysis of studies [2 – 4] shows that the real mechanical properties of sintered materials depend on the particularity and the value of their initial porosity, while the properties of material substance should not depend on its porosity. Therefore, for solving the strength and plastic deformation problems of real PM the theories of strength and plasticity of porous materials [4 – 6] are used. Another particularity of the surface contact deformation is the connection with different tooth surface contact radii of curvature and tips of contact. These problems are very complicated and not much studied by theoretical and engineering methods. Hence, the surface contact deformation of PM gear real process solution has been attempted in this paper.

1. Elastic problem. The formulae for definition of contact pressure p_0 and half the width of the contact area b [1] are introduced as follows:

$$p_0 = 0.418\,(2q\,K_E\,/\,K_R)^{0.5}, \quad (1)$$

$$b = 1.075\,(q\,K_R\,/\,K_E)^{0.5}, \quad (2)$$

* RWTH Aachen, Institute für Werkstoffkunde, Augustinerbach 4, 52062 Aachen, Germany

$$K_E = E_1 E_2 \;/\; (E_1 \;+\; E_2), \qquad (3)$$
$$K_R = R_1 R_2 \;/\; (R_1 \;+\; R_2), \qquad (4)$$

where K_E and K_R are mechanical and geometrical parameters of gear tooth contact, E_1, E_2 - Young's modulus, R_1, R_2 – radii of die and PM gears toot surface curvature, q – load at length unit of contact area.

For calculating the mechanical parameters of gear tooth contact K_E we must use the formulae for defination of Young's modulus of PM [2]. In that case we obtain

$$K_E = E_1 E_0(1 - v)^{3.4} \;/\; (E_1 + E_0 (1 - v)^{3.4}) . \qquad (5)$$

To calculate geometrical parameters of gear tooth contact $K_{R,}$ we must know all the radii of tooth surface curvature in the contact zone. Investigations show that there are four different cases of contacts (Fig. 1):

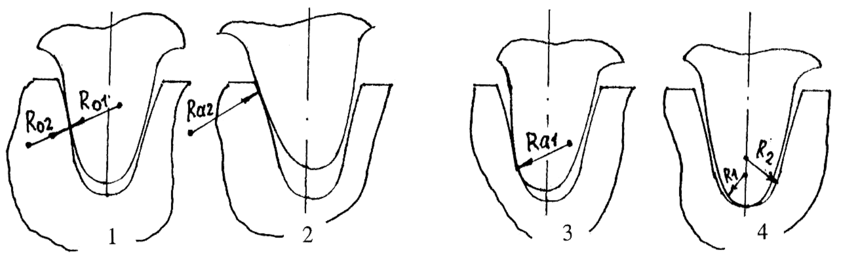

Fig. 1. Different cases of gear tooth contact: 1- external cylindrical contact, 2 – ($R_{c1} = \infty$), 3 – ($R_{c2} = \infty$), 4 – internal cylindrical contact

Fig.1 shows that in all the cases radii of the curvature are changed. Therefore, we must calculate all the radii of the tooth surface curvature.

It is known that the centers of gear tooth surface curvature are on the gear foundation circle. In Fig.2 the centers of gears O_1, O_2, their radii of initial circles $r_1=mz_1/2$, $r_2=mz_2/2$, foundation circles r_{01}, r_{02} and basic radii of tooth surface curvature R_{01}, R_{02} are shown, where m, z -are module and the number of gear teeth. From Fig.2 we may write the following formulae:

$$r_0 = mz\cos 20^o/2, \quad R_0 = mz\sin 20^0/2. \qquad (6)$$

Using formulae (4) and (6) in case of contact number 1 (Fig. 1), if $R_1=R_{01}$ and $R_2=R_{02}$, we may write

$$K_{R0} = 0{,}171 mz_1z_2/(z_1+z_2). \qquad (7)$$

To define radii R_a of surface curvature at the top of tooth, (Fig. 3) is used: $R_a=(r_a^2-r_0^2)^{0.5}$.

Using the well-known formulae for definition of top circle radii $r_a=mz/2+m$ and (6) we obtain

$$R_a=(R_0^2+ m^2(1+z))^{0.5}. \qquad (8)$$

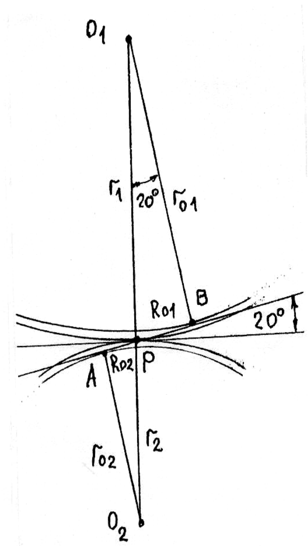

Fig.2. The scheme for definition of r_0 and R_0

Note, that in cases of number 2 and 3 we have correspondingly $R_1=R_{c1}=\infty$, $R_2=R_{a2}$ and $R_1=R_{a1}$, $R_2=R_{c2}=\infty$.

It is very important to give the correct solution to the problems of radii R_1 and R_2 definition in case of number 4 in Fig.1(internal cylindrical contact). In this case we must use the following formula for defining the geometrical parameter K_R of gear tooth contact:

$$K_R= R_1R_2/(R_2- R_1). \quad (9)$$

If $R_1=R_2$, $K_R=\infty$, using formulae (1) and (2) we have $p_0=0$, $b=\infty$. This means that the teeth are wedging. Therefore, R_2 must be greater than R_1 ($R_2 > R_1$).

So, all the formulae for definition of components of elastic contact deformation process of sintered PM gears are established.

2. The first plastic deformation problem. The analysis of the material plasticity conditions ascertain that for the PM the generalized criteria of the shape changing energy can be used by putting in this criteria the porosity as the most dangerous defect. For the sintered PM the plasticity condition of porous material is the most special one [5]

$$\sigma_i^2 + 9\alpha\sigma_0^2 = \beta\sigma_y^2, \quad (10)$$

where σ_i is the effective stress, σ_0 is the hydrostatic pressure, σ_y is the material yield point, α and β are the porosity functions.

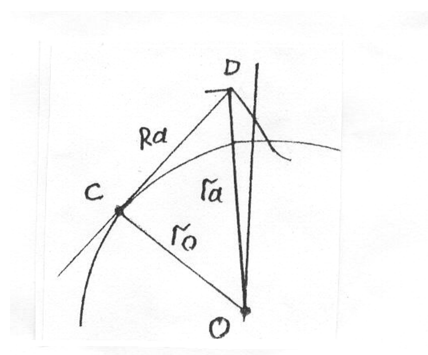

Fig.3.The scheme to define R_a

The analysis of different porosity functions α and β proposed by different authors shows that the selection of the new porosity functions in each particular case of the PM is difficult and inexpedient. Therefore, irrespective of materials specification at first the well known and the most simple for using porosity functions are selected. Then the calculation of porosity particularities of the real sintered materials is realized by introducing into the plasticity condition (10) the constant for the material real porosity parameter n_1 and parameter n_2 for bringing the mechanical properties of porous material to the properties of the nonporous. In this case the plasticity condition will be as follows [4]

$$\sigma_i^2 + 9\alpha^{n_1}\sigma_0^2 = \beta^{2n_2+1}\sigma_y^2. \tag{11}$$

Functions α and β differently influence the stress-strain state of material, the function α takes into account the influence of hydrostatic pressure and together with function β allows to obtain the real mechanical characteristics of the material. In [6] porosity functions are selected

$$\alpha = v, \quad \beta = 1\text{-}v. \tag{12}$$

For sintered steel containing Ni(4%), Cu(1,5%) and M_0(0,5%) in [6] the following average values of porosity parameters n_1 =1,1 and n_2 =1,4 are defined. Note, that $n_{1,}$ n_2 parameter values and functions of porosity (12) are used.

The conception of design stress σ_d [1] permits to write the plasticity condition (11) for contact deformation of PM. Maximum dimensionless value of design stress $\overline{\sigma}_d^{\max}$ for the dangerous point is expressed by the known value $\overline{\sigma}_i$, $\overline{\sigma}_0$ [1] and is determined by the following formula:

$$\overline{\sigma}_d^{\max} = (\overline{\sigma}_i^2 + 9\alpha^m \overline{\sigma}_0^2)^{0.5} \big/ \beta^{n+0.5}, \tag{13}$$

where $\overline{\sigma}_d^{\max} = \sigma_d^{\max} \big/ p_0^{\max}$, $\overline{\sigma}_i = \sigma_i \big/ p_0^{\max}$ and $\overline{\sigma}_0 = \sigma_0 \big/ p_0^{\max}$.

In that case plasticity condition is $\overline{\sigma}_d^{\max} = \sigma_y \big/ p_0^{\max}$.

This formula allows to define the maximum pressure on the area of contact $p_0^{\max}$, as values of $\overline{\sigma}_d^{\max}$, σ_y are known:

$$p_0^{\max} = \sigma_y \big/ \overline{\sigma}_d^{\max}. \tag{14}$$

If we have yield stress of material σ_y and maximum dimensionless value of design stress $\overline{\sigma}_d^{\max}$ from formula (14), we may obtain corresponding maximum value of contact pressure $p_0^{\max}$. If $p_0^{\max}$ is known from formulae (1) and (2) we may get in case of contact deformation the following formulae for defining corresponding load at length unit $q^{\max}$ and half of width $b^{\max}$:

$$q^{\max}=2{,}87(p_0^{\max})^2 \; K_R/K_E, \tag{15}$$

$$b^{\max}=1.82p_0^{\max} \; K_R/K_E. \tag{16}$$

The numerical results and discussion of the contact deformation problem of sintered PM gears are presented. In Table 1 the calculated from formulae (6) and (8) parameters of gears are given.

Table I

Parameters of die and PM gears

Parameters of gear	Die gear (1)	PM gear (2)
Module – m	3.5	3,5
Pressure angle	20^o	20^o
Number of teeth – z	35	21
Pitch (initial) radius – r = mz/2, mm	61.25	36,75
Radius of top circle – r_a= mz/2+m, mm	64.75	40,25
Radius of foundation circle r_0 = (mz/2)cos20^o , mm	57.575	34,545
Basic radius of curvature surface of tooth R_0=(mz/2)sin20^o , mm	20.94	12,57
Radius of curvature surface of tooth top $R_a=(r_a^2-r_0^2)^{0.5}$, mm	29.62	20,66
Porosity of material - v_0	0	0,1
Young's modulus - E , MPa	$2 \cdot 10^5$	$1.48 \cdot 10^5$

If $E_1 = 2 \cdot 10^5$ MPa [3], $E_0 = 2.1 \cdot 10^5$ MPa [2] and v_0=0.1, then $E_2 = 1.48 \cdot 10^5$ MPa and $K_E = 0.85 \cdot 10^5$ MPa. If $E_1 = 2 \cdot 10^5$ MPa, $E_0 = 2.1 \cdot 10^5$ MPa and v_0 = 0, then $K_E = 1 \cdot 10^5$ MPa. If initial porosity of material v = 0.1 and current porosity in case of densification process of PM gear are changed within $0 \le v \le 0.1$ the parameter of K_E is changed within: $0.85 \cdot 10^5 \le K_E \le \le 1 \cdot 10^5$ MPa.

In Tab.2 in case of the value of load unit q=0.01 kn/mm and initial porosity of PM gear v_0=0.1 ($K_E = 0.85 \cdot 10^5$MPa) the calculated from formulae (4), (9), (2) and (1) values of geometrical parameters of tooth contacts K_R, half of width of the contact area b and contact pressure p_0 are given. In case of number 4 (Fig.1) the four different values of R_1 are examined. It is established that if the top radius of die gear R_1 is considerably smaller than the root radius of PM gear R_2(R_1= 0.25R_2 and 0.5R_2), the values of contact pressure p_0 are essentially greater.

Table 2

Parameters of gear contact, when q=0.01 kn/mm, v_0=0.1

Numbers of contacts	Radiuses of contact (Fig.1), mm		K_R mm	b mm	p_o MPa
1	R_{01}=20.94	R_{02}=2.57	7.85	0.0327	195
2	$R_{c1}=\infty$	R_{a2}= 20.66	20.66	0.053	120
3	R_{a1}=29.62	$R_{c2} = \infty$	29.62	0.063	100
4	R_1=0.5	R_2 = 2.0	0.67	0.0095	666
	R_1 =1.0		2.0	0.016	385
	R_1=1.5		6.0	0.029	222
	R_1=1.75		14.0	0.044	146

The numerical examples for defining values of contact pressure p_0 and sizes of the contact area in elastic deformation of gears under given constant external forces are shown. The values of p_0 at all portions of the strip the tooth surface is different. It is established that selecting the values R_1/R_2 the interval of p_0 can be reduced.

In Table 3 in contact number 1 (Fig. 1) for different initial porosity of PM [1] the calculated maximum values of dimensionless design stress $\bar{\sigma}_d^{max}$, corresponding contact pressure P_0^{max} (14), load at length unit q^{max} (15) and half of width b^{max} (16), when K_R =7.85 mm and σ_y =630MPa[6] are given.

Table 3

Parameters of gear contact in case of contact number 1(Fig. 1)

N	Type of Material	Place of dangerous point[1]	$\bar{\sigma}_d^{max}$ [1]	K_E, MPa	P_0^{max}, MPa	q^{max}, n/mm	b^{max}, mm
1	Usually v_{01}=0, μ=0.3	in material	0.56	$1 \cdot 10^5$	1125	277.0	0.156
2	PM v_{02}=0.1, μ=0.27	on the surface of contact	1.04	$0.85 \cdot 10^5$	608	97.8	0.102
3	PM v_{03}=0.2, μ=0.24	on the surface of contact	1.76	$0.66 \cdot 10^5$	358	43.5	0.077

Table 3 shows that for usual material the first plastic deformation appears in the material and for porous material it appears on the surface of the material, where contact pressure acts. High hydrostatic pressure influences the porous material and therefore in the plasticity condition the value σ_0 and functions of porosity α and β are present.

The comparison of calculated data shows that for high porosity material the dimensionless value of maximum design stress calculated by formula (13) is greater than the low porosity and pore-free material. Consequently, for high porosity materials the values p_0^{max}, q^{max} and b^{max} in accordance with (14)-(16) will be low. This means that for porous material the first plastic deformation appears under essentially little external force : $q_1^{max}/q_2^{max} = 2.83$, $q_1^{max}/q_3^{max} = =6.37$.

For usual material the high hydrostatic pressures which arise in this technological process, do not play any role, at the same time for porous material the high hydrostatic pressure has an important influence on the technological process. In this case the values of maximum contact pressure considerably decrease.

Summary. The mechanical K_E and geometrical K_R parameters of die and PM tooth contact are determined. It is shown that in case of porosity the material K_E decreases. To define the maximum value of p_0^{max}, the maximum load at length unit q^{max} and corresponding half of width b^{max} of contact area, when in the PM gear tooth the first plastic deformations arise, the plasticity condition of real porous materials is used. It is shown that taking into account the value of high hydrostatic pressure in porous material calculation, results of the values of p_0^{max}, q^{max} and b^{max} considerably decrease.

The article has been done within the frame work of the DAAD program.

REFERENCES

1. **Petrosyan G., Beiss P.** The investigation of process contact deformation of sintered PM cylinders // Reports of the NAS RA and SEUA. Series of TS.- 2005. - V. 58, N 2. - P. 203-208.
2. **Beiss P., Sander C.** Elastic properties of sintered iron and steel// 1998 PM World Congress.Microstructure.- Shrewsbury, EPMA.- 1998.- V. 2.- P. 552-561.
3. **Roshan Kumar D., Andrew Denise V., Krishna Kumar R., Ashok S.** Simulation of surface densification of PM gears// Society of Automotive Engineers. Inc.- USA.-1990.- P. 55-59.
4. **Petrossian G. L.** Plastic deformation of PM.- Moscow, Metallurgy.- 1988.- 153. (in Russian).
5. **Green R. J.** A plasticity theory for porous solids.- Int. J. Mech. Sci.- 1972.- V. 14.- P. 215.
6. **Petrosyan G., Beiss P., Hambardzumyan A., Badalyan H.** Peculiarities of deformation diagram construction of porous sintered samples from steel powder// Materials of reports at annual scientific conference of SEUA. – Yerevan, Armenia.- 2001.- V.1.- P. 209-210 (in Russian).

SEUA. The material is received on June 15.2004.

Գ.Լ. ՊԵՏՐՈՍՅԱՆ, Պ. ԲԱՅՍ

ԵՌԱԿԱԼՎԱԾ ՓՈՇԵՆՅՈՒԹԵ ԱՏԱՄՆԱՆԻՎՆԵՐԻ ԿՈՆՏԱԿՏԱՅԻՆ ԴԵՖՈՐՄԱՑՄԱՆ ԳՈՐԾԸՆԹԱՑԻ ԱՌԱՆՁՆԱՀԱՏԿՈՒԹՅՈՒՆՆԵՐԸ

Դիտարկվում են փոշենյութե ատամնանվի` մամլամայրային ատամնանվի հետ հպման դեպքում առաձգական դեֆորմացիաների և առաջին պլաստիկ դեֆորմացիաների առաջացման գործընթացների հետազոտման խնդիրներիը: Որոշվել են այդ ատամնանիվների ատամների հպման մեխանիկական և երկրաչափական բնութագրերը: Փոշենյութե ատամնանվի ատամում առաջին պլաստիկ դեֆորմացման առաջացման դեպքում հպման առավելագույն ճնշման և այլ պարամետրերի որոշման համար օգտագործվում է իրական ծակոտկեն նյութերի պլաստիկության պայմանը: Բերված է թվային օրինակ:

Г. Л. ПЕТРОСЯН, П. БАЙС

ОСОБЕННОСТИ КОНТАКТНОГО ДЕФОРМИРОВАНИЯ ЗУБЧАТЫХ КОЛЕС ИЗ СПЕЧЕННОГО ПОРОШКОВОГО МАТЕРИАЛА

Рассматриваются вопросы исследования задач упругого деформирования и появления первой пластической деформации в процессе контакта порошково-металлического и матричного зубчатых колес. Определены механические и геометрические параметры контакта зубьев зубчатых колес. Для определения максимального давления и других параметров контакта зубьев порошково-металлического зубчатого колеса использовано условие пластичности реальных пористых материалов. Приведен численный пример.